



A note to machine Repair Model: Operational Analysis and Numerical Investigation

Rekha Kumari

Assistant Professor, Government Raza P. G. College Rampur

Abstract

In this paper, a different machine repair model is presented which incorporates balking, renegeing and repairmen. Mathematical analysis and numerical investigation has been performed for the proposed machine repair system. Performance analysis is also presented with steady state probability functions. The impact of different parameters have been observed with the help of suitable diagrams

Keywords: Machine repair; balking; renegeing; steady states; optimization;

Introduction

Without waiting in a queue to avail any kind of service, it is almost impossible now days because of increasing population day by day. Everywhere we see around us, there is a long queue for eg., traffic jams, tool booths, market, parlor, offices, etc., [1]. It is very natural phenomenon for a human being to unlike the waiting always. Nobody wants to wait due to very busy lifestyle.

We are living in the modern age and this is the age of technology and e-governance. It needs modeling based innovation. Models are designed, analyzed and if obtained results are matched with the real situation, they are ready to implement [2]. However, from the development of models to implementation, it requires team work. The most complex systems in this age are communication networks. Further, reliability and efficiency of the components are checked [3]. The dynamic behavior of the systems with the process, development of mathematical models is requisite, which tells us the stochasticity service of arriving requests. Queuing theory is the most useful tool for the performance of the systems. Customers flow from infinite or finite population to the service facility from waiting line to serve the all customers at a time [4]. The major characteristics of Queuing theory are Arrival pattern of customers, Service pattern of servers, Customers and customer behavior.

Since decades researchers are giving attention to machine repair problem. The real world problems like communication systems, production line system, machine operating system, etc., have been solved analytically with numerical methods.

The machine repair models like queueing model long with finite capacity has been solved by several authors[5-9]. Furthermore, various applications are presented in the form of banking, traffic flow, vacation model where the machine repair system have been found to be feasible as the real world system reflects.

Machine repair system plays significant role when some units have failed but it will remain in queue while repairing due to unit failure. However, the practical situation refers that queuing process may be continued and damaged part can be left without getting repaired. Such behavior of the system is balking and reneging. These behaviors are observed in real world problems. In this direction, M/M/1/m/m machine interference model with balking and reneging is studied by Shawky [1]. In this study, steady state probability and the cases M/M/1/m/m and M/M/2/m/m models are derived. Further, machine repair system has been studied with reneging, balking as well as warm standbys by Wang and Ke [2]. This investigation portrays reliability characterization like steady state probability and the mean time of first failure with numerical investigation. Also, Ke and Wang gives the M/M/R machine repair model with balking, spares and reneging found the gained cost analysis [8].

However, the real situation encounters no failed units in the system on service time at some random time, known as vacation in the queueing models. Gupta developed the machine repair problem with warm spares. In this study, the server take vacations along with exhaustive service [5]. The steady-state probability distribution are also derived in this study. Further, the machine repair system with reneging, warm spares and the repairman following N-policy discipline was analyzed by Jain in [6]. Reliability performances and few numerical results were obtained.

Here, we present one repair model with balking, reneging, warm spares and the synchronous multiple vacations in the viewpoint of reliability and availability. The steady state probability is derived. The analysis summarizes with the numerical results at last.

The modeling Assumptions:

The formulated model has the following realistic assumptions listed point wise:

1. There are total m operating units and the warm spares are w in the system.
2. The lifetime distributions are exponentially distributed with rates λ, α
3. Taken R are the repairmen and the repair time is taken as μ
4. The vacation time is denoted by θ
5. The impatient unit balk joining queue is a and $1-a$ is the probability.
6. The waiting time is r .
7. All the parameters are independent of each other.

PERFORMANCE ANALYSIS

Steady-state Probability Equations

Let $N(t)$ and $J(t)$ respectively be the number of the failed units and the state of the repairman in the system at time t ,

$J(t)$ is 1, when the server is on at the time of service t and it is zero when server is on vacation at the time t ,

then, the collection $\{N(t), J(t) \ t \geq 0\}$ is a Markov process with state space as following:

$$\Omega = \cup = \{(0,0)\} \cup \{(i,j)\}: i = 1,2, \dots, k \ j = 0,1\}$$

Consider the state space of failed unit as follows

$$F = \{(k,0), (k,1)\}$$

The mean failure rate of the system is give by

$$\lambda_1 = m \lambda + (w - i)\alpha, \lambda_2 = (m + w - i)\lambda b, \lambda_3 = (m + w - i)\lambda b,$$

Now, we present matrix-geometric solution, the method of blocked matrix to get iterative schemes of steady-state probability. Using following theorem the iterative schemes are developed to investigate numerically.

Theorem: Define $P = (P_0, P_1, L, P_i)$ be the steady state probability vector of the transition matrix A , where, $P_0 = (P_{00}), P_i = (P_{i0}, P_{i1}), (i = 1, 2, L, k)$

The proof of above theorem is similar as done by Ma and Zhao.

Theorem: Mean time to first failure of the system is known as following:

$$MTTFF = \int_0^{+\infty} R(t)dt = \lim_{s \rightarrow 0} R^*(s)$$

The proof of above theorem is similar as done by Ma and Zhao.

