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# AXIALLY SYMMETRIC STRING COSMOLOGY IN BIANCHI-I SPACE-TIME

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**Abstract :**

In this paper we have investigated string cosmology in axially symmetric Bianchi-I space time using different assumptions. Various physical parameters of the model have been also found and discussed.

**Key Words** : string cosmology, symmetry model universe, massive string.

## INTRODUCTION

Recently many researchers in theory of relativity have focussed their mind towards the study of string cosomologies [3,4,6-8,10, 14, 15,17].

As a matter of fact at the early stage of the universe a phase transition occurs as the temperature lowers below some critical. temperature and this can give rise to various topologically stable defects of which strings are of most important whose world sheets are two dimensional time-like surfaces (Kibble [5]). It has been noted (Kibble [5]) that the existence of a large scalar network of strings in the early universe does not contradict the present-day observations of the universe and further the vacuum strings (Zeldovich [17]) can generate density fluctuations sufficient to explain the galaxy formation. These strings have stress energy and they couple to the gravitational field so that it may be interesting to study the gravitational effects which arise from strings. This has been already done by several authors, (Vilenkin [15], Gott [3], Garfinkle [2], although the general relativistic treatment of strings was pioneered by Letelier [6] and Stachel [12].

In geometrical string (massless) models, infinite number of degrees of freedom are possessed by each string for which the end points move at the speed of light. This problem is resolved by considering the realistic (massive) string model to Takabayashi [13]. The energy-momentum tensor for the massive strings has been first formulated by Letelier [6], who considered the massive string being formed by geometric string with particles attached along its extension. Its application to general relativity first appeared in Letelier [8], while Stachel [12] considered massless strings. So the total energy momentum tensor for a cloud of massive strings can be written as

(1.1) $T_i^j = \rho u_i u^j - \tau x_i x^j$

Where $\rho$ is the rest energy density for a cloud of strings with particles attached to them (p-strings). Thus we have

(1.2) $\rho = \rho_p + \tau$

$\rho_p$ being the particle energy density and $\tau$ being the string's tension density, $u^i$ is the four velocity for the cloud of particles and $x^i$ is the four vector representing the string's direction which essentially is the direction of anisotropy. Thus

(1.3) $u_i u^i = -1 = -x_i x^i \text{ and } u_i x^i = 0$

in (–, +, +, +) signature (i.e., +2 sign.)

Banerjee et. al. [1] have found some cosmological solutions in Bianchi 1 space time following the technique used by Letelier and Stachel with/without magnetic field. Melvin [9] in his solution for dust and declectromagnetic field argued that the presence of magnetic field is not as unrealistic as it appears to be, because for a large part of the history of evolution matter was highly ionized and matter and field were smoothly coupled. Later during cooling as a result of expansion the ions combined to form neutral matter.

In this paper we have studied the string cosmology in axially symmetric Bianchi 1 space time using different assumption. Various physical parameters of the model have been also evaluated.

## 2. THE FIELD EUUATION

We consider an axially symmetric Bianchi 1 model, which is

(2.1) $ds^2 = \exp(2\lambda)dx^2 + \exp(2v)(dy^2 + dz^2) - dt^2$

where $\lambda = \lambda(t)$ and $v = v(t)$

Now, the energy momentum tensor for the string dust with a magnetic field along the direction of the string i.e., the x direction is given by

(2.2) $T_i^j + E_i^j = \rho u_i u^j - \lambda x_i x^j + \frac{1}{4\pi}\left(F_i^{\alpha} F_{\alpha}^j - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} \delta_i^j\right)$

In the above, $T_i^j$ is the stress-energy tensor for a string dust system, $E_i^j$ is that for the magnetic field and $F_{\alpha\beta}$ is the electromagnetic field tensor. The other terms have already been explained in the previous section. In the co-moving co-ordinate system $u^i = \delta_4^i$ and

(2.3) $T_4^4 - = -\rho,\ T_1^1 = -\tau, T_2^2 = T_3^3 = 0 = T_i^j (\text{for } i \neq j)$

