SOME RESULTS ON METHOD OF GENERATING FUNCTION

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ABSTRACT

The use of generating functions with many applications in number theory and combinatory is a very important and powerful technique. In this section we will describe this technique in more detail. By the manipulation and interpretation of series and product identities, the generating function is an important contribution in proving division functions. The main function of the generating function is to determine the coefficient of y or to get upper or lower limit of coefficient of y. One can also use the generating function to come on asymmetric value of coefficient of y (if possible). A good example of this application is to "show that, P(i) < $e^{3\sqrt{i}}$ for all by (Cohen, 1978).

Definition (Partition) In Z^+, $\sum_{\lambda=1}^{\lambda=r} a_\lambda = a_1 + a_2 + \cdots + a_r = i$ where a_λ are parts or summands of that partition.

For understanding we can take four which can be written as a 1+1+1+1, 1+1+2, 1+3, 2+2 and 4, hence *P* (4)=five.

Definition . Let $f_i\ (i)$ count partitions into the integers $1,2,\ldots,k$, with repetitions allowed.

Definition. Let $f_O(i)$ count partitions (odd integers) of i such that repetitions allowed.

Definition . Let $f_d(i)$ count partitions distinct parts

Definition . Let $f_t(i)$ count partitions distinct powers of 2, (i.e., into $1,2,4,8,\ldots$).

Our discussion above showed (Generating function)

$f_k(i) = \frac{1}{[(1-y)(1-y2)(1-y3)]}$. From Theorem we find immediately

Keywords- Generating Function, Partition, identities , Division.

Introduction:-

Example Find the partitions of five.

Solution The required partitions of five

5
4+1
3+2
3+1+1
2+2+1
2+1+1+1
1+1+1+1+1.
⇒ P_5=7.

It is very difficult to calculate P_i, but there is a possibility to find generating function for them. now itself find Euler's generating function for $\{P(i)\}_{i=0}^{\infty}$.

In other words, Here's we discuss other way to find out how to calculate this function $\sum P(i)y^i$.

Consider $(1 + y + y^2 + y^3 \cdots)(1 + y^2 + y^4 + y^6 \cdots)(1 + y^3 + y^6 \cdots)$

$$(1 + y^4 + y^8 \cdots)$$

Now We claim that we can find the required result by solving this equation, namely $\sum P(i)y^i$. Now It is important how can this possible.

To explain this , let us take y^3 first .Now take randomly coefficients of y^3,y and 1 from 1st ,2nd and remaining brackets respectively ,we have a contribution of one to the coefficient of y^3 .In the same pattern , if we consider y^3 and 1 from the 3rd and other brackets, we can find second contribution of one to the coefficient of y^3 . Now the main concept is how does this type of collection and integer partitions of integers are connected?

First we consider, In particular by an example if an integer partition of i

i = 25 =6+4+4+3+2+2+2+1+1 can be written as

i = 25 $= 1(2) + 2(3) + 3(1) + 4(2) + 5(0) + 6(1)$.

i.e. 25 $= (1^2 2^3 3^1 4^2 5^0 6^1)$.

Clearly here we also see in the product of equation (3.3.1), we observe that each term forms an infinite G.P. So by result using product of an infinite G.P. we have reqd. product is

$$\frac{1}{1-y}\,\frac{1}{1-y^2}\,\frac{1}{1-y^3}$$

These previous observations lead to Euler's[1] Theorem.

Theorem (Euler's)

$$\mathcal{E}(y) \stackrel{\text{def}}{=} \frac{1}{1-y}\frac{1}{1-y^2}\frac{1}{1-y^3}\ldots = \sum_{i=0}^{\infty} P\,(i) y^i$$

Example Prove ,In Particular, For , $i = 5$

$$f_O(i) = f_d(i)$$

Solution We can explain this concept by taking $i = 5$.

Partition	Odd Partition	Distinct Partition
5	$*$	$*$
$4+1$		$*$
$3+2$		$*$
$3+1+1$	$*$	
$2+2+1$		
$2+1+1+1$		
$1+1+1+1+1$	$*$	

Table Partitions $i = 6$

$\Rightarrow f_O(5) = f_d(5)$

In fact, for every i, $f_O(i) = f_d(i)$

Conclusions -

Theorem - The no. of partitions (odd parts) of any integer i = no. of partitions of i (distinct parts) i.e.. $f_O(i) = f_d(i)$

proof. We can write $1 + y = \frac{1-y^2}{1-y}$,

$$1 + y^2 = \frac{1-y^4}{1-y^2},$$

$$1 + y^3 = \frac{1-y^6}{1-y^3},$$

$$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots ..$$

By multiplying all the terms L.H.S. together obtain the generating function for $f_d(i)$ while multiplying all the terms R.H.S. together we get Required generating function $f_O(i)$ (after cancellations)

i.e $f_d(i) = (1+y)(1+y^2)(1+y^3) \ldots \ldots \ldots$

$$= \frac{1-y^2}{1-y} \cdot \frac{1-y^4}{1-y^2} \cdot \frac{1-y^6}{1-y^3} = \frac{1}{(1-y)(1-y^3)(1-y^5)\ldots\ldots} = f_O(i)$$

$\Rightarrow$ we get required result.

As above example there are so many identities in partition theory which

are proved using generating functions this is one of them.

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