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CONNECTIVITY AND HAMILTONIAN PATH

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ABSTRACT

The connectivity of a graph is an important measurement for the fault-tolerance of the network. To provide more accurate measures for the fault –tolerance of networks than the connectivity some generalizations connectivity have been introduced. The study of n-dimensional hyper cubes can be embedded with (n-1-f). Mutually independent Hamiltonian cycles when f≤n-2 faulty edges. The connectivity of a network the minimum number of nodes whose removal will disconnect the network is directly related to its reliability and fault tolerability hence an important indicator of the networks robustness. We extend the notion of connectivity by introducing two new kinds of connectivity called structure connectivity and substructure connectivity.

KEYWORDS

Structure connectivity, cycles, hypercubes, interconnection network, fault tolareance, hamiltational cycle, substructure connectivity, hamiltation, pancake network, star network

INTRODUCTION

In broad casting, a data set is copied from one node to all other nodes, or a subset thereof broadcasting from a single source to all other nodes, one-to-all broadcasting, and concurrent broadcasting is used in a variety of linear algebra algorithms matrix -vector multiplication. LU- factorization and Householder transformations

Graphs of minimum height have minimum propagation time which is the overriding concern for small data volumes or a high overhead for each communication action. For large data volumes it is important to use the bandwidth of a Boolean cube effectively, in particular, if each processor is able to communication on all its parts concurrently.

We purpose three new spanning single graphs for Boolean n-cubes of $N=2^n$ nodes. For scheduling disciplines for the four different communications as follows:

1) One-to –all broadcasting

2) Personalized communication

3) all-to-all broadcasting; and

4) Personalized communication.

A factor of four of the best known lower bound, also for concurrent communication on all portsthe ability of a system to continue operations correctly in the presence of failures in one or many of itscomponents is known as fault tolerance.

The connectively is one of the essential parameters to evaluate the fault tolerance of a network. In the last decades this problem was extensively for those cayley graphs for which the existence of Hamiltonian cycles, further properties related to this problem, such as edge Hamilton city Hamilton- connectivity and Hamilton lace ability are investigated. In particular mutually independent Hamiltonian cycles of pancake graphs p_n and star graphs s_n are studied.

In many parallel computer systems, processors are connected on the basis of interconnection networks such as hypercubes, star graph, meshes, bubble-sort networks etc… For the sake of simplicity,a network topology is usually represented by a graph, in which edges correspond to connections or communication.

The faults in a network may take. Various forms such as hardware failures, software errors, or even missing of transmitted packets.

In this paper, faulty edges, one kind of hardware failures, are defined. More precisely, a set F of faulty edges in a graph G contains those edges which with be removed from G. The basic topology of an interconnection network can be modeled, and the edges in E represent the communication betweenprocessors.

We use the terms graph and network interchangeably. The connectivity is an important indicator of the reliability and fault tolerability of a network.

STRUCTURE CONNECTIVITY OF HYPERCUBES

THEOREM: 2.1

Let $n \geq 4$ be an integer .Then for each $1 \leq m \leq (n-2)$, $\kappa(Q_n; Q_m) = n - m$and $(Q_{,2m}) \leq n - m$.

PROOF:

Let Q_n=$Q_m Q_{n-m}$.as Q_{n-m} is $(n-m)$ regular and $(n-m)$connected, every vertex in (Q_{n-m}) is adjacent to exactly $n-m$ number of vertices.

Let $t \in (Q_{n-m})$ be adjacent to $t_{1,2.....}t_{(n-m)} \in V(Q_{n-m})$.

Hence,

Induced subgraph $(Q_n$,t) of Q_n is adjacent to exactly$(n - m)$ subcubes namely $(Q_m,t_1),(Q_m,t_2)\ldots(Q_m,t_{(n-m)})$.

Clearly,

Removal of $\bigcup_{i=1}^{n-}(Q_m,t_i)$ disconnects Q_n .

Thus,

$(Q_n;Q_m) \leq n-m$.if κ$(Q_n;Q_m) < n-m$,

then without loss of generality one can assume that removal of $\bigcup_{i=2}^{n-m}(Q_m,t_i)$ disconnects Q_n.Hence

The removal of $\bigcup_{i=2}^{n-}$ will disconnected $Q_{n-m.}$, a contradiction to Q_{n-m} is $(n-m)$ connected .hence κ $(Q_n;Q_m) = n-m$.Since

$Q_{n-m.}$ is Hamiltonian, it contains a spanning cycle of length 2^{n-m} .