Further, since the magnetic field is being assumed in the x-direction $F_{23}$ is the only non-zero component of the electromagnetic field tensor. Maxwell equation $F_{[ij,\alpha]} = 0$ and $\left(F^{ij}(-g)^{1/2}\right) = 0$ , now lead to the result

(2.4) $F_{23} = k$

k being a constant quantity. So the components of stress energy tensor for the electromagnetic field are

(2.5) $$E_4^4 = E_1^1 = -E_2^2 = -E_3^3 = -\frac{k^2}{8\pi}\exp(-4\beta)$$

Now, choosing units such that $8\pi G = 1$, the surviving components of Einstein field equations

(2.6) $$R_i^j - \frac{1}{2}\delta_i^j R = -\left(T_i^j + E_i^j\right)$$

are

(2.7) $$2\dot{\lambda}\dot{\nu} + \dot{\nu}^2 = \rho + \frac{k^2}{8\pi}\exp(-4v)$$

(2.8) $$2\ddot{\nu} + 3\dot{\nu}^2 = \tau + \frac{k^2}{8\pi}\exp(-4v)$$

(2.9) $$\ddot{\lambda} + \dot{\nu}^2 + \ddot{\nu} + \dot{\nu}^2 + \dot{\lambda}\dot{\nu} = -\frac{k^2}{8\pi}\exp(-4\nu)$$

The proper volume $R^3$, expansion scalar $\theta$ and shear scalar $\sigma^2$ are respectively given by

$$R^3 = \exp(\lambda + 2v)$$

(2.10) $$\theta = u^i_{;i} = \dot{\lambda} + 2\dot{\nu} = 3\frac{\dot{R}}{R}$$

(2.11) $$\sigma^2 = \sigma_{ij}\sigma^{ij} = \dot{\lambda}^2 + 2\dot{\nu}^2 - \frac{1}{3}\theta^2$$

where

(2.12) $$\sigma_{ij} = \frac{1}{2}\left[u_{ij} + u_{ji} + u_i u^\alpha u_{j\alpha} + u_j u^\alpha u_{i\alpha}\right] - \frac{1}{3}\theta\left(g_{ij} + u_i u_j\right)$$

Now, one can directly obtain the Ray Chaudhuri's equation (Ray Chaudhuri [11] from the above set of field equations [2.7] to [2.9] and using (2.11) and (2.13) as

$$(2.13) \quad \dot{\theta} = \frac{1}{3}\theta^2 - 2\sigma^2 = \frac{1}{2}\rho_p - \frac{k^2}{8\pi}\exp(-4\nu)$$

where

$$(2.14) \quad R_{ij}u^i u^j = -\frac{\rho_p}{2} - \frac{k^2}{8\pi}\exp(-4v)$$

Now in view of all the three (strong, weak and dominant) energy conditions (Wald [16]), one finds $\rho \geq 0$ and $\rho_p > 0$, together with the fact that the sign of $\tau$ is unrestricted. It may take values positive, negative or zero as well. This implies in view of [2.13] that even the existence of the strings is unable to halt the collapse. From the above energy conditions we find that $\tau$ might even take the negative value and therefore. Einstein's equation (2.6) with $\tau < 0$, is the equation for an anisotropic fluid with pressure different from zero along the direction of x.

## 3. SOLUTION OF THE FIELD EQUATIONS :

We have three equations (2.7) – (2.9) in four unknowns $\lambda$, v, $\rho$ and thus the system is indeterminate. To make the system determinate, we require one more relation. For this we use different assumptions in the following cases.

