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Mathematical models for Machine Learning Techniques

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Abstract: The goal of the machine learning approaches is to automatically detect complex patterns in a given dataset, which enables inference or prediction to be made using those techniques on fresh datasets. The use of algorithms that are based on machine learning makes it possible to recognise similar groupings within an input dataset (unsupervised learning). Machine learning is a method for data analytics that trains computers to perform what comes naturally to both people and animals, which is to learn from experience. Algorithms that are used in machine learning make use of computational techniques in order to "learn" directly from data. These algorithms do not depend on a preconceived equation as a model. The number of samples that may be used for learning rises, and the algorithm is able to modify itself to get better results. Deep learning is a subfield of machine learning with its own unique set of challenges. A function is able to match input-output pairs via the process of supervised learning, which is an automated activity. A supervised learning algorithm's goal is to provide a function that maps the input-output (vector-supervision signal) pair as its output. This function can then be used for further learning. The technique makes it possible, in the best possible case, to accurately label the data in order to establish the classes. In the related field of human psychology, this phenomenon is referred to as conceptual learning. The Convolutional Neural Network (CNN), Support Vector Machines (SVM), Logistic Regression (LR), Naive Bayes (NB), Linear Discriminant Analysis (LDA), Decision Trees (DT), and Random Forest (RF) supervised learning algorithms and associated mathematical models were investigated in this research.

Keywords: Machine Learning Approaches, Machine Learning Techniques, supervised learning algorithms, Support Vector Machines (SVM), Logistic Regression (LR), Naive Bayes (NB), Linear Discriminant Analysis (LDA)

I. Introduction

Machine learning is a data analytics technique that teaches computers to do what comes naturally to humans and animals: learn from experience. Machine learning algorithms use computational methods to directly "**learn**" from data without relying on a predetermined equation as a model. As the number of samples available for learning increases, the algorithm adapts to improve performance. Deep learning is a special form of machine learning.

Machine Learning Work

Machine learning uses two techniques: supervised learning, which trains a model on known input and output data to predict future outputs, and unsupervised learning, which uses hidden patterns or internal structures in the input data.

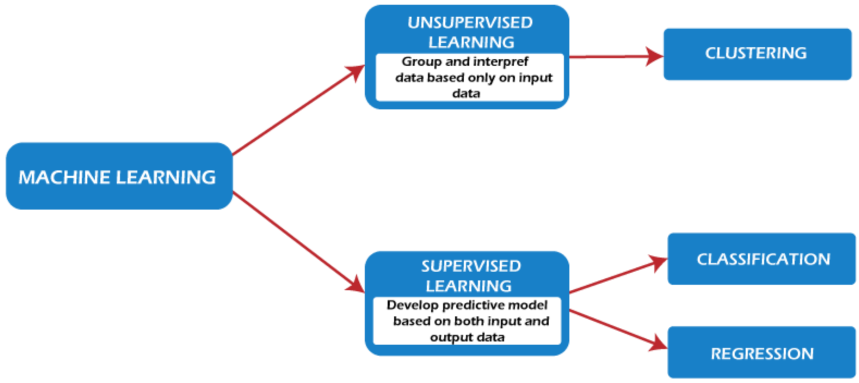

Fig. 1: Machine Learning Work

Machine learning approaches

The purpose of machine learning techniques is to automatically recognize complex patterns in a given dataset, allowing therefore for inference or prediction in new datasets.5 Machine learning techniques allow for the identification of homogeneous groups in the input dataset (unsupervised learning). When a group or category label is available for each case (supervised learning), machine learning techniques allow for the creation of a classifier, or a regression function, predicting the category membership of new cases (Figure 2). In order to ensure a good performance of the machine learning techniques in a given dataset, all possible sources of bias should be identified, removed or minimized. Therefore, before any machine learning process, the representativeness

