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Bio-Physical Interpretation of a Mathematical Model on two Phase blood flow in Coronary Arteries during Angina

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In this paper, we have studies the two phase blood flow in coronary arteries during angina in human cardiovascular system. We have collected clinical data in case of angina for haemoglobin v/s blood pressure. We have considered the two phase blood flow, one phase which is plasma and other is red blood cells. We have applied the Non-Newtonian Power law model and are shown by the equation of continuity and equation of motion. Using real model and numerical method, hematocrit and blood pressure drop have been derived. The graphical presentation for hematocrit v/s blood pressure drop in much closed to the clinical observation.

Keywords- Two Phase, Coronary, Power law, Hematocrit, Blood pressure drop.

1. **Introduction-** The heart structure has four chambers. Each top cambers are called atrium and the bottom chambers are called ventricles. The coronary circulatory system consists of the blood vessels which are responsible for the blood flow within the heart muscle. Although blood fills the chambers of the heart, even it is so thick that is requires coronary blood vessels to deliver blood deep into the heart. The vessels that supply oxygen rich blood to the heart are known as coronary arteries. The vessels that remove the deoxygenated blood from the heart muscle are known as cardiac veins. The right and left coronary arteries run on the surface of the heart. These arteries, when healthy are capable of auto regulation to maintain coronary blood flow at levels appropriate to the needs of the heart muscle.The right coronary artery supply blood right side of the heart and the left coronary artery supply blood left side of the heart. With the help of mathematical model we will also be capable of developing an inside in to angina related coronary artery blood flow. Coronary artery disease is a blockage or narrowing of the arteries that supply blood to the heart muscle, often due to a buildup of fatty plaque inside the arteries. Angina is a type of coronary artery disease. Angina is chest pain or discomfort that occurs if an area of heart muscle does not get oxygen rich blood. It is well known that the fluid dynamical parameters particularly the high wall shear stress play an important role is angina.

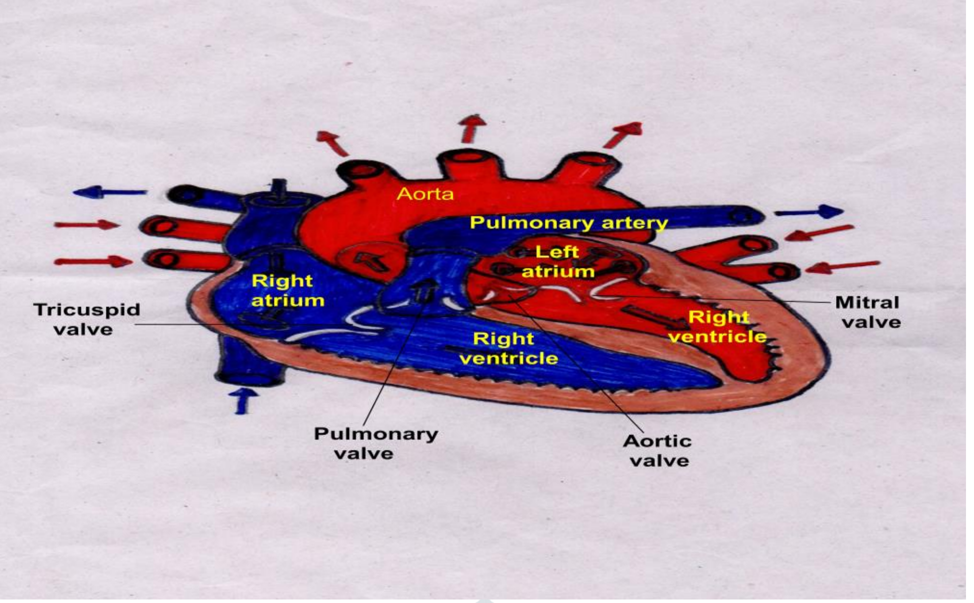


Fig.1 Structure and Function of the heart

2. Real Model

1. **Choice of frame of reference** – We select generalized three dimensional orthogonal curvilinear co-ordinate system, briefly prescribed called as 3-dim Euclidean space. We interpret the quantities related to blood flow in tensorial form which is comparatively realistic, the biophysical laws thus expressed fully hold good in any co-ordinate system [1].

