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Rainbow detour connectionsofgraphs and Hamilton-Connected Properties

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Abstract

Let G be a nontrivial connected graph on which is defined a coloring c: E(G) → {1, 2, . . . , k}, k ∈ N, of the edges of G, where adjacent edges may be colored the same. A path P in G is a rainbow path if no two edges of P are colored the same. If the graph G contains a rainbow u v detour path for each of its two vertices u and v, it is rainbow detour-connected. The rainbow detour connection number $rdc(G)$ of G is the minimum k for which such a k-edge coloring exists. If G contains a Rainbow detour for every pair u, v of distinct vertices, then G is strongly Rainbow detour -connected. The strong rainbow detour connection number $srdc(G)$ of G is the smallest k for which a k-edge colouring of G results in a strongly rainbow detour-connected graph. Thus, for every nontrivial connected graph G, $rdc(G) \leq srdc(G)$. In this paper, we calculate the rainbow detour and strong rainbow detour connection number on various types of graphs.

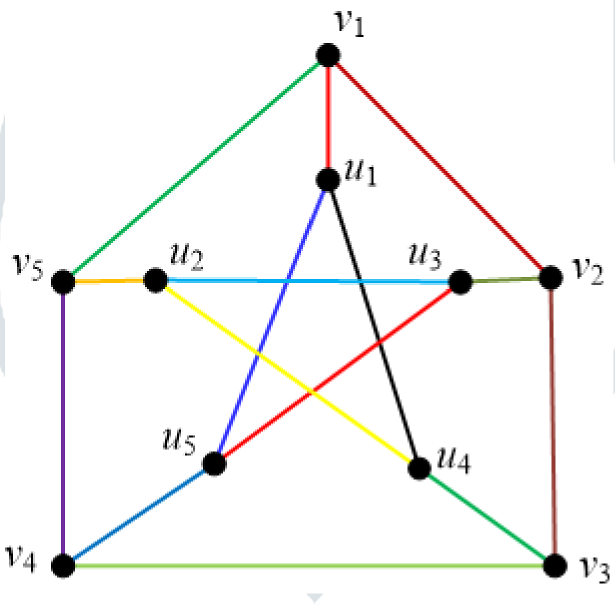
Keywords:edge coloring, rainbow detour coloring, strong rainbow detour coloring

AMSSubjectClassification2000:05C15.

Introduction

Let G be a nontrivial connected graph with a coloring c: E(G) → {1, 2, . . ., k}, k ∈ N of its edges, where adjacent edges can be colored the same. A rainbow path is an u − v path P in G in which no two edges of P are the same color. The graph G is rainbow detour-connected (with respect to c) if it contains a rainbow u−v detour path for each of its two vertices u and v.In this case, the coloring c is referred to as a rainbow detour coloring of G. If k colors are used, c is a rainbow k-coloring. The rainbow detour connection number $rdc(G)$ of G is the minimum k for which a rainbow k-coloring of G's edges exists.If every $x, y \in V(G)$ is connected by a Hamilton path (path containing all vertices of G), then the graph G is a HamiltonConnected graph. All HC graphs are Hamiltonian, but the converse need not be true. For $n \geq 3$, the complete graph K_n is both HC and Hamiltonian, whereas all bipartite graphs are not. A rainbow path is one with distinct colors on all of its edges.

Let G be a nontrivial connected graph. If any two vertices of G are connected by a rainbow path, then an edge coloring of G is rainbow connected. We introduce the detour rainbow connected path in this paper if any two vertices of G are connected by a rainbow detour path.Let c be a rainbow detour coloring of a connected graph G. A rainbow u–v detour in G is a rainbow u–v detour path of length $D(u, v)$,where $D(u, v)$ is the longest distance between u and v (the length of the longest u–v path in G). The graph G is strongly detour-connected if it contains a rainbow u–v detour for each of its two vertices u and v. In this case, the coloring c is known as a strong rainbow detour coloring of.The strong rainbow detour connection number $srdc(G)$ of G is the minimum k for which there exists a coloring c: E(G) → {1, 2, . . ., k} of the edges of G such that G is strongly rainbow detour-connected. A minimum strong detour rainbow coloring of G is a strong rainbow detour coloring of G using $srdc(G)$colors. Thus, for each connected graph G, $rdc(G) \leq srdc(G)$.Since every coloring that assigns distinct colors to the edges of a connected graph is both a rainbow detour coloring and a strong rainbow detour coloring, every connected graph is rainbow detour -connected and strongly rainbow detour-connected with respect to some coloring of G's edges. Furthermore, if G is a nontrivial connected graph of size m whose diameter (the largest distance between two vertices of G) is diam(G), then (1) $diam(G) \leq rdc(G) \leq srdc(G) \leq m$.



