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SOME NEW TYPES OF $\tilde{g}$(1,2)*-CONTINUOUS MAPS IN BITOPOLOGICAL SPACES

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Abstract: In this paper, we introduce the new type of $\tilde{g}$ (1,2)*-continuous maps. As application to $\tilde{g}$ (1,2)*-open sets. We also discuss some properties of $\tilde{g}$ (1,2)*-continuous maps and $\alpha\tilde{g}$ (1,2)*-continuous maps.

Key words and Phrases: $\tilde{g}$ (1,2)*-open, $\tilde{g}$ (1,2)*-continuous, $\alpha\tilde{g}$ (1,2)*-continuous.

1.INTRODUCTION

Levine [7], introduced the generalized closed sets in topology. Kelly [6] introduced the concepts of bitopological spaces. Recently Sheik John [18] introduced and studied another form of generalized continuous maps called ω-continuous maps respectively. Abd El-Monsef and et al. [1], introduced the β-open sets and β-continuous mapping, Andrijevic [2], introduced semi-preopen sets. Arya and et al. [3], introduced the characterization of s-normal spaces. Bhattacharya [4], introduced semi-generalized closed sets in topology. Duszynski [5], introduced a new generalization of closed sets in bitopology. Rajamani and et al. [9], introduced on αgs-closed sets in topological spaces. Ravi and et al. [13], on stronger forms of (1,2)*-quotient mappings in bitopological spaces.

In this paper, we introduce the new type of $\tilde{g}$ (1,2)*-continuous maps. As application to $\tilde{g}$ (1,2)*-open sets. We also discuss some properties of $\tilde{g}$ (1,2)*-continuous maps and $\alpha\tilde{g}$ (1,2)*-continuous maps.

2.PRELIMINARIES

Throughout this paper, (X, τ_1, τ_2) (briefly, X) will denote bitopological space (briefly, BTPS).

Definition 2.1 Let H be a subset of X. Then H is said to be $\tau_{1,2}$-open [11] if $H = P \cup Q$ where $P \in \tau_1$ and $Q \in \tau_2$.

The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed.

Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.

Definition 2.2 [11] Let H be a subset of a bitopological space X. Then

(i) the $\tau_{1,2}$-closure of H, denoted by $\tau_{1,2}$-cl(H), is defined as $\cap \{F : H \subseteq F$ and F is $\tau_{1,2}$-closed$\}$.

(ii) the $\tau_{1,2}$-interior of H, denoted by $\tau_{1,2}$-int(H), is defined as $\cup \{F : F \subseteq H$ and F is $\tau_{1,2}$-open$\}$.

Definition 2.3 A subset H of a BTPS X is called:
(i) (1,2)*-semi-open set [10] if $H \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(H))$;
(ii) (1,2)*-preopen set [10] if $H \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(H))$;
(iii) (1,2)*-α -open set [8] if $H \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(H)))$;
(iv) (1,2)*-β-open set [12] if $H \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(H)))$;
(v) regular (1,2)*-open set [10] if $H = \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(H))$.

The complements of the above-mentioned open sets are called their respective closed sets.

Definition 2.4

A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called

(i) (1,2)*-$\ddot{g}_\alpha$-continuous [17] if $f^{-1}(V)$ is an (1,2)*-$\ddot{g}_\alpha$-closed set of X for every $\sigma_{1,2}$-closed set V of Y.

(ii) (1,2)*-ψ -continuous [15] if $f^{-1}(V)$ is a (1,2)*-ψ -closed set of X for every $\sigma_{1,2}$-closed set V of Y.

(iii) (1,2)*-ĝ-continuous [15] if $f^{-1}(V)$ is a (1,2)*-ĝ-closed set of X for every $\sigma_{1,2}$-closed set V of Y.

(iv) (1,2)*-g-continuous [16] if $f^{-1}(V)$ is a (1,2)*-g-closed set of X for every $\sigma_{1,2}$-closed set V of Y.

(v) (1,2)*-$\ddot{g}$ -continuous [17] if $f^{-1}(V)$ is an (1,2)*-$\ddot{g}$ -closed set of X for every $\sigma_{1,2}$-closed set V of Y.

(vi) (1,2)*-α g-continuous [10] if $f^{-1}(V)$ is an (1,2)*-α g-closed set of X for every $\sigma_{1,2}$-closed set V of Y.

(vii) (1,2)*-gs-continuous [14] if $f^{-1}(V)$ is a (1,2)*-gs-closed set of X for every $\sigma_{1,2}$-closed set V of Y.

(viii) (1,2)*-gsp-continuous [14] if $f^{-1}(V)$ is a (1,2)*-gsp-closed set of X for every $\sigma_{1,2}$-closed set V of Y.

(ix) (1,2)*-sg-continuous [11] if $f^{-1}(V)$ is a (1,2)*-sg-closed set of X for every $\sigma_{1,2}$-closed set V of Y.

(x) (1,2)*-semi-continuous [11] if $f^{-1}(V)$ is a (1,2)*-semi-open set of X for every $\sigma_{1,2}$-open set V of Y.

