JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

On Reduced Fuzzy Automaton

Mo Suphiyan khan

Department of Mathematics

M.L.K.(P.G.) College, Balrampur

Dr. Veena Singh

Department of Mathematics

M.L.K.(P.G.) College, Balrampur

Abstract: In this paper, first we show that every fuzzy automaton has at least one strongly connected subautomaton and if M is a cyclic fuzzy automaton, then M has a unique maximal layer. After that we show that it is possible to construct a fuzzy automaton having singleton as a unique minimal layer from a given fuzzy automaton with a unique minimal layer and so, we get a reduced fuzzy automaton.

Keywords: Fuzzy automaton, Subautomaton, Layer, Reducible fuzzy automaton.

1. INTRODUCTION

The theory of fuzzy sets was introduced by Zadeh [8, 9]. In [2], Ito Masami has defined the concept of layers of an automaton and also, characterized the subautomaton of an automaton in terms of layers. In [2], Ito Masami has shown that for any finite poset P, there exists an automaton, whose poset of layers is isomorphic to P. Subsequently, Wee [7] has introduced the idea of fuzzy automata and the algebraic study of fuzzy automata has been initiated by Malik [3] (cf., [4] for details). In [5], S.P. Tiwari, Vijay K. Yadav, Anupam K. Singh have defined the concept of layers of a fuzzy automaton. characterized the subautomaton of a fuzzy automaton in terms of layers. In [5], S.P. Tiwari, Vijay K. Yadav, Anupam K. Singh have shown that the maximal layer of a cyclic fuzzy automaton and minimal layer of a directable fuzzy automaton are unique. In this paper, we show that every fuzzy automaton has at least one strongly connected subautomaton and if M is a cyclic fuzzy automaton, then M has a unique maximal layer. We have also shown that any fuzzy automaton can be reduced.

2. PRELIMINARIES

All fuzzy-theoretic and lattice-theoretic notions and results used here, but not defined or explained, are fairly standard by now (and can be found in [1],[6]). However, for convenience, we recall some of the notions used in the sequel.

Definition 2.1([1]): A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function, which assigns to each object a grade of membership lies between zero to one. i.e., A fuzzy set is a pair (X, μ), where X is a non - empty set and $\mu: X \to [0, 1]$, a membership function, then a fuzzy set $A = (X, \mu)$ is defined as

$$A = \{(x, \mu(x)):\ x \in X\}$$

Also, denoted as

$$A = \left\{\frac{\mu(x)}{x} : x \in X\right\}$$

Let $x \in X$. Then x is called

(i) Not included in the fuzzy set A = (X, μ), if μ(x) = 0 (no member)
(ii) Fully included in the fuzzy set A = (X, μ), if μ(x) = 1 (Full Member)
(iii) Partially included in the fuzzy set A = (X, μ), if 0 < μ(x) < 1 (fuzzy member)

Other definitions –

(i) A fuzzy set A = (X, μ) is empty (A = ∅) if and only if $\mu_A(x) = 0, \forall x \in X$

(ii) Two fuzzy sets A and B are equal (A = B) if and only if $\mu_A(x) = \mu_B(x), \forall x \in X$:

(iii) A fuzzy set A is included in a fuzzy set B (A ⊆ B) if and only if $\mu_A(x) \le \mu_B(x), \forall x \in X$:

Fuzzy set Operations –

(i) For a given fuzzy set A, its **complement** A^c (or cA) is defined by the following membership function:

$$\mu_{cA}(x) = 1 - \mu_A(x), \forall x \in X$$

(ii) For given a pair of fuzzy sets A, B, their **intersection** A ∩ B is defined by

$$\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}, \forall x \in X$$

(iii) For given a pair of fuzzy sets A, B, their **union** A ∪ B is defined by

$$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}, \forall x \in X$$

Definition 2.2([5]): A relation R defined on a set 'S' is called a partial ordering or partial order if it reflexive, anti-symmetric and transitive. A set 'S' together with a partial ordering R is called a partially ordered set or poset and is denoted by (S, R).

Definition 2.3([6]): Let (P, ≤) be a poset. Then an element $a \in P$ is called minimal if $a \le b, \forall b \in P$. Similarly, $b \in P$ is called maximal if $a \le b, \forall a \in P$.

Also, a, b ∈ P and a ≠ b, then a is called predecessor of b and b is called successor of a, if $a \le c \le b$ and $c \in P$ imply that c = a or c = b.

