JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

ON SEMI HEYTING ALMOST DISTRIBUTIVE LATTICES

M.V.RATNAMANI AND V.V.V.S.S.P.S.SRIKANTH
Department of Basic science and Humanities, Aditya Institute of Technology and Management,Tekkali, India.

Abstract : In this paper, we exhibit that the algebra semi Heyting almost distributive lattices is not equationally definable with Heyting almost distributive lattices and almost semi Heyting algebras.

IndexTerms : Almost distributive lattice, Heyting almost distributive lattice, semiHeyting almost distributive lattice and almost semi Heyting algebra.

2010 Mathematics Subject Classification: Primary 06D99; Secondary 06D20.

I. Introduction

In [4], Sankappanavar categorized a class of algebras (semi-Heyting algebras) as an abstraction from Heyting algebras and observed that they are distributive, pseudo-complemented and congruences on them are determined by filters. In [1], authors generalized the structure Heyting algebras and introduced Heyting almost distributive lattices by considering an almost distributive lattice with a maximal element. In [2], authors introduced the class of semi-Heyting almost distributive lattices as a generalization of the class of semi-Heyting algebras by having an almost distributive lattice with maximal elements, which are not lattices. Later, in [3], authors generalized the class of Heyting algebras using the structure almost distributive lattice with a maximal element and introduced the class of almost semi-Heyting algebras which are not lattices and not Heyting algebras. In this paper we observe that the varieties of semi-Heyting almost distributive lattices are not equationally definable with Heyting almost distributive latticesand almost semi-Heyting algebras. Also Semi Heyting almost distributive lattice is not associative and commutative with respect to the binary operation $\rightarrow$.

II. Preliminaries

Let us recall the definition of an almost distributive lattice and semi-Heyting almost distributive lattice, Heyting almost distributive lattice and almost semi Heyting algebra and certain necessary results which are required in the sequel.

Definition 2.1. [5] An almost distributive lattice (ADL) is an algebra (L, ∨, ∧, 0) of type (2, 2, 0) which satisfies the following;

(i) $a_1 \vee 0 = a_1$

(ii) $0 \wedge a_1 = 0$

(iii) $(a_1 \vee b_1) \wedge c_1 = (a_1 \wedge c_1) \vee (b_1 \wedge c_1)$

(iv) $a_1 \wedge (b_1 \vee c_1) = (a_1 \wedge b_1) \vee (a_1 \wedge c_1)$

(v) $a_1 \vee (b_1 \wedge c_1) = (a_1 \vee b_1) \wedge (a_1 \vee c_1)$

(vi) $(a_1 \vee b_1) \wedge b_1 = b_1$, for all $a_1, b_1, c_1 \in L$.

Example 2.2. [5] Let L be a non-empty set. Fix $a_0 \in L$. For any $a_1, b_1 \in L$. Define $a_1 \wedge b_1 = b_1, a_1 \vee b_1 = a_1$ if $a_1 \neq a_0$, $a_0 \wedge b_1 = a_0$ and $a_0 \vee b_1 = b_1$. Then $(L, \vee, \wedge, x_0)$ is an ADL and it is called as discrete ADL.

In this section L stands for an ADL $(L, \vee, \wedge, 0)$ unless otherwise specified.

Given $a_1, b_1 \in L$, we say that a_1 is less than or equal to b_1 if and only if $a_1 = a_1 \wedge b_1$; or equivalently $a_1 \vee b_1 = b_1$, and it is denoted by $a_1 \leq b_1$. Hence $\leq$ is a partial ordering on L. An element $m \in L$ is said to maximal if for any $a_1 \in L, m \leq a_1$ implies $m = a_1$.

Theorem 2.3. [5] For any $m \in L$, the following are equivalent;

(i) m is a maximal element

(ii) $m \vee a_1 = m$, for all $a_1 \in L$.

(iii) $m \wedge a_1 = a_1$, for all $a_1 \in L$.