2. **Choice of parameters -** We are given five quantities, namely the three components of velocity, the pressure p and the density ρ.

3. **Two phase description -** Blood is a complex fluid consisting of particulate corpuscles suspended in a non-Newtonian fluid. The particulate solids are red blood cells (RBCs), white blood cells (WBCs) and platelets. 55% of the plasma and 45% of the blood cells in a whole blood and approximately 98% of RBCs in 45% of blood cells and there are a few parts (approximately 2%) of the other cells, which are ignorable in view of volume percentage, so one phase is of the blood plasma and 2nd phase is of blood RBCs [1].

4. **Constitutive Equations -** Generally blood is non-homogeneous mixture of plasma and blood cells. The constitutive equations proposed for whole blood mixture are as follows:

- **Newtonian equation**

$\tau = \eta e^n$ When n = 1 then the nature of fluid is Newtonian.

Where, η is viscosity coefficient. This is found to hold good in the broad blood vessels where there is low hematocrit.

- **Non-Newtonian Power law equation**

$\tau = \eta e^n$ When $n \neq 1$ then the nature of fluid is Non-Newtonian.

This is found to be conformable for strain rate between 5 and 200 sec^{-1}

5. **Boundary Conditions**

- 1. The velocity of blood flow on the axis of blood vessels at r = 0 will be maximum and finite, say v_0 = maximum velocity.
- 2. The velocity of blood flow on the wall of blood vessels at r= R, where, R is the radius of blood vessels, will be zero. This condition is well known as no-slip condition.

3. Mathematical Formulation-

3.1 Equations for two phase blood flow

(a) Equation of continuity

We observed that there is no source or sink in the whole circuit of the human blood circulatory system, the heart behaves like a pumping station, so the conservation of mass can well be applied to hemodynamic.

Fig.2 Blood vessel

Mass of enter the blood = Mass of outer the blood

Therefore law of conservation of mass applied for coronary circulatory system. The flow of blood is affected by presence of blood cells. This effect is directly proportional to the volume occupied by blood cells. Let the volume portion covered by the blood cells in unit volume be X, this X be replaced by $\frac{H}{100}$, where H is the hematocrit the volume percentage of blood cells.If the mass of blood cells to plasma is r then clearly.

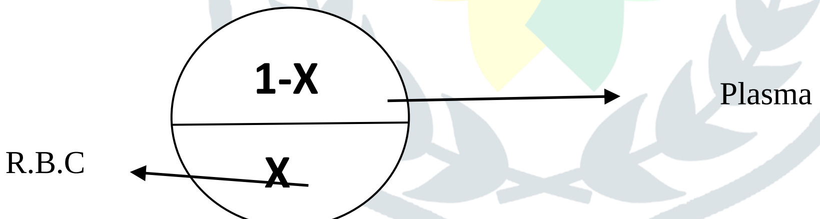


Fig. 3 Unit volume of blood

$$r = \frac{X\,\rho_c}{(1-X)\rho_p} \qquad \text{.............................. (3.1)}$$

Where $\acute{\rho}_c$ and $\acute{\rho}_p$ are densities of blood cells and plasma respectively. Usually this mass ratio is not a constant. But in present context we have treated it constant. The both phase of blood, i.e. blood cells and plasma move with the common velocity. Campbell and Pitcher has presented a model for two phase of blood flow [2]. Hence the principle of conservation mass of coronary vessels holds good.

We get the equation of continuity for two phases as follows:-

$$\frac{\partial (X\,\rho_c)}{\partial t} + (Xp_c v^i)_{,i} = 0 \quad \text{for blood cells phase} \quad \text{.................................. (3.2)}$$

$$\text{and } \frac{\partial (1-X)\rho_p}{\partial t} + (1-X)\rho_p\, v^i), i) = 0 \quad \text{for plasma phase} \quad \text{..............................(3.3)}$$

Where, v is the common velocity of two phase blood cells and plasma. If we define the uniform density of the blood ρ_m as follow

$$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p} \quad \ldots\ldots (3.4)$$