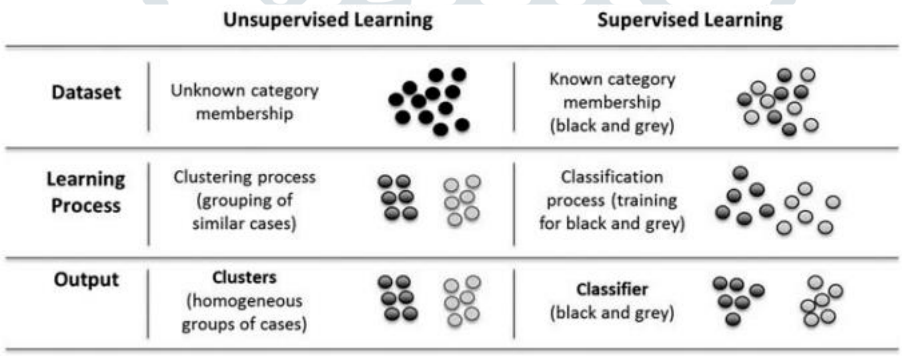

Fig 2: Unsupervised learning vs. Supervised Learning

Figure 2. Illustration of the two machine learning approaches, the unsupervised approach, when the category membership is unknown, and the supervised approach, when the category membership is known of the input dataset for the true population (i.e., the population under study) should be confirmed, the noise should be removed, the missing data should be treated, and the data dimensionally (i.e., the number of parameters/features and the number of cases in the dataset) should be adjusted.

II. Background

Churkin et al. (2022), Mathematical models for hepatitis C virus (HCV) dynamics have provided a means for evaluating the antiviral effectiveness of therapy and estimating treatment outcomes such as the time to cure. Recently, a mathematical modeling approach was used in the first proof-of-concept clinical trial assessing in real-time the utility of response-guided therapy with direct-acting antivirals (DAAs) in chronic HCV-infected patients. Several retrospective studies have shown that mathematical modeling of viral kinetics predicts time to cure of less than 12 weeks in the majority of individuals treated with sofosbuvir-based as well as other DAA regimens. A database of these studies was built, and machine learning methods were evaluated for their ability to estimate the time to cure for each patient to facilitate real-time modeling studies. Data from these studies exploring mathematical modeling of HCV kinetics under DAAs in 266 chronic HCV-infected patients were gathered. Different learning methods were applied and trained on part of the dataset ('train' set), to predict time to cure on the untrained part ('test' set). Our results show that this machine learning approach provides a means for establishing an accurate time to cure prediction that will support the implementation of individualized treatment.

Wilmott (2022), Every technical subject has its own jargon, and that includes machine learning (ML). Principal components analysis is a technique for finding common movements in data. Maximum likelihood estimation is a common method for estimating parameters in a statistical/probabilistic model. A confusion matrix is a simple way of understanding how well an algorithm is doing at classifying data. In ML a cost function or loss function is used to represent how far away a mathematical model is from the real data. One adjusts the mathematical model, usually by varying parameters within the model, so as to minimise the cost function. Some of the methods the authors look at are often used for natural language processing (NLP). NLP is about how an algorithm can understand, interpret, respond to, or classify text or speech.

Masum et al. 2022), The COVID-19 pandemic has caused a global crisis with 47,209,305 confirmed cases and 1,209,505 confirmed deaths worldwide as of November 2, 2020. Forecasting confirmed cases and understanding the virus dynamics is necessary to provide valuable insights into the growth of the outbreak and facilitate policy-making regarding virus containment and utilization of medical resources. In this study, they applied a mathematical epidemic model (MEM), statistical model, and recurrent neural network (RNN) variants to forecast the cumulative confirmed cases. They proposed a reproducible framework for RNN variants that addressed the stochastic nature of RNN variants leveraging z-score outlier detection. They incorporated heterogeneity in susceptibility into the MEM considering lockdowns and the dynamic dependency of the transmission and identification rates which were estimated using Poisson likelihood fitting. While the experimental results demonstrated the superiority of RNN variants in forecasting accuracy, the MEM presented comprehensive insights into the virus spread and potential control strategies.

Pavlyutin et al. (2022), To predict the spread of the new coronavirus infection COVID-19, the critical values of spread indicators have been determined for deciding on the introduction of restrictive measures using the city of Moscow as an example. A model was developed using classical methods of mathematical modeling based on exponential regression, the accuracy of the forecast was estimated, and the shortcomings of mathematical methods for predicting the spread of infection for more than two weeks. As a solution to the problem of the accuracy of long-term forecasts for more than two weeks, two models based on machine learning methods are proposed: a recurrent neural network with two layers of long short-term memory (LSTM) blocks and a 1-D convolutional neural network with a description of the choice of an optimization algorithm. The forecast accuracy of ML models was evaluated in comparison with the exponential regression model and one another using the example of data on the number of COVID-19 cases in the city of Moscow.