Consider the Peterson graph G in Figure 1, which includes a rainbow detour 8-coloring of P to demonstrate these concepts. As a result, $rdc(G) \leq 8$ If, on the other hand, there are adjacent vertices, then the rainbow detour coloring of G is $rdc(G) = 8$.As a result, any rainbow detour coloring of path P uses at least 8- colors,$rdc(G) \geq 8$. If P has detour path from u_1 to $u_5$is $D(u_1, u_5) = u_1 v_1 v_5 u_2 u_3 v_2 v_3 v_4 u_5 = 8$.

If there exists a non-adjacent vertex, then we have the rainbow detour coloring of G is$rdc(G) = 9$, and there is a detour path from u_1 to $u_2$is $D(u_1, u_2) = u_1 u_5 u_3 v_2 v_1 v_5 v_4 v_3 u_4 u_2 = 9$.

Proposition 1.1

Let G be a nontrivial connected graph of size m. Then

(i) $srdc(G) = m - 1$ if and only if G is a complete graph.
(ii) $rdc(G) \leq m$ if and only if G is a Tree.
Proof
We first verify(i). If G is a complete graph, then all the vertices are adjacent to each other, then the coloring that assigns $m - 1$ to every edge of G is a strong rainbow $detour$ $(m - 1)$ $-$coloring of G and so $srdc(G) = m - 1$.

We now verify (ii). Suppose first that G is not a tree. Then G contains a cycle C $v_1, v_2, v_3 \ldots\ldots v_k v_1$, where$k \geq 3$. Then the $(m - 1)$ $-$coloring of the edges of G that assigns 1 to the edges $v_1 v_2$ and v_2, v_3 and assigns the $(m - 2) -$ distinct colors from {2, 3, . . ., m − 1} to the remaining $(m - 2)$ edges of G is a rainbow coloring. Thus$rdc(G) \leq m - 1$. Next, let G be a tree of size m. Assume, to the contrary, that $rdc(G) \leq m - 1$.

Proposition 1.1 also implies that the only connected graphs G for which $rdc(G) = m - 1$ are the complete graphs and that the only connected graphs G of size m for which $srdc(G) = m$ are trees.

2. Some rainbow detour connection numbers of graphs

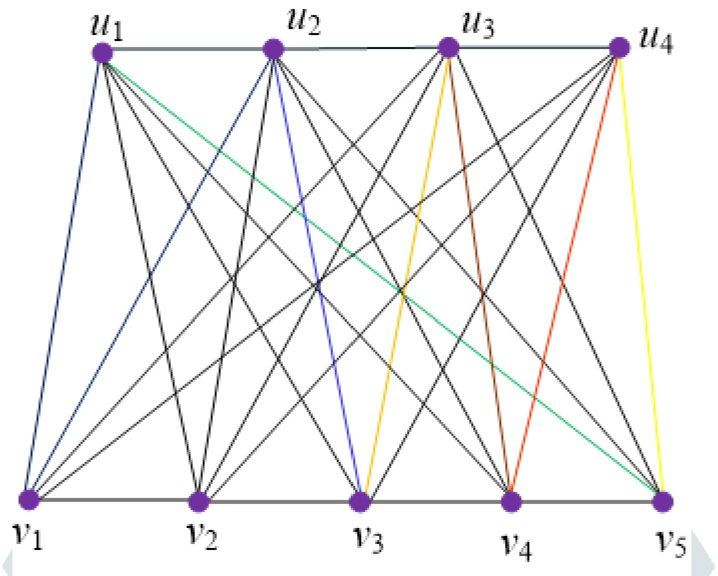
In this section, we determine the rainbow connection numbers of some well-known graphs. We refer to the book [1] for graph-theoretical notation and terminology not described in this paper. We begin with cycles of order n. Since $diam(C_n) = \left\lfloor \frac{n}{2} \right\rfloor$, it follows by (1) that $srdc(C_n) \geq$, $srdc(C_n) = \left\lfloor \frac{n}{2} \right\rfloor$. This lower bound for $rc(C_n)$ and $src(C_n)$is nearly the exact value of these numbers.For a Hamiltonian-connected graph G, an edge coloring $c : E(G) \longrightarrow [k]$ = {1, 2, . . . , k} is called a Hamiltonian-connected rainbow k-coloring if every two vertices of G are connected by a rainbow Hamiltonian path in G. An edge coloring c is a Hamiltonianconnected rainbow coloring if c is a Hamiltonian-connected rainbow k -coloring for some positive integer k.The minimum k for which G has a Hamiltonian-connected rainbow k -coloring is the rainbow Hamiltonian-connection number of G, denoted by hrc(G).