Removal of isomorphic Hamiltonian cycles of all sub cubes$(Q_m,_1)$,(Q_m,t_2)….$(Q_m,t_{(n-m)})$.Disconnect Q_n.

Hence ,$(Q_{n\,,}C_{2m}) \leq n-m$

THEOREM: 2.2

Let $n \geq 5$ be an integer .then for any integer m with $2 \leq m \leq n-2$ and any for even integer k with $2^m < k < 2^{m+1}$, κ$(Q_n;C_k) \leq n-m$.

PROOF:

Let Q_n=$Q_m Q_{n-m.}(2 \leq m \leq n-2)$.

Let $t \in (Q_{n-m})$ be adjacent to $n-m$ number of vertices say$t_{1,2.....}t_{(n-m)}$.

It is well known that hypercube is a bipartite graph so, t_i is not adjacent t_j for all $i \neq (1 \leq i,j \leq n-m)$.also,every pair of adjacent edges lies in exactly one 4-cycle.hence we name vertices say $u_{1,2,\,\dots\,(n-m)}$ from $(Q_{n-m}\ )$

Such that

t_i is adjacent to say u_i for all $1 \leq i \leq n-m$

The construction of cycle of length$2^m + l$ for every integer l $(2 \leq l \leq 2^m - 2)$ produceed asfollows.

We start by choosing an arbitrary Hamiltonian cycles say C^i of $(Q_m,)$ for every $i(1 \leq i \leq n-m)$.Then

We choose any edge say e^i on C^i and its corresponding edge f^i in (Q_m, u_i) is an edges-bipancyclic graph there exist cyclesC_i^i for every integer l $(2 \leq l \leq 2^m - 2)$ Containing.

THEOREM: 2.3

$\mathcal{JH}\surd(Q_{n)=}n-1 if\ n \in \{1,2,3\}$ And f$\mathcal{H}\surd(Q_{N)} = n$ if n≥ 4.

PROOF

Let F be a set of faulty edges ofQ_n. Suppose that $Q_n$is partitioned along dimension n into $\{Q^0_n, Q^1_n\}$ and E_c is the set of crossing edges between Q^0_n andQ^1_n. Then we define$F_0 = F \cap (Q^0_n)$, =F∩ E_c . Moreover, we get $\delta = n - 1 - |F|$ in the reminder of this paper.

MUTUALLY INDEPENDENT HAMILTONIAN CYCLESDEFINITION: 5.1

Let V be a finite set and E a subset of $\{(u, v)|(u, v) is\ an\ unordered\ paitr\ of\ V\}$.

Then

G= (V, E) is a graph with vertex set V and edges set E.

The order of G, that is, the cardinality of the set V, is denoted by n(G).

For a subset S of V the graph G[S] induced by S is a graph with vertex set

$V(G[S] = S\ and\ edges\ set\ E(G[S] = \{(x, y)|(x, y) \in E(G) and\ x, y \in S\}$.

Two vertices u and v are adjacent if (u, v) is an edge of G. For a vertex u the set $N_G(u) = \{u|(u, v) \in\in E\}$ is called the set of neighbors of u

. The degree$de(u)$ of a vertex u in G, is the cardinality of the set $N_G(u)$.

DEFINITION: 5.2

The minimum degree of G (G) , is $\min\{deg_G(x)|x \in V\}$.

A graph G is K-regular if $de(u) = K$ for every vertex u in G. the connectivity of G is theminimum number of vertices whose removal leaves the remaining graph disconnected or trivial.

THEROEM: 5.1

Let$\{a_1 a_2 \dots \dots . a_r\}$ be a subset of $\langle n\rangle$ for some positive integer $r \in \langle n\rangle$ with$n \geq 5$Assume that **u** and **v** are two distinct vertices of P_n with $\boldsymbol{u} \in P^{\{ar\}}_{\boldsymbol{n}}$ **and** $\boldsymbol{v} \in P^{\{ar\}}_n$ .Then , there is a Hamiltonian path $\langle \boldsymbol{u} = \boldsymbol{x_1}, \boldsymbol{h_1}, \boldsymbol{y_1} \dots \dots \boldsymbol{x_r}, \boldsymbol{h_r}, \boldsymbol{y_r} = \boldsymbol{v}\rangle of \bigcup^r_{i=1} {}^{\{ar\}}_n$ joining **u and v** such that $\boldsymbol{x_1} = u, \boldsymbol{y_r} = \boldsymbol{v}$ and Hamiltonian path of ${}^{\{ar\}}_n$ joining $\boldsymbol{x_i}$ $\boldsymbol{and\ y_i}$for every $i, 1 \leq i \leq r$.