**Case 1** : Here we choose $\lambda = 0$

In this case equation (2.9) reduces to

$$(3.1) \quad \ddot{v} + \dot{v}^2 = -\frac{k^2}{8\pi}e^{-4v}$$

Equation (3.1) can be written as an integral equation

(3.2) $\int d\left(\dot{v}^2 e^{2v}\right) = -\frac{k^2}{4\pi} e^{-2v} dv + k_1$

where $k_1$ is constant of integration. So, we get

(3.3) $\dot{v}^2 = k_1 e^{-2v} + \frac{k^2}{8\pi} e^{-4v}$

which can again be written as an integral form as

(3.4) $\int \frac{e^{2v} dv}{\left[k_1 e^{2v} + \frac{k^2}{8\pi}\right]^{1/2}} = \pm(t - t_0)$

where $t_O$ is another integration constant. Integrating (3.4) we get

(3.5) $e^{2v} = k_1 (t - t_0)^2 - \frac{k^2}{8\pi k_1}$

$\rho$ and $\tau$ can now be found from (2.7) and (2.8) respectively as

(3.6) $\rho = \frac{k_1}{\left[k_1 (t - t_0)^2 - \frac{k^2}{8\pi k_1}\right]}$

(3.7) $\tau = \frac{\left[k_1^2 (t - t_0)^2 - 3, \frac{k^2}{8\pi}\right]}{\left[k_1 (t - t_0)^2 - \frac{k^2}{8\pi k_1}\right]^2}$

and

(3.8) $\rho_p = \rho - \tau = \frac{\left(\frac{k^2}{4\pi}\right)}{\left[k_1 (t - t_O)^2 - \frac{k^2}{8\pi k_1}\right]^2}$

Finally, the proper volume $R^3$, expansion scalar $\theta$ and shear scalar $\sigma$ can be obtained

from (2.10) to (2.13) respectively as

(3.9) $R^3 = k_1 (t - t_0)^2 - \frac{k^2}{8\pi k_1}$

(3.10) $\theta = \frac{2k_1 (t - t_O)}{R^3}$

(3.11) $\sigma^2 = \frac{1}{6}\left[\frac{2k_1(t-t_O)}{R^3}\right]^2$

From the above solutions we observe that at the initial epoch.

$$(t-t_O)^2 = \frac{k^2}{8\pi k_1^2};$$

the string model starts with an initial singularity $R^3 \to 0$, while $\rho, \rho_p, \tau, \theta, \sigma^2$ etc. diverge.

This is a line singularity, since exp $(2\lambda) \to 1$ exp (2v) $\to$ 0. At a later instant when

$(t-t_0)^2 = \frac{3k^2}{8\pi k_1}$ we have $\tau_0 = 0$ and $\rho = \rho_p$. So at this epoch string vanish and we are left with a dust filled universe with a magnetic field. At this stage

$$3.12 \quad \begin{cases} \rho = \frac{4\pi k_1^2}{k^2} \\ R^3 = \frac{k^2}{4\pi k_1} \\ \theta = \frac{2}{k}(6\pi)^{1/2} \\ \text{and} \\ \sigma^2 = \frac{2\pi k_1^2}{3k^2} \end{cases}$$

i.e., all these parameters are of finite magnitude. In this solution matter is directly related with the magnetic field as is noted in (3.8). When the magnetic field is absent, the matter is also absent and the solution reduces to that of pure geometric string distribution.

**Case II** : Here we use the condition

(3.13) $\lambda = yv$

where y is a constant. Using (3.13) equation (2.9) is reduces to

(3.14) $(y+1)\ddot{v} + (y^2+y+1)\dot{v}^2 = -\frac{k^2}{8\pi}e^{-4v}$

This equation can be written as an integral equation

(3.15) $\int d\left[\dot{v}^2 \exp\left(2\left(\frac{y^2+y+1}{y+1}\right)v\right)\right]$

$$= -\frac{k^2}{4\pi(y+1)}\int \exp\left(2\left(\frac{y^2-y-1}{y+1}\right)\nu\right)d\nu + B$$

where B is constant of integration. So we have

(3.16) $\dot{\nu}^2 = B\exp\left[-2\left(\frac{y^2+y+1}{y+1}\right)\nu\right] - \frac{k^2}{8\pi\left(y^2-y-1\right)}\exp(-4\nu)$

Which can again be written as an integral form as

(3.17) $\int \frac{e^{2\nu}d\nu}{\left[B\exp\left(-2\left(\frac{y^2-y-1}{y+1}\right)\nu\right) - \frac{k^2}{8\pi\left(y^2-y-1\right)}\right]^{1/2}} \pm (t - t_0)$

where $t_o$ is another constant of integration. To solve (3.17) we choose y such that

(3.18) $y^2 - y - 1 = 2(y+1)$

$$\Rightarrow y^2 - 3y - 3 = 0$$

which is quadratic equation in y.