Combined the equation (3.2) and (3.3)

$$\frac{\partial \rho_m}{\partial t} + (\rho_m v^i)_{,j} = 0 \quad \ldots\ldots (3.5)$$

As we know that blood is incompressible fluid, hence ρ_m will be a constant quantity. Thus the equation of continuity (3.5) for blood flow takes the following form:

$$v^i_{,i} = 0$$

i.e. $$\frac{\partial v^i}{\partial x^i} + \frac{v^i \partial \sqrt{g}}{\sqrt{g}\, \partial x^i} = \frac{1}{\sqrt{g}}(\sqrt{g} v^i)_{,i} = 0 \quad \ldots\ldots (3.6)$$

(b) Equation of Motion

According to the conservation of momentum, the total momentum of any fluid is covered in absence of external force. So the law of conservation of momentum will apply to coronary circulatory system.

The hydrodynamic pressure p be supposed to be uniform because the both phases i.e. blood cells and plasma are always in equilibrium state in blood. Taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum in coronary vessels.

Pressure p  Viscous force τ

Fig. 4 Direction of Pressure and Viscous force

$$\frac{dp}{dt} + p - \tau = 0$$

$$\frac{dp}{dt} = - p + \tau$$

Where, $\frac{dp}{dt}$ = rate of change of momentum

P = internal pressure, τ = viscous force

We get equation of motion for the two phases of blood cells as follows:-

$$X\rho_c \frac{\partial v^i}{\partial t} + (X\rho_c v^j) v^i_{,j} = -X p_{,j}\, g^{ij} + X\eta_c (g^{jk} v^i_{k})_{,j} \quad \ldots\ldots (3.7)$$

Similarly, taking the viscosity coefficient of plasma to be .the equation of motion for plasma will be as follows:-

$$(1 - X)\rho_p \frac{\partial v^i}{\partial t} + \{(1 - X)\rho_p v^i v^i_{,j} = -(1 - X) p_{,j} g^{ij} + (1 - X)(g^{ik} v^i_{,k})_{,j} \quad \ldots\ldots (3.8)$$

Now adding equation (3.7) and (3.8) and using relation (3.4), the equation of motion for blood flow with the both phases will be as follows:-

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^j) v^i_{,j} = -p_{,j} g^{ij} + \eta_m (g^{jk} v^i_{k})_{,j} \quad \ldots\ldots (3.9)$$

Where, $\eta_m = X\eta_c + (1 - X)\eta_p$ is the viscosity coefficient of blood as a mixture of two phases.

3.2 Two phase Non - Newtonian Power Law

3.2 (a) Mathematical Modelling

We right the equation of continuity in tensorial form as follows:

$$\frac{1}{\sqrt{g}}(\sqrt{g}v^i)_{,i} = 0 \qquad \text{.. (3.10)}$$

Again, we write down the equation of motion as follows:-

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^j)\, v^i_{,j} = -p_{,j}g^{ij} + \eta_m\,(g^{jk}v^i_k)_{,j} \qquad \text{...................... (3.11)}$$

Where, $\rho_m = X\rho_c + (1 - X)\rho_p$, is the density of blood as mixture of blood cells and plasma. While

$\eta_m = X\eta_c + (1 - X)\eta_p$ is the viscosity of mixture of the blood. Other symbols have their usual meanings.

Now we have to transform the equations (3.10) and (3.11) in cylindrical form. As we know for cylindrical co-ordinates,

$x^1 = r,\ x^2 = \Theta,\ x^3 = z$

The determinant of metric tensor is $g = r^2$

The components of metric tensor are $g_{11} = 1, g_{22} = r^2,\ g_{33} = 1$, rests are zeros. Again the components of the conjugate metric tensor are $g^{11} = 1, g^{22} = \frac{1}{r^2}, g^{33} = 1$ rest are zeroes.

The chritoffel's symbol of 2nd kind are as follow:-

$$\left\{{1 \atop 2\,2}\right\} = -r\ ,\ \left\{{1 \atop 2\,2}\right\} = \left\{{1 \atop 2\,2}\right\} = \frac{1}{r}$$

Remaining others is zero.