(xi) (1,2)*-α -continuous [10] if $f^{-1}(V)$ is an (1,2)*-α -closed set of X for every $\sigma_{1,2}$-closed set V of Y.

(xii) (1,2)*-continuous [11] if $f^{-1}(V)$ is a $\tau_{1,2}$-closed set of X for every $\sigma_{1,2}$-closed set V of Y.

Remark 2.5 [16]

(i) Every $\tau_{1,2}$-open is (1,2)*-$\ddot{g}$ -open but not conversely.

(ii) Every (1,2)*-$\ddot{g}$ -open is (1,2)*-$\ddot{g}_\alpha$-open but not conversely.

(iii) Every (1,2)*-$\ddot{g}$ -open is (1,2)*-ψ -open but not conversely.

(iv) Every (1,2)*-$\ddot{g}$ -open is (1,2)*-ĝ-open but not conversely.

(v) Every (1,2)*-$\ddot{g}$ -open is (1,2)*-g-open but not conversely.

(vi) Every $\tilde{g}$ (1,2)*-open is (1,2)*-αg -open but not conversely.

(vii) Every (1,2)*-$\ddot{g}$ -open is (1,2)*-α g-open but not conversely.

(viii) Every (1,2)*- $\ddot{g}$ -open is (1,2)*-gs-open but not conversely.

(ix) Every (1,2)*- $\ddot{g}$ -open is (1,2)*-gsp-open but not conversely.

(x) Every (1,2)*- $\ddot{g}$ -open is (1,2)*-sg-open but not conversely.

3. $\tilde{g}$ (1,2)*-CONTINUOUS MAPS

We introduce the following definitions:

Definition 3.1

A map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called

(i) $\tilde{g}$ (1,2)*-continuous if the inverse image of every $\sigma_{1,2}$-closed set in Y is $\tilde{g}$ (1,2)*-closed set in X.

(ii) $\alpha\tilde{g}$ (1,2)*-continuous if the inverse image of every (1,2)*- α-closed set in Y is $\tilde{g}$ (1,2)*-closed in X.

Example 3.2

(i) Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {c}} and τ_2 = {ϕ, X, {a, c}}. Then the sets in {ϕ, X, {c}, {a, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b}, {a, b}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {b}} and σ_2 = {ϕ, Y, {a, b}}. Then the sets in {ϕ, Y, {b}, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}, {a, c}} are called $\sigma_{1,2}$-closed. We have (1,2)*- $\ddot{G}$ C(X) = {ϕ, {b}, {a, b}, X}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is (1,2)*- $\ddot{g}$ -continuous.

(ii) Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a}}, τ_2 = {ϕ, X, {a}, {b, c}}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {b, c}}. Then the identity function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is $\tilde{g}$ (1,2)*-continuous.

Proposition 3.3

Every (1,2)*-continuous map is (1,2)*- $\ddot{g}$ -continuous but not conversely.

Proof

The proof follows from Remark 2.5 (i).

Example 3.4

The map f in Example 3.4.2 (i) is (1,2)*- $\ddot{g}$ -continuous but not (1,2)*-continuous, since f^{-1}({a}) = {a} is not $\tau_{1,2}$- open in X.

Proposition 3.5

Every (1,2)*- $\ddot{g}$ -continuous map is (1,2)*- $\ddot{g}_\alpha$ -continuous but not conversely.

Proof

The proof follows from Remark 2.5 (ii).

Example 3.6

Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {b}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, X, {b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {a, c}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {b, c}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {b, c}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {a}} are called $\sigma_{1,2}$-closed. We have (1,2)*- $\ddot{G}$ C(X) = {ϕ, {a, c}, X} and (1,2)*- $\ddot{G}_\alpha$ C(X) = {ϕ, {a}, {c}, {a, c}, X}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is (1,2)*- $\ddot{g}_\alpha$ -continuous but not (1,2)*- $\ddot{g}$ -continuous, since f^{-1}({a}) = {a} is not (1,2)*- $\ddot{g}$ -closed in X.

Proposition 3.7

Every (1,2)*- $\ddot{g}$ -continuous map is (1,2)*- ψ -continuous but not conversely.

Proof

The proof follows from Remark 2.5 (iii).

Example 3.8

Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, X, {a}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b, c}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {a, c}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {a, c}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {b}} are called $\sigma_{1,2}$-closed. We have (1,2)*-$\ddot{G}$ C(X) = {ϕ, {b, c}, X} and (1,2)*-ψ C(X) = {ϕ, {b}, {c}, {b, c}, X}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is (1,2)*-ψ -continuous but not (1,2)*-$\ddot{g}$ -continuous, since f^{-1}({b}) = {b} is not (1,2)*-$\ddot{g}$ -closed in X.

Proposition 3.9

Every (1,2)*-$\ddot{g}$ -continuous map is (1,2)*-ĝ-continuous but not conversely.

Proof

The proof follows from Remark 2.5 (iv).