We denote this relation by < a, b >. An element b ∈ P is called atomic if there exists a minimal element a ∈ P with < a, b >. Let a ∈ P. Then o(a) element | {b ∈ A: < b, a >} |. Moreover, by o(P), we denote max{o(a) | a ∈ P}([5]).

Definition 2.4: Let A = (Q, X, δ), where Q and X are non-empty finite sets, called a state set and an alphabet, respectively and δ is a function called a state transition function such that δ (q, a) ∈ Q, for some q ∈ Q and any a ∈ X. Then A is called a finite automaton.

Note that the above δ can be extended to the following function in a natural way i.e., δ (q, e) = q and δ (q, au) = δ (δ (q, a), u) for some q ∈ Q, u ∈ X^* and a ∈ X, where X^* is the set of all strings on X, obtained by concatenation.

Let A = (Q, X, δ) be an automaton. We define an equivalence relation ~ on Q as follows:

for q, p ∈ Q, q~ p if and only if there exist u, u ∈ X such that δ (p, u) = q hold.

Definition 2.5: Let A = (Q, X,δ) be an automaton. For p ∈ Q, we define a subset T_p of Q by {q ∈ Q | p ~ q}. This subset T_p is called a layer of A.

For two layers T_p and T_q, we define a partial order $\preccurlyeq_A$ as follows:

$T_p \preccurlyeq_A T_q$ if and only if there exists a word u ∈ X such that δ (q, u) = p.

We denote the poset ($\{T_p \mid p \in Q\}, \preccurlyeq_A$) by P(A).

Definition 2.6: Let A = (Q, X, δ) and B = (T, X, θ) be two automata, then B is called a sub automaton of A if the following conditions are satisfied:

(i) T ⊆ Q
(ii) $\theta = \delta|_{T \times X}$, i.e., θ is the restriction of δ to T × X.

3. REDUCIBLE FUZZY AUTOMATA

Recall that X^* denotes the set of all finite words over a non-empty set X. We shall denote the identity of X by e. Also, |P| denotes the cardinality of a finite set P.

Definition 3.1: A fuzzy automaton is a tuple M = (Q, X, δ), where Q is non empty finite set, called the set of states and X is non-empty finite set, called the set of inputs and δ is a fuzzy subset of Q x X x Q, i.e., a map δ: Q x X x Q → [0, 1] such that ∀ p, q ∈ Q, ∀ u ∈ X and x ∈ X,

$$\delta(p, e, q) = \begin{cases} 1, & \text{if } p = q \\ 0, & \text{if } p \neq q \end{cases}$$

and $$\delta(p, ux, q) = \vee \{\delta(p, u, r) \wedge \delta(r, x, q) : r \in Q\}$$

Also, it has been observed that $\delta(p, uv, q) = \vee\{\delta(p, u, r) \wedge \delta(r, v, q) : r \in Q\}$ ∀ p, q ∈Q, u, v ∈ X

Definition3.2: Let M= (Q, X, δ) be a fuzzy automaton and A⊆Q. The source and the successor of A are respectively the sets

$$\sigma_Q(A) = \{q \in Q : \delta(q, u, p) > 0, \text{ for some } (u, p) \in X^* \times A\}$$

and $$s_Q(A) = \{p \in Q : \delta(q, u, p) > 0, \text{ for some } (u, q) \in X^* \times A\}.$$

We shall frequently write $\sigma_Q(A)$ and $s_Q(A)$ as just $\sigma(A)$ and $\sigma(\{q\})$ and s({q}) as just $\sigma(q)$ and s(q).

Definition 3.3: A fuzzy automaton N = (R, X, λ) is called a subautomaton of a fuzzy automaton M = (Q, X, δ), if R⊆Q and s(Q) = R and $\delta|_{R \times X \times R} = \lambda$.

Definition 3.4: Let M = (Q, X,δ) be a fuzzy automaton and N = (R, X, λ) be a fuzzy subautomaton of automaton M, i.e., N⊆M. Then, N is called separated, if $s_Q(Q-R) \cap R = \emptyset$, where R⊆Q

Definition 3.5: A fuzzy automaton M = (Q, X, δ) is called strongly connected if

$$p \in s(q), \ \forall p, q \in Q,$$

Definition 3.6: A fuzzy automaton M= (Q, X, δ) is called cyclic, if for all p ∈ Q, there exists q_0 ∈ Q and u ∈ X* such that $\delta(q_0, u, p) > 0$.

Definition 3.7: A fuzzy automaton M = (Q, X, δ) is called directable if for all p, q ∈ Q, there exist r ∈ Q and u ∈ X* such that δ(p, u, r)>0 and δ (q, u, r)>0.