The relation between physical component and covariant components of the velocity of the blood flow are as follows:

$$\sqrt{g_{11}}\ v^1 = v_r \Rightarrow v_r = v$$

$$\sqrt{g_{22}}\ v^2 = v_\theta \Rightarrow v_\theta = rv^2$$

$$\sqrt{g_{33}}\ v^1 = v_z \Rightarrow v_z = v^3$$

Again the physical components of $-p_{,j}g^{ij}$ is $-\sqrt{g_{ii}}\ p_{,j}g^{ij}$

Now, we are in a position to write down the governing equation of blood flow in cylindrical form as follows:

The equation of continuity –

$$\frac{1}{r}\frac{\partial(rV_r)}{\partial r} + \frac{1}{r}\frac{\partial(rV_\theta)}{\partial \theta} + \frac{\partial(V_z)}{\partial z} = 0 \qquad \text{... (3.12)}$$

The equation of motion-

Components of equations of motion

r- Component

$$\rho_m\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+\frac{\partial v_r}{\partial z}\right)=\frac{\partial p}{\partial r}+\eta_m\left[\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right\}+\frac{1}{r^2}\frac{\partial^2(v_r)}{\partial\theta^2}-\frac{2}{r^2}\frac{\partial(v_\theta)}{\partial\theta}+\frac{\partial^2(v_z)}{\partial z^2}\right] \quad .....(3.13)$$

θ – Component:

$$\rho_m\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{V_\theta}{r}\frac{\partial v_\theta}{\partial \theta}-\frac{V_\theta v_r}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=\frac{1}{r}\frac{\partial p}{\partial r}+\eta_m\left[\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial(rv_\theta)}{\partial r}\right\}+\frac{1}{r^2}\frac{\partial^2(v_\theta)}{\partial\theta^2}-\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right] \quad (3.14)$$

Z – Component:

$$\rho_m\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{V_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial r}+\eta_m\left[\frac{1}{r}\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial(v_z)}{\partial r}\right\}+\frac{1}{r^2}\frac{\partial^2(v_z)}{\partial\theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right] \quad ....(3.15)$$

The appropriate boundary conditions are is follows-

(1) The velocity of blood flow on the axis of artery i.e. r = 0, will be maximum and finite, say V_0
(2) The velocity of blood flow on the wall of the blood vessels, i.e. at r = R. Where R is radius of the traverse section of the artery, will be zero, this condition is well known as no-slip condition.

Solution

The blood flow in artery is symmetric w.r.t. axis. Hence $v_\theta = 0$ and also v_r , v_z and p do not depend upon θ.

Since only one component of the velocity which is effective, we have $v_r = 0$, $v_\theta = 0$ and $v_z = v$ say

The flow is steady, we have

$$\frac{\partial p}{\partial t}=\frac{\partial v_r}{\partial t}=\frac{\partial v_\theta}{\partial t}=\frac{\partial v_z}{\partial t}=0$$

Keeping in view these facts, we obtain the following result

Equation of continuity reduces to

$$\frac{\partial v_z}{\partial z}=0\Rightarrow v_z=v(r) \quad .... (3.16)$$

The r^{th} component of equation of motion reduces to

$$\rho_m(0)=-\frac{\partial p}{\partial r}+\eta_m(0)\Rightarrow\frac{\partial p}{\partial r}=0\Rightarrow p=p(z) \quad .... (3.17)$$

θ – Component of equation of motion reduces to

$$\rho_m(0)=-(0)+\eta_m(0)\Rightarrow 0=0 \quad .... (3.18)$$

Similarly, the z^{th} component of equation of motion reduces to

$$\rho_m v_z\frac{\partial V_z}{\partial t}=-\frac{\partial p}{\partial z}+\eta_m\left[\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial(V_z)}{\partial r}\right\}+\frac{\partial^2 V_z}{\partial z^2}\right] \quad .... (3.19)$$

With the help of equation (3.16) and (3.17) we get

$$0=-\frac{\partial p}{\partial z}+\eta_m\left[\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial V(r)}{\partial r}\right\}\right] \quad .... (3.20)$$