PROOF:

We set x_1 as **u** and y_r as **v**.

We know that,

${}^{\{ar\}}_n$ is isomorphic to P_{n-1} for every $i \in \langle r\rangle$.

This statement holds for $r = 1$.Thus,

Assume that, $r \geq 2.|E_n^{a_i a_{i+1}}|$ for every $i \in \langle r-1 \rangle$ with $\boldsymbol{y}_1 \neq \boldsymbol{x}_1$ and $\boldsymbol{x}_r \neq \boldsymbol{y}_r$.

There is a Hamiltonian path H_i $o^{\{ai\}}$ joining $\boldsymbol{x}$ to $\boldsymbol{y}$ for every $i \in \langle r \rangle$.

Then

$\langle \boldsymbol{u} = \boldsymbol{x}_{1,,} \boldsymbol{y}_1 \ldots \ldots \boldsymbol{x}_{r,} \boldsymbol{h}_{r,} \boldsymbol{y}_r = \boldsymbol{v} \rangle$ is the desired path.

LEMMA: 5.1

Let a and b be any two distinct elements in $\langle \boldsymbol{n} \rangle$ with $n \geq 4$ and let x be a vertex of P_n.then is a Hamiltonian path P of $P_n - \{x\}$ joining a vertex u with $(u)_1 = a$ .to vertex vwith $(v)_1 = b$.

PROOF:

Suppose that $n = 4$. Since P_4 is vertex transitive, we may assume that, $x = 1234$.

Without loss of generality,We may assume that $a < b$.the required paths of $P_4 - \{1234\}$.are listed below:

a=1 and b=2
⟨1423,4123,3214,2314,1324,3124,4213,2413,3142,4132,1432,3412,2143,1243,3421,4321,2341,4312,2134⟩
a=1 and b=3 ⟨1423,4123,2143,1243,4213,2413,3142,1342,2431,3421,4231,2341,3241,4231,1324,3124,2134,4312,3214⟩
a=1 and b=4 ⟨1423,2413,3142,1342,2413,3421,4321,2341,3241,4231,1324,2314,3214,4123,2143.1243,4213,4312,4132⟩
a=2 and b=3 ⟨2134,4312,1342,3142,2413,4213,1243,2143,3412,1432,4123,2314,3214,4123,1423,3214,2341.4321,3124⟩
a=2 and b=4 ⟨2134,3124,1324,2314,3214.4123,2143,1243,4213,2413,1423,3241,4231,2431,3421,4321,2341,1432,3412⟩
a=3 and b=4 ⟨3214,4123,2143,1243,4213,3124,2134,4312,3412,1432,2341,4321,3421,2431.1342,3142,2413,1423⟩

We can find the required Hamiltonian path on P_n for every n, $n \geq 5$.

THE STAR GRAPHS

Let n be a positive integer. The n dimensional star graph,,is a graph with the vertex set$V(S_n) = \{u_1 \ldots \ldots u_n | u_i \in \langle n \rangle$ and $u_j \neq u_k$ for j$\neq k\}$.

The adjacency is defined as follows:$_1$ $\ldots . u_i$ $\ldots . u_n$ is adjacent to v_1 $\ldots . v_i$ $\ldots . v_n$ through an edge of dimension i with 2≤ i ≤ n if $v_j =$ for every j∈ $\langle n \rangle$-{1,i},$v_1 = u_1$, and $v_i = u_1$.

The graphs S_2, S_3 and S_4 are illustrated in fig,it showed that the connectivity of S_n is (n-1).

We use boldface to denoted vertices in S_n.

Hence

U_1, U_2 .... U_n denotes a sequence of vertices in S_n.

By definition,

S_n is an (n-1)-regular graph with n! vertices. We use e to denote the vertex 12… . n.