Definition 3.8: A homomorphism from a fuzzy automaton M = (Q, X, δ) to a fuzzy automaton N = (R, Y, λ) is a pair (f, g) of maps, where f: Q→R and g: X→Y are functions such that λ (f(q), g(x), f(p) ≥ δ (q, u, p) ∀ (q, u, p) ∈ Q × X ×Q.

Remark 3.1: In the above definition if X = Y and g is the identity map on X, then we say that f is homomorphism from M to N.

Definition 3.9: Let M = (Q, X, δ) be fuzzy automata. We define a relation R on Q as follows:

(p, q) ∈ R if and only if δ(p, u, q) > 0 and δ(q, v, p) > 0 for some u, v, ∈ X^*.

Theorem 3.1: The relation R, defined on Q in definition 3.9, is an equivalence relation on Q.

Proof: (i) Reflexivity: As δ(q, e, q) = 1 > 0, ∀ q ∈ Q, so, ${}_qR_q$ or (q, q) ∈ R, i.e., R is reflexive.

(ii) Symmetry: Let ${}_qR_p$ i.e., $\delta(q, x, p) > 0$ and $\delta(p, y, q) > 0$ for some $x, y \in X^*$

⇒ $\delta(p, y, q) > 0$ and $\delta(q, x, p) > 0$, for some $x, y \in X^*$

⇒ ${}_pR_q$ i.e., (p, q) ∈ R

Thus, R is symmetric on Q.

(iii) Transitivity: Let ${}_qR_p$, ${}_pR_r$. Then we have δ(q, x, p) > 0, δ(p, y, q) > 0, δ(p, z, r) > 0 and δ(r, w, p) > 0, for some x, y, z, w ∈ X^* and q, r, p ∈ Q.

We can easily see that δ(q, xz, r) > 0 and δ(r, wy, q) > 0, for some xz, wy ∈ X^*

⇒ ${}_qR_r$, i.e., (q, r) ∈ R

Thus, R is transitive on Q.
Hence R is equivalence relation on Q.

Definition 3.10: Let M = (Q, X, δ) be a fuzzy automaton and R be the relation on Q. Then for p ∈ Q, the set L_p = {q ∈ Q: (p, q) ∈ R} a layer of M.

For two layers L_p and L_q of Q, we define $L_p \preccurlyeq_M L_q$, if and only if δ(q, u, p) >0, for u ∈ X^*.

We shall denote the set {L_p: p ∈ Q} by E_m.

Theorem 3.2: The set (E_m, $\preccurlyeq_M$) is a poset or ({L_p: p ∈ Q}, $\preccurlyeq_M$) is a poset.

Proof: We show $\preccurlyeq_M$ is a partial order relation on the set {L_p: p ∈ Q}.

(i) Reflexivity: As δ(q, e, q) = 1 > 0 ⇒ $L_p \preccurlyeq_M L_q$

Thus $\preccurlyeq_M$ is Reflexive on $\preccurlyeq_M$.

(ii) Anti-symmetry: Let $L_p \preccurlyeq_M L_q$ and $L_q \preccurlyeq_M L_p$, ∀ p, q, r ∈ Q.

⇒ δ(q, x, p) > 0, δ(p, y, q) > 0 for some x, y ∈ X^*

i.e., ${}_pR_q$ or p ~ q

i.e., p and q are of some equivalence class, thus $L_p = L_q$.

Hence $\preccurlyeq_M$ is anti-symmetric on E_M.

(iii) Transitivity: Let $L_p \preccurlyeq_M L_q$, $L_q \preccurlyeq_M L_r$, ∀ p, q, r ∈ Q

$L_p \preccurlyeq L_q$ ⇒ δ(q, x, p) > 0, for some x ∈ X^*

$L_q \preccurlyeq L_r$ ⇒ δ(r, y, q) > 0, for some y ∈ X^*

As δ(r, y, q) > 0 and δ(q, x, p) > 0 imply that δ(r, yx, p) > 0
⇒ $L_p \preccurlyeq_M L_r$

Thus $\preccurlyeq_M$ is transitive on E_M.
Therefore, $\preccurlyeq_M$ is partial ordering on E_M then, the set ({L_p: p ∈ Q}, $\preccurlyeq_M$) is a poset.

Theorem 3.3: Every fuzzy automaton has at least one strongly connected sub automaton.