Whereas, the equation (3.17) expresses the fact that the pressure p depends only on z. We also retain the fact that pressure gradient $-\frac{\partial p}{\partial z}$ in the arteries remote from heart is constant, say p then the equation (3.20) takes the following form

$$0 = P + \eta_m \left[\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial V(r)}{\partial r}\right\}\right] \qquad \ldots\ldots (3.21)$$

Integrating the equation (3.21)

We get,

$$r\frac{dv}{dr} = -\frac{Pr^2}{2\eta_m} + A \qquad \ldots\ldots (3.22)$$

Where A be the constant of integration

Applying the first boundary condition (3.22)

We get A = 0

Hence equation (3.22) reduces to

$$r\frac{dv}{dr} = -\frac{Pr^2}{2\eta_m} \qquad \ldots\ldots (3.23)$$

Again integrating the equation (3.23),

We get,

$$v = -\frac{Pr^2}{4\eta_m} + B \qquad \ldots\ldots (3.24)$$

Again using 2nd boundary condition on the equation (3.24), we can evaluate the integration constant as follows:

$$B = \frac{PR^2}{4\eta_m}$$

Inserting the value of B in the equation (3.24), we obtain the velocity of blood flow in the arteries remote from the heart as follows:

$$v = \frac{P}{4\eta_m}(R^2 - r^2) \qquad \ldots\ldots (3.25)$$

3.2 (b) Mathematical Modelling

Equation of continuity for power law flow will be as follows:

$$\frac{1}{\sqrt{g}(\sqrt{g}v^i)_{,i}} = 0 \qquad \ldots\ldots (3.26)$$

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^i)\, v^i_{,j} = T^{ij}_{,j} \qquad \ldots\ldots (3.27)$$

Where, T^{ij} is taken from constitutive equation of power law flow. Since the blood vessels are cylindrical, the above governing equations have to be transformed into cylindrical co-ordinates. As we know earlier:

Now we have to transform the equations (3.26) and (3.27) in cylindrical form. As we know. For cylindrical

$x^1 = r,\ x^2 = \Theta,\ x^3 = z$

Matrix of metric tensor in cylindrical co-ordinates is as follows:

$$[\,g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

While matrix of conjugate matrix tensor is as follows:

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Whereas the chritoffel's symbol of 2nd kind are as follow:-

$\{{1 \atop 2\,2}\}$ = -r , $\{{1 \atop 2\,2}\}$ = $\{{1 \atop 2\,2}\}$ = $\frac{1}{r}$

Remaining others is zero.

Relation between contra variant physical components of the blood flow will be as follows:

Again the physical components of $p_{,j}g^{ij}$ is $-\sqrt{g_{ii}}\ p_{,j}g^{ij}$

The matrix of the physical components of shearing stress-tensor

$T^{ij} = \eta_m(e^{ij})^n = \eta_m(g^{ik}v^i_{,k} + g^{jk}v^j_{,k})^n$ Will be as follows

$$\sqrt{g_{11}}\ v^1 = v_r \implies v_r = v$$

$$\sqrt{g_{22}}\ v^2 = v_\theta \implies v_\theta = rv^2$$

$$\sqrt{g_{33}}\ v^1 = v_z \implies v_z = v^3$$

$$\begin{bmatrix} 0 & 0 & \eta_m(\frac{dv}{dr})^n \\ 0 & 0 & 0 \\ \eta_m(\frac{dv}{dr})^n & 0 & 0 \end{bmatrix}$$

The covariant derivative of T^{ij}

$$T^{ij}_{,j} = \frac{1}{\sqrt{g}}\frac{\partial(\sqrt{g}T^{ij})}{\partial x^j} + \left\{{i \atop j\ k}\right\}T^{ij}$$

Keeping in view the above fact, the governing tensorial equation can be transformed into cylindrical form which

Are as follows; the equation of continuity

$\frac{\partial v}{\partial z} = 0$ .... (3.28)

Equation of motion

r – Component

$-\frac{\partial p}{\partial r} = 0$ .... (3.29)

θ = component, $0 = 0$ .... (3.30) z component $0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r}[r(\frac{\partial vz}{\partial r})^n]$ ....(3.31)

Here, this fact has been taken in view that the blood flow is axially Symmetric in arteries concerned, i.e.