Shanmugam et al. (2022), The availability of low-grade coal with a high amount of ash has urged the improvisation of separation equipment with minimal or no water utilization. The present work addresses the study on the separation equipment performance with different moisture coal. The experimental results were obtained in terms of separation efficiency. After obtaining the experimental results, the mathematical modeling results were obtained using different techniques. The cubic regression and cascade neural network models were considered to study the mathematical correlation with experimental results. The R-squared value of each mathematical modeling technique was correlated with the model fitting to check the model's validity. The results clearly showed that the cubic model fitting for the experimental condition had provided an excellent R-squared value varying from 92% to 99%. The cascade model fitting for the experimental condition has provided a higher R-squared value, i.e., more than 99%. Results show that for all experimental conditions, the cascade model fitting of the neural network technique provides the significant mathematical modeling technique suitable for predicting the separation equipment's performance compared to the cubic model of the regression technique.

Varshneya et al. (2021), At present, the QT interval on the electrocardiographic (ECG) waveform is the most common metric for assessing an individual's susceptibility to ventricular arrhythmias, with a long QT, or, at the cellular level, a long action potential duration (APD) considered high risk. However, the limitations of this simple approach have long been recognized. Here, they sought to improve prediction of arrhythmia susceptibility by combining mechanistic mathematical modeling with machine learning (ML). Simulations with a model of the ventricular myocyte were performed to develop a large heterogenous population of cardiomyocytes (n = 10,586), and they tested each variant's ability to withstand three arrhythmogenic triggers: 1) block of the rapid delayed rectifier potassium current (IKr Block), 2) augmentation of the L-type calcium current (ICaL Increase), and 3) injection of inward current (Current Injection). Eight ML algorithms were trained to predict, based on simulated AP features in preperturbed cells, whether each cell would develop arrhythmic dynamics in response to each trigger. They found that APD can accurately predict how cells respond to the simple Current Injection trigger but cannot effectively predict the response to IKr Block or ICaL Increase. ML predictive performance could be improved by incorporating additional AP features and simulations of additional experimental protocols. Importantly, they discovered that the most relevant features and experimental protocols were trigger specific, which shed light on the mechanisms that promoted arrhythmia formation in response to the triggers. Overall, our quantitative approach provides a means to understand and predict differences between individuals in arrhythmia susceptibility.

Chaudhary et al. (2021), Social media is popular in our society right now. People are using social media platforms to purchase various products. They collected the data from various social media platforms. They analysed the data for prediction of the consumer behaviour on the social media platform. They considered the consumer data from Facebook, Twitter, Linked In and YouTube, Instagram, and Pinterest, etc. There are diverse and high-speed, high-volume data which are coming from social media platform, so they used predictive big data analytics. In this paper, they have used the concept of big data technology to process data and analyse it to predict consumer behaviour on social media. They have analysed consumer behaviour on social media platforms based on some parameters and criteria. They analysed the consumer perception, attitude towards the social media platform. To get good quality of result, they pre-process data using various data pre-processing to detect outlier, noises, error, and duplicate record. They developed mathematical modeling using machine learning to predict consumer behaviour on the social media platform. This model is a predictive model for predicting consumer behaviour on the social media platform. 80% of data are used for training purposes and 20% for testing.

Iantovicsm(2021), Current machine intelligence metrics rely on a different philosophy, hindering their effective comparison. There is no standardization of what is machine intelligence and what should be measured to quantify it. In this study, they investigate the measurement of intelligence from the viewpoint of real-life difficult-problem-solving abilities, and they highlight the importance of being able to make accurate and robust comparisons between multiple cooperative multiagent systems (CMASs) using a novel metric. An important property of the proposed metric is the universality, as it can be applied as a black-box method to intelligent agent-based systems (IABSs) generally, not depending on the aspect of IABS architecture. To demonstrate the effectiveness of the MetrIntPairII metric, they provide a representative experimental study, comparing the intelligence of several CMASs composed of agents specialized in solving an NP-hard problem.