Proof: Let $M = (Q, X, \delta)$ be a fuzzy automaton and E_M be the collection of all layers of M of different classes. Then $(E_M, \preccurlyeq_M)$ is a poset. Let $q \in Q$, and $Lq \in E_M$ be a minimal layer. We know that –

$$Lq \subseteq s(Lq) \text{ ----- } (1)$$

By definition, $s(Lq) = \{p \in Q: \delta(q, u, p) > 0, \text{ for some } (u, q) \in X^* \times Lq\}$

Then, for $p \in s(Lq)$, there exist $u \in X^*$ and $t \in Lq$ such that $\delta(t, u, p) > 0$, Now $t \in Lq$ implies there exists $v \in X^*$ such that $\delta(q, v, t) > 0$, thus,

$\delta(q, vu, p) \geq \delta(q, v, t) \wedge \delta(t, u, p) > 0$, i.e., $\delta(q, vu, p) > 0$.

Also, by minimality of Lq, $Lq \preccurlyeq_M Lp$, which shows that $\delta(p, w, q) > 0$

as $\delta(q, vu, p) > 0$, $\delta(p, w, q) > 0$ implies that $(q, p) \in R$ i.e., $p \in Lq$. Thus, for all $p \in s(Lq)$, $p \in Lq$ implies

$$s(Lq) \subseteq Lq \text{ ----- } (2)$$

from (1) and (2), $s(Lq) = Lq$

So, (Lq, X, δ') is a sub automaton of M, where $\delta' = \delta|_{Lq \times X \times Lq}$

Further, let $p, r \in Lq$ i.e., $(q, p), (q, r) \in R$.

Then there exists $u, v \in X^*$ such that $\delta(q, u, p) > 0$ and $\delta(r, v, q) > 0$, or that $\delta(r, vu, q) > 0$, i.e., $p \in s(r)$ where by the subautomaton (Lq, X, δ') is strongly connected.

Hence, every fuzzy automaton has at least one strongly connected sub automaton.

Theorem 3.4: Let M be a cyclic fuzzy automaton. Then M has a unique maximal layer which is maximum is E_M.

Proof: Let $M = (Q, X, \delta)$ be a cyclic fuzzy automaton and Lq be a maximal layer in E_M. As $q \in Lq$, $Lq \subseteq Q \Rightarrow q \in Q$ and M is cyclic, then $\exists\, q_o \in R$ such that $\delta(q_o, u, q) > 0$, for some $u \in X^*$, and therefore $Lq \preccurlyeq_M Lq_o$. As Lq is maximal layer in E_M, then, $Lq_o \preccurlyeq Lq$, then $Lq = Lq_o$. Hence $Lq_o \in E_M$ is a unique maximal layer.

4. CONSTRUCTION OF REDUCED FUZZY AUTOMATON

The following is toward the construction of fuzzy automaton having singleton as a unique minimal layer from a given fuzzy automaton with a unique minimal layer.

Let $M = (Q, X, \delta)$ be a fuzzy automaton having unique minimal layer Lq. Construction a fuzzy automaton $M' = (((Q - L_q) \cup \{r\}), X, \lambda)$, where r is a new state and $\lambda: (Q - L_q) \cup \{r\} \times X \times ((Q - L_q) \cup \{r\}) \longrightarrow [0, 1]$ is a map

i.e. $\lambda: ((Q - L_q) \cup \{r\}) \times X \times ((Q - L_q) \cup \{r\}) \longrightarrow [0, 1]$

$$\lambda(q, u, t) = \begin{cases} \delta(q, u, t), & \text{if } t, q \in (Q - L_q) \\ 1, & \text{if } t \in \{r\} \text{ i.e., } t = r, q \in \{Q - L_q\} \cup \{r\} \\ 0, & \text{if } t \in (Q - L_q), q \in \{r\} \text{ i.e., } q = r \end{cases}$$

The from the definition of M', it is clear that {r} is a unique minimal layer of M'.

Theorem 3.5: Let $M = (Q, X, \delta)$ be a fuzzy automaton having unique minimal layer L_0, and $M' = (((Q - L_0) \cup \{q_0\})\ X, \lambda)$ be a construction fuzzy automaton from M, having unique minimal layer $\{q_0\}$, where λ is defined as follows:

$$\forall\, (q, u, p) \in (Q - L_0) \cup \{q_0\} \times X \times (Q - L_0) \cup \{q_0\}$$

$$\lambda\,(q, u, p) = \begin{cases} \delta\,(q, u, p), & p, q \in (Q - L_0) \\ 1, & p \in \{q_0\}, \text{i.e.}, p = q_0, q \in (Q - L_0) \cup \{q_0\} \\ 0, & q \in \{q_0\} \text{i.e.}, q = q_0,\ p \in (Q - L_0) \end{cases}$$

then, M' is homomorphic image of M.