It is know that S_n is a bipartite graph with one partite set containing the vertices corresponding to odd permutations and the other partite set containing those vertices correspond to even permutations.

We use white vertices to represent those even permutation vertices and we use black vertices to represent those odd permutation vertices.

Let u=$u_1 u_2 \ldots \ldots u_n$ be an arbitrary vertex of the star graph S_n.

We say that

u_i is the ith coordinate of u,(u),for 1≤ i ≤n.For 1≤ i ≤n, Let $S_n^{\{i\}}$ be the subgraph of S_n induced by those vertices u with $(u)_n$ =

i.Then

S_n can be decomposed into n subgraph $S_n^{\{i\}}$,$1 \leq i \leq n$,and each $S_n^{\{i\}}$ is isomorphic to S_{n-1}.

Thus,

The star graph can also be constructed recursively. Let I be any subset of $\langle n\rangle$.We use S_n^I to denote the subgraph of S_n indused by $\cup_{i\in I}$ $(S_n^{\{i\}})$. For any two distinct elements i and j in $\langle n\rangle$, We use $E_n^{i,j}$ to denote the set of edges between $S_n^{\{i\}}$ and $S_n^{\{1\}}$.

By the definition of S_n, there is exactly one neighbor v of uSuch that

U and v are adjacent through an i-dimensional edge with 2≤i≤n. For this reason,we use $(u)^i$ to denote the unique i-neighbor of u.

We have

$((u)^i)^i$ = u and $(u) \in S_n^{\{(u)1\}}$.

LEMMA: 5.2

Let a and b be any two distinct element in $\langle n\rangle$ with n≥ 4.Assume that x is a white vertex of S_n,and assume that $x_1 and\ x_2$ are two distinct neighbors of x. Then there is a Hamiltonian path p of $S_n -$ $\{x, x_1, x_2\}$ joining a white vertex u with $(u)_1 = a$ to a white vertex v with $(v)_1 = b$.

PROOF:

Since

S_n is vertex transitive and edge transitive, we many assume that x = e,$_1 = (e)^2$, and $x_2 = (e)^3$. Without loss of generality,

We may also assume that a < b. We have a≠ n and b ≠ 1.

We prove this statement by induction on n.

For n = 4, the required paths of S_4 - {1234, 2134, 3214} are listed below:

a=1 and b=2	⟨1324,3142,4132,1432,3412,4312,2314,1324,3124,4123,2143,1243,4213,2413,1423,3241⟩
a=1 and b=3	⟨1423,2413,4213,1243,2143,4123,3124,1324,2314,4312,3412,1432,4132,2431,4231,3241⟩
a=1 and b=4	⟨3142,4132,1432,3412,4312,2314,1324,3124,4123,1423,3421,2431,4231,3241,2341,4321⟩
a=2 and b=3	⟨2314,1324,3124,4123,2143,1243,4213,2341,3241,4231,2431,1432,4132,3142,1342,4312⟩
a=2 and b=4	⟨2314,1324,3124,4123,2143,1243,4213,2413,1423,3412,4321,2341,4312,1342,3142,4132⟩
a=3 and b=4	⟨3124,1324,2314,4312,3412,1432,4132,3142,1342,2341,4321,2143,4123,1423,2413,4213⟩

Suppose that

This statement holds for S_k for every k, 4≤ $k \leq n$ -1.

Let c be element in $\langle n-1\rangle - \{a\}$.

By induction there is a Hamiltonian path H of ${}^{\{n\}}_{n} - \{e, (e)^2, (e)^3\}$ joining a white vertex u of $(u)_1 = a$ to a white vertex z with $(z)_1 = c$.

We choose a white vertex v in ${}^{\{1\}}_{n}$ with $(v)_1 = b$.

There is a Hamiltonian path R of $S^{\langle n-1\rangle}_{n}$ joining the black vertex (z) to v.

Then

$\langle u, H, z, (z), R, v\rangle$is the desired path of $S - \{e, (e)^2, (e)^3\}$.

CONCLUSION:

In this paper i have studied about a concept A structure connectivity and substructure connectivity of hypercubes by using optimum broadcasting and mutually independent Hamiltonian path and fault free cycles of fault , The H-substructure connectivity of Q_n, applications, of the concept of mutually independent Hamiltonian cycles and star graph are induced in this paper.

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