3. Join of Graphs

The graph $G_i + G_j$ is formed by adding all edges between the graphs G_i and G_j. The union of the disjoint edge sets of the graphs $G_i, G_j$and the complete bipartite graph $K_{p,q}$is the edge set $G_i + G_j$.

Theorem 1.1
Let $G \in P_m + P_n$, $m < n$. Then $rdc(P_m + P_n) = m + n - 1$.
Proof:
Suppose that $G = P_m + P_n$
Without loss of generality, we let $m < n$, To prove the theorem, it is enough if we prove G is Hamilton connected graph. Let $U = \{u_1, u_2, u_3 \ldots\ldots u_m\}$ and $V = \{v_1, v_2, v_3 \ldots\ldots v_n\}$ such that $V(G) = U \cup V$ and $|V(G)| = m + n$. Then we have a rainbow detour connected between every pair of vertices. For example, the rainbow detour connection number from
$u_1 \longrightarrow v_1 = u_1 v_5 u_4 v_4 u_3 v_3 v_2 u_2 v_1 = 8$

$v_1 \longrightarrow v_2 = v_1 u_1 u_2 u_3 u_4 v_5 v_4 v_3 v_2 = 8$



$P_4 + P_5$

Table 1.1 exhibits Hamilton-paths between all pairs of vertices of $P_m + P_n$

Table 1: Different cases which exhibits the Hamilton-path for all pairs of vertices of $P_m + P_n$

S. No.	Cases	Hamilton-path
1	$u = u_i$ $v = v_j$	$\{u = u_i, u_{i-1}, u_{i-2} \dots u_1, v_1, v_2 \dots v_{j-1}, u_{i+1}, u_{i+2} \dots u_m, v_n, v_{n-1} \dots v_j = v\}$
2	$u = u_i$ $v = u_j$	$\{u = u_i, u_{i-1}, u_{i-2} \dots u_1, v_1, v_2 \dots v_i, u_{i+1}, v_{i+1} \dots u_{j-1}, v_{j-1}, v_j, v_{j+1} \dots v_n\}$ $\circ\{u_m, u_{m-1} \dots u_j = v\}$
3	$u = v_i$ $v = v_j$	$\{u = v_i, v_{i-1}, v_{i-2} \dots v_1, u_1, u_2 \dots u_i, u_{i+1}, v_{i+1}, v_{i+2} \dots v_{j-1}, u_{i+2}, u_{i+3} \dots u_m\}$ $\circ\{v_n, v_{n-1} \dots v_j = v\}$

Proposition 2.2.
For $n \geq 3$, the rainbow detour connection number of the wheel W_n is
$rdc(W_n) = n - 1.$
Proof.
Suppose that W_n consists of an n-cycle C_n : v_1, v_2 , . . ., v_n, $v_{n+1} = v_1$ and another vertex v joined to every vertex of C_n. Since $W_n = K_4$ it follows by Proposition 1.1 that $rdc(W_n) = 3$. For $n \geq 3$, the wheel W_n is not complete and so $rdc(W_n) = n - 1$.
Theorem

If H is a Hamiltonian-connected spanning subgraph of a graph G, then $hrc(\mathrm{G}) < hrc(\mathrm{H})$.

Let G be a Hamiltonian-connected graph of order $n \geq 4$. Because every Hamiltonianconnected rainbow coloring of G is a rainbow colouring, there is no Hamiltonianconnected rainbow coloring of G using fewer than $n-1$ colors, and the edge coloring that assigns distinct colours to distinct edges of G is a Hamiltonian-connected rainbow colouring.

Conclusion

In this present study we obtained the rainbow detour and strong rainbow detour connection number of a graph. The rainbow detour coloring and other graph operations are under investigation.

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