$v_\Theta = 0$ and $v_r = 0$, ...(3.32)

V_z and p do not depend upon Θ. Also the blood flow steadily, i.e.

$$\frac{\partial p}{\partial t} = \frac{\partial v_r}{\partial t} = \frac{\partial v_\theta}{\partial t} = \frac{\partial v_z}{\partial t} = 0 \quad \text{.................................... (3.33)}$$

Solution-

On integrating equation (3.28) we get

$v_z = v(r)$ because v does not depend upon Θ

The integrating of equation of motion (3.30) yields:

P = p (z) since p does not depend upon Θ

Now, with the help of equation (3.32) and (3.33) the equations of motion (3.31) convert in the following form

$$0 = -\frac{dp}{dz} + \frac{\eta_m}{r}\frac{d}{dr}\left\{r\left(\frac{dv}{dr}\right)^n\right\} \quad \text{.. (3.34)}$$

The pressure-gradient $-\frac{\partial p}{\partial z} = P$ of blood flow in the arteries remote the heart can be supposed to be constant and hence the equation (3.33) takes the following form

$$\frac{d}{dr}\left\{r\left(\frac{dv}{dr}\right)^n\right\} = -\frac{pr}{\eta_m} \quad \text{... (3.35)}$$

On integrating equation (3.35), we get

$$r\left(\frac{dv}{dr}\right)^n = \frac{pr}{2\eta_m} + A \quad \text{.. (3.36)}$$

We know that the velocity of the blood flow on the axis of cylindrical arteries is maximum and constant. So that

We apply the boundary condition at r = 0, $V = V_0$ (constant), on equation (3.36) takes the following form

$$r\left(\frac{dv}{dr}\right)^n = \frac{pr}{2\eta_m} \Rightarrow \frac{-dv}{dr} = [\frac{pr}{2\eta_m}]^{\frac{1}{n}} \quad \text{... (3.37)}$$

Integrating equation (3.37) once again, we get

$$v = -[\frac{P}{2\eta_m}]^{\frac{1}{n}} \frac{r^{\frac{1}{n}+1}}{n+\frac{1}{n}} + B \quad \text{... (3.38)}$$

To determine the arbitrary constant B, we will apply the non-slip condition on the inner wall of the arteries at r = R,

v = 0, where R = radius of vessel, on equation (3.38) so as to get

$B = [\frac{P}{2\eta_m}]^{\frac{1}{n}} \frac{nR^{\frac{1}{n}+1}}{n+1}$

Hence the equation (3.38) takes the following form

$$v = [\frac{P}{2\eta_m}]^{\frac{1}{n}} \frac{n}{n+1} [R^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}] \qquad \text{.....................} (3.39)$$

Which determine the velocity of the blood flow in the artery remote from heart where, P is gradient of blood pressure.

And η_m is the velocity of blood mixture.

3.3 Clinical data consultation

We went to Jabalpur Hospital and Research Centre Jabalpur for clinical data consultancy. The Dr. Deepak Bahlani provided the clinical data of haemoglobin and blood pressure of the angina patients.

4. Result and Discussion -

Table 1: Patient case history (Mr. Diwaker, 62year/male)

S.No	Date	Blood Pressure(mmhg)	Haemoglobin (gm %)
1.	18/05/16	150/90	11.2
2.	20/05/16	150/90	11.3
3.	22/05/16	150/80	11.5
4.	25/05/16	140/80	11.6

Q = 250 ml/m = 0.004166 m^3/s [3]

Length of coronary artery $\Delta z = 0.024$ [6]

Radius of coronary artery R = 0.2 cm. = 0.002 m [6]

$\eta_m = 0.035 \text{pas. sec.}, \eta_p = 0.0015 \text{pas. sec.}$ [5]

Hematocrit (H) = 33.6 gm. = 0.031699 liter

We know that, $\eta_m = \eta_c X + \eta_p(1 - X)$, where X = H/100

$\Rightarrow \eta_c = 105.6831$ pas.sec.