For any binary operation → in an ADL $(L, \vee, \wedge, 0)$ with a maximal element m, let us denote the following identities for all $a_1, b_1, c_1 \in L$,

$I(1)$ $[(a_1 \wedge b_1) \rightarrow b_1] \wedge m = m$

$I(2)$ $a_1 \wedge (a_1 \rightarrow b_1) = a_1 \wedge b_1 \wedge m$

$I(3)$ $a_1 \wedge (b_1 \rightarrow c_1) = a_1 \wedge [(a_1 \wedge b_1) \rightarrow (a_1 \wedge c_1)]$

$I(4)$ $(a_1 \wedge m) \rightarrow (b_1 \wedge m) = (a_1 \rightarrow b_1) \wedge m$

$I(5)$ $a_1 \rightarrow a_1 = m$

$I(6)$ $(a_1 \rightarrow b_1) \wedge b_1 = b_1$

$I(7)$ $a_1 \rightarrow (b_1 \wedge c_1) = (a_1 \rightarrow b_1) \wedge (a_1 \rightarrow c_1)$

$I(8)$ $(a_1 \vee b_1) \rightarrow c_1 = (a_1 \rightarrow c_1) \wedge (b_1 \rightarrow c_1)$

Now, we have the following identities which are the consequences of $I(1), I(2), I(3)$ and $I(4)$

$CI(1)$ $(a_1 \rightarrow a_1) \wedge m = m$

$CI(2)$ $[a_1 \wedge (a_1 \rightarrow b_1)] \wedge m = a_1 \wedge b_1 \wedge m$

$CI(3)$ $[a_1 \wedge (b_1 \rightarrow c_1)] \wedge m = \big[a_1 \wedge [(a_1 \wedge b_1) \rightarrow (a_1 \wedge c_1)]\big] \wedge m$

$CI(4)$ $[(a_1 \wedge m) \rightarrow (b_1 \wedge m)] \wedge m = (a_1 \rightarrow b_1) \wedge m$

Definition 2.4. [1] L with a maximal element m is said to be a Heyting almost distributive lattice (abbreviated:HADL), if it holds $I(2), I(5), I(6), I(7)$ and $I(8)$.

Definition 2.5. [2] L with a maximal element m is said to be a semi-Heyting almost distributive lattice (abbreviated: SHADL), if it holds $CI(1), I(2), I(3)$ and $I(4)$.

Definition 2.6. [3] L with a maximal element m is said to be an almost semi-Heyting algebra (abbreviated: ASHA), if it holds $I(1), CI(2), CI(3)$ and $CI(4)$.

III. REMARK ON SHADL, HADL AND ASHA.

In this section we present a counter example that semi-Heyting almost distributive lattices is not equationally definable with respect to Heyting almost distributive lattices and almost semi-Heyting algebras.

Remark 3.1. Every SHADL need not be an ASHA and as well as HADL. For, see the following example.

Example 3.2. Let $L = \{0, x_1, m\}$ be the three element chain. Define a binary operation $\rightarrow$ on L as follows

$\rightarrow$	0	x_1	m
0	m	0	0
x_1	0	m	m
m	0	x_1	m

Then $(L, \vee, \wedge, \rightarrow, 0, m)$ is an SHADL.

Here put $a_1 = 0$ and $b_1 = m$ in $(a_1 \rightarrow b_1) \wedge b_1 = (0 \rightarrow m) \wedge m$

$$= 0 \wedge m$$

$$= 0$$

which is a contradiction to $(a_1 \rightarrow b_1) \wedge b_1 = b_1$ $(I(6))$. Thus $(L, \vee, \wedge, \rightarrow, 0, m)$ is not an HADL

Also, consider $[(a_1 \wedge b_1) \rightarrow b_1] \wedge m = [(0 \wedge m) \rightarrow m] \wedge m$

$$= [0 \rightarrow m] \wedge m$$

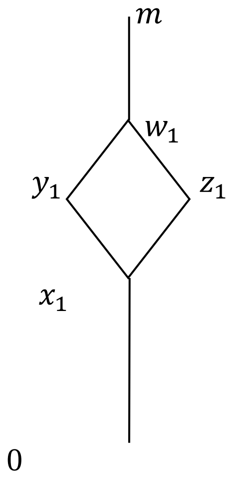
$$= 0 \wedge m$$

$$= 0$$

which is a contradiction to which $[(a_1 \wedge b_1) \rightarrow b_1] \wedge m = m$ $(I(1))$. Thus $(L, \vee, \wedge, \rightarrow, 0, m)$ is not an ASHA.