Gupta et al. (2021), Currently, the entire world is fighting against the Corona Virus (COVID-19). As of now, more than thirty lacs of people all over the world were died due to the COVID-19 till April 2021. A recent study conducted by China suggests that Chest CT and X-ray images can be used as a preliminary test for COVID detection. This paper proposes a transfer learning-based mathematical COVID detection model, which integrates a pre-trained model with the Random Forest Tree (RFT) classifier. As the available COVID dataset is noisy and imbalanced so Principal Component Analysis (PCA) and Generative Adversarial Networks (GANs) is used to extract most prominent features and balance the dataset respectively. The Bayesian Cross-Entropy Loss function is used to penalize the false detection differently according to the class sensitivity (i.e., COVID patient should not be classified as Normal or Pneumonia class). Due to the small dataset, a pre-trained model like VGGNet-19, ResNet50 and Inception_ResNet_V2 were chosen to extract features and then trained them over the RFT for the classification task. The experiment results showed that ResNet50 gives the maximum accuracy of 99.51%, 98.21%, and 97.2% for training, validation, and testing phases, respectively, and none of the COVID Chest X-ray images were classified as Normal or Pneumonia classes.

Clement et al. (2021), COVID-19 is a life-threatening disease which has a enormous global impact. As the cause of the disease is a novel coronavirus whose gene information is unknown, drugs and vaccines are yet to be found. For the present situation, disease spread analysis and prediction with the help of mathematical and data driven model will be of great help to initiate prevention and control action, namely lockdown and quarantine. There are various mathematical and machine-learning models proposed for analysing the spread and prediction. Each model has its own limitations and advantages for a particular scenario. This article reviews the state-of-the art mathematical models for COVID-19, including compartment models, statistical models and machine learning models to provide more insight, so that an appropriate model can be well adopted for the disease spread analysis. Furthermore, accurate diagnose of COVID-19 is another essential process to identify the infected person and control further spreading. As the spreading is fast, there is a need for quick automated diagnosis mechanism to handle large population. Deep-learning and machine-learning based diagnostic mechanism be more appropriate for this purpose. In this aspect, a comprehensive review on the deep learning models for the diagnosis of the disease is also provided in this article.

Bolte & Pauwels (2020), Automatic differentiation, as implemented today, does not have a simple mathematical model adapted to the needs of modern machine learning. In this work they articulate the relationships between differentiation of programs as implemented in practice, and differentiation of non-smooth functions. To this end they provide a simple class of functions, a non-smooth calculus, and show how they apply to stochastic approximation methods. They also evidence the issue of artificial critical points created by algorithmic differentiation and show how usual methods avoid these points with probability one.

Dutta et al. (2020), Machine learning algorithm has brought the augmenting change in the field of artificial intelligence, which espoused human discerning power in a splendid manner. The algorithm has various categories among which classification is the most popular part. Support vector machine algorithm, logistic regression, naïve bays algorithm, decision tree, boosted tree, random forest and k nearest neighbor algorithm are all under classification of algorithms. Classification process needs some pre-defined method, which leads for choosing the train data from the sample data given by the user. Decision-making is the heart of any classification algorithm as supervised learning stands out on the decision of the users. Hence, a strong mathematical model based on conditional probability lies behind each algorithm.

This paper is a study of those mathematical models and logic behind various classification algorithms, which help to create strong decision criteria for users to make the training dataset based on which machine can predict the proper output.

Boso et al. (2020), They address the problem of determining from laboratory experiments the data necessary for a proper modeling of drug delivery and efficacy in anticancer therapy. There is an inherent difficulty in extracting the necessary parameters, because the experiments often yield an insufficient quantity of information. To overcome this difficulty, they propose to combine real experiments, numerical simulation, and Machine Learning (ML) based on Artificial Neural Networks (ANN), aiming at a reliable identification of the physical model factors, e.g. the killing action of the drug. To this purpose, they exploit the employed mathematical-numerical model for tumor growth and drug delivery, together with the ANN - ML procedure, to integrate the results of the experimental tests and feedback the model itself, thus obtaining a reliable predictive tool. The procedure represents a hybrid data-driven, physics-informed approach to machine learning. The physical and mathematical model employed for the numerical simulations is without extracellular matrix (ECM) and healthy cells because of the experimental conditions they reproduce.