Due to many complications arise in analytical calculations. The numerical investigation is presented.

Numerical Investigation

The analytical results from the analysis of the model obtained are realistic results but it is very difficult to draw the real world conclusion from the analytical results. Here, in this section numerical results are developed. Mean time of first failure of the system and steady state probability is discussed by figure. The impact of distinct parameters has been portrayed. Fig. 1 portrays the steady-state probability A increases as failure rate depletion increases and then comes to saturation. Fig.2 shows the effect of different values of λ on MTTFF.

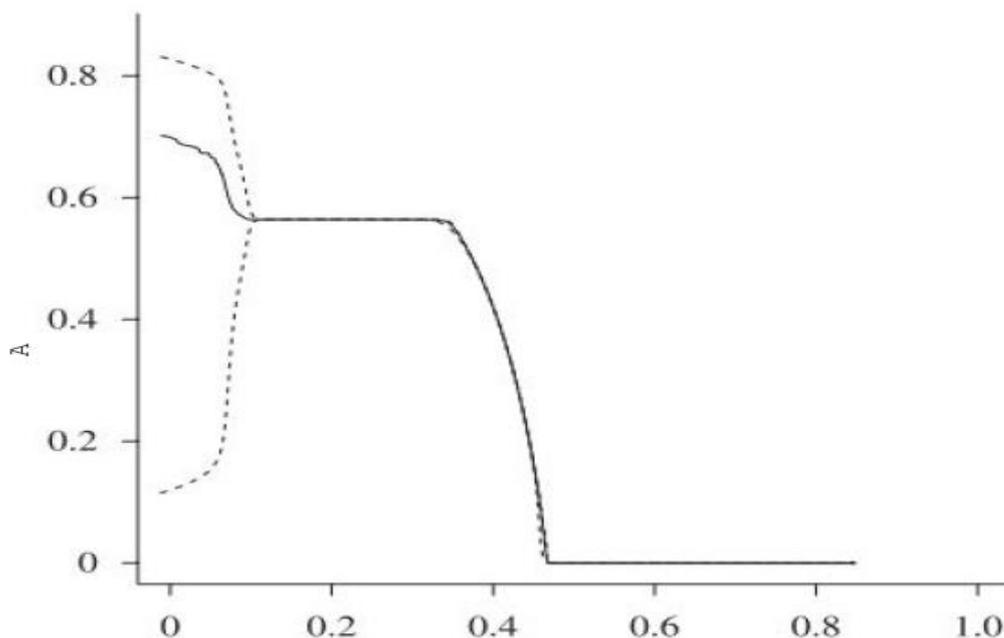
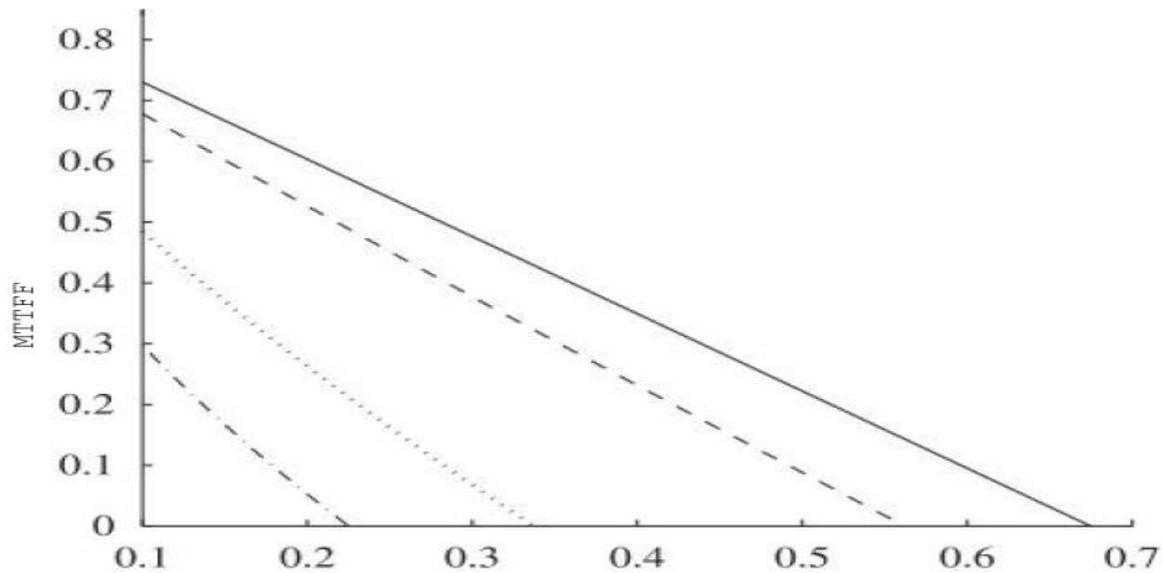


Fig.1 Effect of different values of λ on α .



Effect of different values of λ on MTTF.

Conclusion

The present paper deals with machine repair problem with balking, warm spares, and reneging by considering vacations. Analytical results are presented in the form of two theorems. The numerical analysis is presented the impact of parameters on steady state probability and mean time to failure the system. The model can be optimized based on the numerical investigation.

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