Example 3.10

Let X = Y = {a, b, c, d}, τ_1 = {ϕ, X, {d}, {b, c, d}} and τ_2 = {ϕ, X, {b, c}}. Let σ_1 = {ϕ, Y, {b}} and σ_2 = {ϕ, Y}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is (1,2)*-ĝ-continuous but not (1,2)*-$\ddot{g}$ -continuous, since f^{-1}({a, c, d}) = {a, c, d} is not (1,2)*-$\ddot{g}$ -closed in X.

Proposition 3.11

Every (1,2)*-$\ddot{g}$ -continuous map is (1,2)*-g-continuous but not conversely.

Proof

The proof follows from Remark 2.5 (v).

Example 3.12

Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {b, c}}. Then the sets in {ϕ, X, {a}, {b, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {a}, {b, c}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {c}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {c}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {a, b}} are called $\sigma_{1,2}$-closed. We have (1,2)*-$\ddot{G}$ C(X) = {ϕ, {a}, {b, c}, X} and (1,2)*-G C(X) = P(X). Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is (1,2)*-g-continuous but not (1,2)*-$\ddot{g}$ -continuous, since f^{-1}({a, b}) = {a, b} is not (1,2)*-$\ddot{g}$ -closed in X.

Proposition 3.13

Every $\tilde{g}$ (1,2)*-continuous map is (1,2)*-αg -continuous but not conversely.

Proof

The proof follows from Remark 2.5 (vi).

Example 3.14

Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {b, c}}. Then the sets in {ϕ, X, {a}, {b, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {a}, {b, c}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {b}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {a, c}} are called $\sigma_{1,2}$-closed. We have (1,2)*-$\tilde{G}$ C(X) = {ϕ, {a}, {b}, {c}, {b, c}, X} and (1,2)*-αG C(X) = P(X). Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is (1,2)*-αg -continuous but not $\tilde{g}$ (1,2)*-continuous, since f^{-1}({a, c}) = {a, c} is not $\tilde{g}$ (1,2)*-closed in X.

Proposition 3.15

Every (1,2)*-$\ddot{g}$ -continuous map is (1,2)*-α g-continuous but not conversely.

Proof

The proof follows from Remark 2.5 (vii).

Example 3.16

Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a, b}} and τ_2 = {ϕ, X, {c}}. Then the sets in {ϕ, X, {a, b}, {c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}, {a, b}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {b}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {a, c}} are called $\sigma_{1,2}$-closed. We have (1,2)*-$\ddot{G}$ C(X) = {ϕ, {c}, {a, b}, X} and (1,2)*-αG C(X) = P(X). Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is (1,2)*-α g-continuous but not (1,2)*-$\ddot{g}$ -continuous, since f^{-1}({a, c}) = {a, c} is not (1,2)*-$\ddot{g}$ -closed in X.

Proposition 3.17

Every (1,2)*-$\ddot{g}$ -continuous map is (1,2)*-gs-continuous but not conversely.

Proof

The proof follows from Remark 2.5 (viii).

Example 3.18

Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, X, {a}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b, c}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {a, b}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}} are called $\sigma_{1,2}$-closed. We have (1,2)*-$\ddot{G}$ C(X) = {ϕ, {b, c}, X} and (1,2)*-GS C(X) = {ϕ, {b}, {c}, {a, b}, {a, c}, {b, c}, X}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is (1,2)*-gs-continuous but not (1,2)*-$\ddot{g}$ -continuous, since f^{-1}({c}) = {c} is not (1,2)*-$\ddot{g}$ -closed in X.

Proposition 3.19

Every (1,2)*-$\ddot{g}$ -continuous map is (1,2)*-gsp-continuous but not conversely.

Proof

The proof follows from Remark 2.5 (ix).

Example 3.20

Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {b}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, X, {b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {a, c}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {a, b}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}} are called $\sigma_{1,2}$-closed. We have (1,2)*-$\ddot{G}$ C(X) = {ϕ, {a, c}, X} and (1,2)*-GSP C(X) = {ϕ, {a}, {c}, {a, b}, {a, c}, {b, c}, X}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is (1,2)*-gsp-continuous but not (1,2)*-$\ddot{g}$ -continuous, since f^{-1}({c}) = {c} is not (1,2)*-$\ddot{g}$ -closed in X.

Proposition 3.21

Every (1,2)*-$\ddot{g}$ -continuous map is (1,2)*-sg-continuous but not conversely.

Proof

The proof follows from Remark 2.5 (x).

Example 3.22

Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {b, c}}. Then the sets in {ϕ, X, {a}, {b, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {a}, {b, c}} are called $\tau_{1,2}$-closed. Let σ_1 = {ϕ, Y, {a, b}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}} are called $\sigma_{1,2}$-closed. We have (1,2)*-$\ddot{G}$ C(X) = {ϕ, {b, c}, X} and (1,2)*-SG C(X) = {ϕ, {b}, {c}, {b, c}, X}. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is (1,2)*-sg-continuous but not (1,2)*-$\ddot{g}$ -continuous, since f^{-1}({c}) = {c} is not (1,2)*-$\ddot{g}$ -closed in X.

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