Khalafalla et al. (2020), This paper contributes to the study of PUFs vulnerability against modeling attacks by evaluating the security of XOR BR PUFs, XOR TBR PUFs, and obfuscated architectures of XOR BR PUF using a simplified mathematical model and deep learning (DL) techniques. DL modeling attacks were invoked against PUFs with different stage sizes (e.g., 64, 128, 256) and all are implemented on FPGA chips. Obtained results show that DL modeling attacks could easily break the security of 4-input XOR BR PUFs and 4-input XOR TBR PUFs with modeling accuracy ~99%. Similar attacks were executed using single-layer neural networks (NN) and support vector machines (SVM) with polynomial kernel and the obtained results showed that single NNs failed to break the PUF security. Furthermore, SVM results confirmed the same modeling accuracy reported in previous research ~50%. For the first time, this research empirically shows that DL networks can be used as powerful modeling techniques against these complex PUF architectures for which previous conventional machine learning techniques had failed. Furthermore, a detailed scalability analysis is conducted on the DL networks with respect to PUFs' stage size and complexity. The analysis shows that the number of layers and hidden neurons inside every layer has a linear relationship with PUFs' stage size, which agrees with the theoretical findings in deep learning. Consequently, A new obfuscated architecture is introduced as a first step to counter DL modeling attacks and it showed significant resistance against such attacks (16% - 40% less accuracy). This research provides an important step towards prioritizing the efforts to introduce new PUF architectures that are more secure and invulnerable to modeling attacks. Moreover, it triggers future discussions on the removal of influential bits and the level of obfuscation needed to confirm that a specific PUF architecture is resistant against powerful DL modeling attacks.

Fokas et al. (2020), They introduce a novel methodology for predicting the time evolution of the number of individuals in a given country reported to be infected with SARS-CoV-2. This methodology, which is based on the synergy of explicit mathematical formulae and deep learning networks, yields algorithms whose input is only the existing data in the given country of the accumulative number of individuals who are reported to be infected. The analytical formulae involve several constant parameters that were determined from the available data using an error-minimizing algorithm. The same data were also used for the training of a bidirectional long short-term memory network. They applied the above methodology to the epidemics in Italy, Spain, France, Germany, USA and Sweden. The significance of these results for evaluating the impact of easing the lockdown measures is discussed.

III. Artificial intelligence

Supervised learning is an automatic task allowing a function to match input-output pairs. The purpose of a supervised learning algorithm is to produce a function which maps the input-output (vector-supervision signal) pair. The algorithm will allow in an optimal scenario to correctly label the data to determine the classes. In the parallel world of human psychology, it is called conceptual learning. Among the supervised learning algorithm are Convolutional Neural Network (CNN), Support Vector Machines (SVM), Logistic Regression (LR), Naive Bayes (NB), Linear Discriminant Analysis (LDA), Decision Trees (DT) and Random Forest (RF)

3.1 Convolutional neural network (CNN)

The principle of Neural Network (NN) is based on the collection of nodes (called artificial neurons), which freely model neurons in the brain. Based on examples, without any prior knowledge, without being programmed, this system automatically generates identification characteristics. When the algorithm uses multiple layers of neurons it is known as Deep learning. A Convolutional Neural Network (CNN) is a Deep Learning algorithm which takes an image as input, assign learnable weights to various features (objects) in the image so as to be able to differentiate one image from the other.

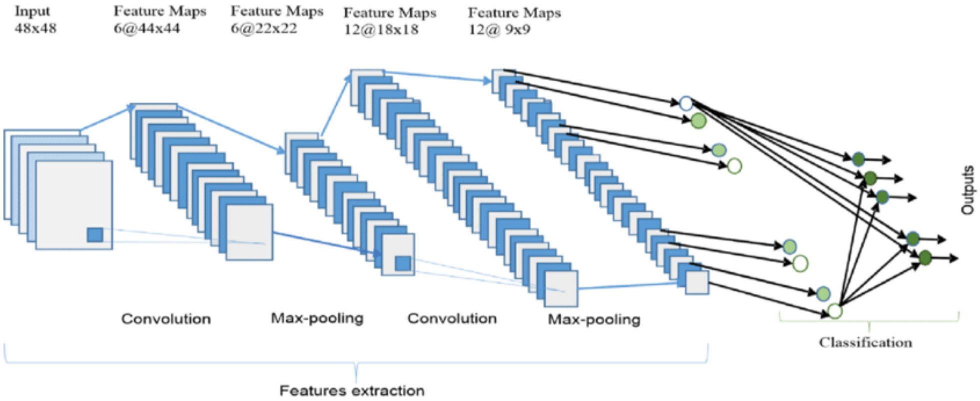

Fig 3: The overall architecture of the Convolutional Neural Network (CNN)

3.2 Support vector machines (SVM)

Support Vector Machines (SVM) are supervised learning methods used for regression, classification and also outlier detection. The aim of SVM is to find a hyperplane in an N-dimensional space (where N is the number of features) that markedly classifies the input data. In other words, SVM will work to find a plane that has the maximum distance between data points of separate classes. Support vectors are those data points that are closest to the hyperplane. These data points affects the position and orientation of the hyperplane.