Proof: Let f: M → M' be a map such that ∀ q ∈ Q,

$$F(q) = \begin{cases} q, & \text{if } q \in (Q - L_0) \\ q_0, & \text{if } q \in L_0 \end{cases}$$

We will discuss if in following four cases –

Case (i), when $q, p \in (Q - L_0)$, then $\lambda\,(f(q), q(u), f(p)) = \lambda\,(q, u, p) = \delta\,(q, u, p)$

Case (ii), when $p \in L_0$, $q \in (q - L_0)$, then $\lambda\,(f(q), g(u), f(p)) = \lambda\,(q, u, q_0) = 1 \geq \delta\,(q, u, p)$

Case (iii), when, $p \in L_0$, $q \in L_0$, then $\lambda\,(f(q), g(u), f(p)) = \lambda\,(q_0, u, q_0) = 1 \geq \delta\,(q, u, p)$

Case (iv), when $q \in L_0$, $p \in (Q - L_0)$, then $\lambda\,(f(q), g(u), f(p)) = \lambda\,(q_0, u, p) = 0$, And

As, $p \in (Q - L_0) \Rightarrow p \notin L_0$, Let $q \in L_0$. Then, $(q, p) \notin R$

So, either $\delta\,(q, u, p) > 0$, $\delta\,(p, v, q_1) = 0$ or $\delta\,(q, u, p) = 0$, $\delta\,(p, v, q_1) > 0$ for some $u, v \in X^*$

If $\delta\,(q_1, u, p) > 0$, $\delta\,(p, v, q_1) = 0$, i.e. $\delta\,(q_1, u, p) > 0$, this gives $Lp \preccurlyeq_M Lq_0 = L_0$ i.e. $Lp \preccurlyeq_M L_0$

which contradicts that L_0 is a minimal layer

So, $\delta\,(p, v, q_1) > 0$ and $\delta\,(q, u, p) = 0$

Thus, $\lambda\,(r(q), g(v), f(p)) = 0 = \delta\,(q, x, p)$

Thus, $\forall\,(q, x, p) \in ((Q - L_0) \cup \{q_0\}) \times X \times ((Q - L_0) \cup \{q_0\})$

$$\lambda\,(f(g), g(x), f(p)) \geq \delta\,(q, x, p).$$

Also, from definition of 'f' it is clear that f is onto. Hence, M′ is a homorphic image of M.

5. Conclusion

In this paper, which is mainly inspired from [5], we find all subautomaton of a fuzzy automaton by using the concept of successor. As mentioned in [5], we have shown that the set of all layers E_M of a fuzzy automaton M forms a poset. We have shown that every fuzzy automaton has at least one strongly connected sub automaton. We have ultimately shown that we can construct a fuzzy automaton M' having singleton as a unique minimal layer from a given fuzzyautomaton M with a unique minimal layer, i.e., any fuzzy automata can be reduced.

References

[1] Klir, George J. and Yuan Bo, "Fuzzy Sets and Fuzzy Logic: Theory and Applications", Vol. 4. New Jersey: Prentice Hall, (1995).

[2] Lto, Masami, "Algebraic structures of automata", Theoretical computer science429 (2012) 164-168.

[3] Malik, D.S., Mordeson, J.N., Sen, M.K. “Submachines of fuzzy finite state machine”, J Fuzzy Math 2:781–792 (1994).

[4] Mordeson, J.N., Malik, D.S., “Fuzzy automata and languages: theory and applications”, Chapman and Hall/CRC, London/Boca Raton, (2002).

[5] Tiwari, S.P, Yadav, Vijay K., Singh, Anupam K., “On algebraic study of fuzzy automata”, Springer-Verlag Berlin Heidelberg 2014.

[6] Tremblay, J.P. and Manohar, R., “Discrete Mathematical Structures with Applications to Computer Science”, Tata McGraw Hill (1975).

[7] Wee, W.G., “On generalizations of adaptive algorithm and application of thefuzzy sets concept to pattern classification”, Ph.D. Thesis, Purdue University (1967).

[8] Zadeh, L. A., “Fuzzy Sets”, Information and Control, 8 (1965) 338-353.

[9] Zadeh, L.A., Fuzzy sets and systems, Proc. Symp. System Theory, Polytechnique Institute of Brooklyn (1965) 29-35.