Remark 3.3. In a SHADL the associative property $[(a_1 \rightarrow b_1) \rightarrow c_1] \wedge m = [a_1 \rightarrow (b_1 \rightarrow c_1)] \wedge m$ and commutative $(a_1 \rightarrow b_1) \wedge m = (b_1 \rightarrow a_1) \wedge m$ of the binary operation $\rightarrow$ fails.

Example 3.4. Let $L = \{0, x_1, y_1, z_1, m\}$ be a 5-element algebra with the following lattice reduct and the $\rightarrow$ operation:



$\rightarrow$	0	x_1	y_1	z_1	w_1	m
0	m	m	m	m	m	m
x_1	0	m	m	m	m	m
y_1	0	z_1	m	z_1	m	m
z_1	0	y_1	y_1	m	m	m
w_1	0	x_1	y_1	z_1	m	m
m	0	x_1	y_1	z_1	w_1	m

Then $(L, \vee, \wedge, \rightarrow, 0, m)$ is an SHADL.

Put $a_1 = x_1, b_1 = y_1$ & $c_1 = z_1$ in $[(a_1 \rightarrow b_1) \rightarrow c_1] \wedge m = [a_1 \rightarrow (b_1 \rightarrow c_1)] \wedge m$

$$\Rightarrow [(x_1 \rightarrow y_1) \rightarrow z_1] \wedge m = [x_1 \rightarrow (y_1 \rightarrow z_1)] \wedge m$$

$$\Rightarrow [m \rightarrow y_1] \wedge m = [x_1 \rightarrow z_1] \wedge m$$

$$\Rightarrow y_1 \wedge m = m \wedge m$$

$$\Rightarrow y_1 \neq m.$$

Which is a contradiction to the associative property.

Hence associative property in SHADL does not hold.

Put $a_1 = x_1$& $b_1 = y_1$in $(a_1 \rightarrow b_1) \wedge m = (b_1 \rightarrow a_1) \wedge m$

$$\Rightarrow (x_1 \rightarrow y_1) \wedge m = (y_1 \rightarrow x_1) \wedge m$$

$$\Rightarrow m \wedge m = z_1 \wedge m$$

$$\Rightarrow m \neq z_1.$$

Which is a contradiction to the commutative property.

Hence commutative property in SHADL does not hold.

IV. CONCLUSION:

In this paper we have showed that the algebra semi Heyting almost distributive lattices is different from Heyting almost distributive lattices and almost semi Heyting algebras although all the three algebras are generalizations of almost distributive lattices and observed that the associative and commutative properties does not hold in semi Heyting almost distributive lattices.

REFERENCES:

[1] Rao G.C., Berhanu Assaye and M.V.Ratnamani.: *Heyting almost distributive lattices*, Inter-national Journal of Computational Cognition, 8, 89-93(2010).

[2] Rao G.C., Ratnamani M.V., Shum K.P. and Berhanu Assaye *Semi-Heyting almost distributive lattices*, Lodachevskii Journal of Mathematics, 36, 184-189(2015).

[3] Rao.G.C., Ratnamani M.V., Shum K.P. *Almost semi-Heyting algebra* Southeast Asian Bulletin of Mathematics 42,95-110(2018).

[4] Sankappanavar H.P.: *Semi-Heyting algebras: an abstraction from Heyting algebras* ACTASDEL IX CONGRESO, 33-66 (2007).

[5] Swamy U.M. and Rao G.C.: *Almost Distributive Lattices*, Jour.Aus.Math.Soc., 31,77-91(1981).