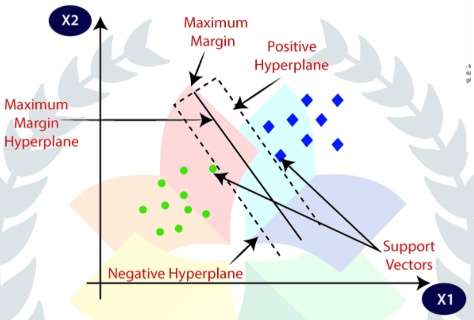

Fig 4: SVM

3.3 Logistic regression (LR)

In statistics, logistic regression is used to model the probability, each sample is assigned a probability between 0 and 1. It can be extended to model several classes of events in order to determine for example different objects in an image .

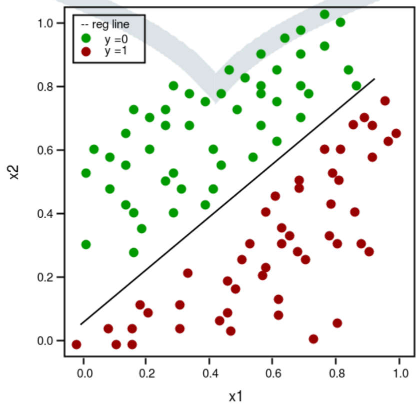

Fig 5: Logistic regression (LR)

3.4 Naive Bayes (NB)

Naive Bayes classifiers are among the simplest Bayesian network models from the family of probabilistic classifiers. Coupled with the Kernel density estimation, they can reach high levels of precision in digital images classification.

Bayes' Theorem finds the probability of an event occurring given the probability of another event that has already occurred. Bayes' theorem is stated mathematically as the following equation:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

where A and B are events and $P(B) \neq 0$.

- Basically, we are trying to find probability of event A, given the event B is true. Event B is also termed as **evidence**.
- P(A) is the **priori** of A (the prior probability, i.e. Probability of event before evidence is seen). The evidence is an attribute value of an unknown instance (here, it is event B).
- P(A|B) is a posteriori probability of B, i.e., probability of event after evidence is seen.

Now, with regards to our dataset, we can apply Bayes' theorem in following way:

$$P(y|X) = \frac{P(X|y)P(y)}{P(X)}$$

where, y is class variable and X is a dependent feature vector (of size *n*) where:

$$X = (x_1, x_2, x_3, ..., x_n)$$

3.4 Linear discriminant analysis (LDA)

Linear discriminant analysis (LDA) is used to find a linear combination of features that characterizes or separates classes of objects or events in pattern recognition and machine learning. This resulting combination can be used as a linear classifier for dimensionality reduction before the final classification.

3.5 Decision trees (DT) and random forest (RF)

Decision trees is a technique that helps analysing decisions by identifying the most likely strategy leading to the goal. Random Forest on its part is essentially a collection of Decision Trees whose results are accumulated into a final result. They have the ability to limit variance without increasing error due to bias. In medical practice, it is used to classify patient images.

IV. Conclusion and future Work

The conclusive statement of machine learning methodologies is to automatically find complex patterns in a given dataset, allowing inferences or predictions to be made on new datasets using these techniques. 5 Utilizing machine learning-based algorithms makes it feasible to identify comparable groups within an input dataset (unsupervised learning). Machine learning is a data analytics technique that teaches computers to execute what comes naturally to both humans and animals, namely learning from experience. Machine learning algorithms "learn" directly from data using computational approaches. These algorithms do not rely on a predetermined model equation. The number of samples that may be utilised for learning increases, and the algorithm is able to improve its own performance by modifying itself. Deep learning is a branch of machine learning that has its own distinct issues. A function is capable of matching input-output pairs via the automated process of supervised learning. The objective of a supervised learning algorithm is to provide an output function that maps the input-output (vector-supervision signal) pair. This feature may then be used for more study. In the best-case scenario, the approach makes it feasible to precisely label the data in order to define the classes. This phenomenon is referred to as conceptual learning within the area of human psychology. This study studied the Convolutional Neural Network (CNN), Support Vector Machines (SVM), Logistic Regression (LR), Naive Bayes (NB), Linear Discriminant Analysis (LDA), Decision Trees (DT), and Random Forest (RF) supervised learning techniques and related mathematical models.

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