Its solution is

$$y = \frac{3+\sqrt{9+12}}{2}$$

(3.19) $y = \frac{3+\sqrt{21}}{2}$

With this value of y, equation (3.17) can at once be integrated to yield

(3.20) $r^{2\nu} = \left[\frac{8\pi B}{k^2}\left(5\pm\sqrt{21}\right) - \frac{k^2\left(t-t_0\right)^2}{2\pi\left(5\pm\sqrt{21}\right)}\right]^{\frac{1}{2}}$

From (3.20) it is clear that arbitrary constant B must be +ve in this case. So we replace B by $D^2$ in the following. The other parameters can be found as before.

$$(3.21)\quad \rho=\frac{\frac{\left(9+2\sqrt{21}\right)}{\left(5+\sqrt{21}\right)^2}\frac{k^2}{16\pi^2}\left(t-t_0\right)^2-\left(5+\sqrt{21}\right)D^2}{\left[\frac{8\pi D^2}{k^2}\left(5+\sqrt{21}\right)-\frac{k^2\left(t-t_0\right)^2}{2\pi\left(5+\sqrt{21}\right)}\right]^2}$$

From energy conditions $\rho > 0$ which demands positive sign before $\sqrt{21}$ in equation (3.20).

With this choice other parameters are found explicitly as follows.

$$(3.22)\quad \tau=\frac{\left(9+\sqrt{21}\right)D^2-\frac{\left(4+\sqrt{21}\right)}{\left(5+\sqrt{21}\right)^2}\frac{k^2}{16\pi^2}\left(t-t_0\right)^2}{\left[\frac{8\pi D^2}{k^2}.\left(5+\sqrt{21}\right)-\frac{k^2\left(t-t_0\right)}{2\pi\left(5+\sqrt{21}\right)}\right]^2}$$

$$(3.23)\quad \rho_p=\frac{4D^2+\frac{k^2\left(t-t_0\right)^2}{16\pi^2\left(5+\sqrt{21}\right)}}{\left[\frac{8\pi D^2\left(5+\sqrt{21}\right)}{k^2}-\frac{k^2\left(t-t_0\right)^2}{2\pi\left(5+\sqrt{21}\right)}\right]^2}$$

$$(3.24)\quad R^3=\left[\frac{8\pi D^2\left(5+\sqrt{21}\right)}{k^2}-\frac{k^2(t-t_0)^2}{2\pi\left(5+\sqrt{21}\right)}\right]\times\frac{\left(7+\sqrt{21}\right)}{8}$$

$$(3.25)\quad \theta=-\left(\frac{7+\sqrt{21}}{5+\sqrt{21}}\right)\frac{k^2}{8\pi}\left[\frac{\left(t-t_0\right)}{\frac{8\pi D^2\left(5+\sqrt{21}\right)}{k^2}-\frac{k^2\left(t-t_0\right)^2}{2\pi\left(5+\sqrt{21}\right)}}\right]$$

$$(3.26)\quad \sigma^2 = \frac{7+\sqrt{21}}{\left(5+\sqrt{21}\right)^2}\left(\frac{k^2}{48\pi^2}\right)\left[\frac{t-t_0}{\frac{8\pi D^2}{k^2}(5+\sqrt{21}-\frac{k^2\left(t-t_0\right)^2}{2\pi\left(5+\sqrt{21}\right)}}\right]$$

Now it is clear from (3.20) that

$$(3.27)\ T^2 < \frac{16\pi^2 D^2}{k^4}\left(5+\sqrt{21}\right)^2$$

where $T = t_0 - t$. When $t < t_0$ we have $T > 0$ and clearly from equation (3.25) $\theta > 0$

Hence we have the expanding model.

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