Again using this relation and change in to the hematocrit

$\eta_m = \eta_c X + \eta_p(1 - X) \Rightarrow \eta_m = 1.056846H + 0.0015$

From equation P = -dp/dz , We get $Q = \left[\frac{\Delta p}{2\eta_m \Delta z}\right]^{\frac{1}{n}} \frac{n\pi R^{\frac{1}{n}+3}}{3n+1}$

Pressure drop $\Delta P = \frac{S+D}{2} - S = -3993.96 \text{pas. sec.}$

Put the value of Q, ΔP, ΔZ and R in equation (1)

$$0.004166 = [237735.143]^{\frac{1}{n}}(\frac{n}{3n+1})3.14 \times (0.002)^3 \times (0.002)^{\frac{1}{n}}$$

$$165843.949 = \left(\frac{n}{3n+1}\right) \times (4754.714286)^{\frac{1}{n}}$$

by using trial method, we get the value of n is

n = 62510269

Again use from equation $Q = \left[\frac{\Delta P}{2\eta_m \Delta Z}\right]^{\frac{1}{n}} \frac{n\pi R^{\frac{1}{n}+3}}{3n+1}$

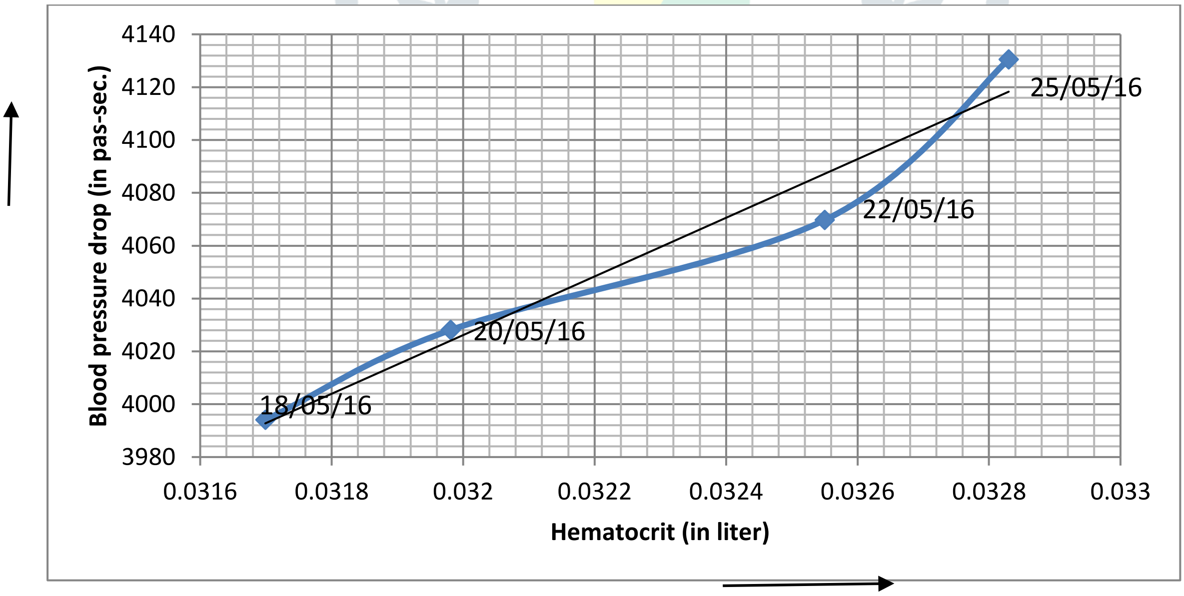
$$\Delta p = 1.056846H + 0.0015) \times (114113.15)$$

$$\Delta p = 120600H + 171.169725$$

Table 2 for Hematocrit v/s Pressure drop

Date	18/05/16	20/05/16	22/05/16	25/05/16
Hematocrit (in liter)	0.031699	0.0319811	0.03255	0.032830
B. P. drop (in pascal)	4130.49133	4028.091220	4096.700575	4130.46858265

Graphical Presentation



Graph

We have taken a clinical data of patients who are suffering from angina heart disease. We get a relationship between blood pressure drop and hematocrit for Non-Newtonian flow and draw a graph between blood pressure drop and hematocrit, this graph shows from 0.031699 to 0.03255 lower convex curve and from 0.03198 to 0.03283 upper concave curve.

Acknowledgement-

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