



# FUZZY rh-SUPER IRRESOLUTE MAPPING

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**Abstract:** In this paper the concept of fuzzy rh -super irresolute mappings have been introduced and explore some of its basic properties in fuzzy Topological Space.

**Keywords:** fuzzy topology, fuzzy super closure, Fuzzy Super Interior fuzzy rh - super closed sets and fuzzy rh - super open sets, fuzzy rh -super continuous and fuzzy rh -super irresolute mappings

## I.INTRODUCTION

Let  $Y$  be a non empty set and  $I = [0,1]$ . A fuzzy set on  $Y$  is a mapping from  $Y$  in to  $I$ . The null fuzzy set  $0$  is the mapping from  $X$  in to  $I$  which assumes only the value is  $0$  and whole fuzzy sets  $1$  is a mapping from  $Y$  on to  $I$  which takes the values  $1$  only. The union (resp. intersection) of a family  $\{A_\alpha: \alpha \in \Lambda\}$  of fuzzy sets of  $X$  is defined by  $\cup A_\alpha$  (resp.  $\cap A_\alpha$ ) to be the mapping  $\sup P_\alpha$  (resp.  $\inf P_\alpha$ ). A fuzzy set  $P$  of  $Y$  is contained in a fuzzy set  $R$  of  $Y$  if  $P(y) \leq R(y)$  for each  $x \neq y$ . A fuzzy point  $y_\alpha$  in  $Y$  is a fuzzy set defined by  $x_\beta(x) = \alpha$  for  $y = x$  and  $y(x) = 0$  for  $x \neq y$ ,  $\alpha \in [0,1]$  and  $x \in y$ . A fuzzy point  $y_\alpha$  is said to be quasi-coincident with the fuzzy set  $P$  denoted by  $x_{\alpha q} P$  if and only if  $\alpha + P(y) > 1$ . A fuzzy set  $P$  is quasi coincident with a fuzzy set  $R$  denoted by  $P_q R$  if and only if there exists a point  $y \in Y$  such that  $P(y) + R(y) > 1$ .  $P \leq R$  if and only if  $(P_q R^c)$ . A family  $\tau$  of fuzzy sets of  $y$  is called a fuzzy topology [2] on  $Y$  if  $0,1$  belongs to  $\tau$  and  $\tau$  is super closed with respect to arbitrary union and finite intersection. The members of  $\tau$  are called fuzzy super open sets and their complement are fuzzy super closed sets. For any fuzzy set  $P$  of  $Y$  the closure of  $P$  (denoted by  $cl(P)$ ) is the intersection of all the fuzzy super closed super sets of  $P$  and the interior of  $P$  (denoted by  $int(P)$ ) is the union of all fuzzy super open subsets of  $P$ .

**Definition 1.1 [5,10,11,12]:** Let  $(Y, \tau)$  fuzzy topological space and  $P \leq Y$  then

1. Fuzzy Super closure  $scl(P) = \{y \in Y : cl(U) \cap P \neq \emptyset\}$
2. Fuzzy Super interior  $sint(P) = \{y \in Y : cl(U) \leq P \neq \emptyset\}$

**Definition 1.2 [5, 10,11,12]:** A fuzzy set  $P$  of a fuzzy topological space  $(Y, \tau)$  is called:

- (a) Fuzzy super closed if  $scl(P) \leq P$ .
- (b) Fuzzy super open if  $1-P$  is fuzzy super closed  $sint(P) = P$

**Remark 1.1 [5, 10,11,12]:** Every fuzzy closed set is fuzzy super closed but the converses may not be true. **Remark 1.2 [5, 10,11,12]:** Let  $P$  and  $Q$  are two fuzzy super closed sets in a fuzzy topological space  $(Y, \tau)$ , then  $P \cup Q$  is fuzzy super closed.

**Remark 1.3 [5]:** The intersection of two fuzzy super closed sets in a fuzzy topological space  $(Y, \tau)$  may not be fuzzy super closed.

**Definition 1.3:** A fuzzy set  $P$  of an fuzzy topological space  $(Y, \tau)$  is said to be :-

- (a) fuzzy regular super open if  $P = int(cl(P))$  [7].
- (b) fuzzy h-super closed if  $cl(P) \leq O$  whenever  $P \leq O$  and  $O$  is an fuzzy super open set. [14]
- (c) fuzzy h-super open if  $P^c$  is fuzzy g-closed. [14]
- (d) fuzzy rh-super closed if  $cl(P) \leq O$  whenever  $P \leq O$  and  $O$  is an fuzzy regular super open set. [16]
- (e) fuzzy rh-super open if  $P^c$  is fuzzy mg-closed. [16]

**Remark 1.3:** Every fuzzy super closed set is fuzzy h-super closed and every fuzzy h-super closed set is fuzzy rh-super closed but the converse may not be true. [14,16]

**Definition 1.4:** A mapping  $f : (Y, \tau) \rightarrow (X, \mu)$  is said to be :

1. Fuzzy h-super continuous if the pre image of every fuzzy super closed set of  $X$  is fuzzy h-super closed in  $Y$ . [15].
2. Fuzzy rh-super continuous if the pre image of every fuzzy super closed set of  $X$  is fuzzy rh-super closed in  $Y$ . [17]

**Remark 1.4:** Every fuzzy super continuous mapping is fuzzy h-super continuous and every fuzzy h-super continuous mapping is fuzzy rh-super continuous but the converse may not be true. [17]

**Definition 1.5:** A collection  $\{F_\beta: \beta \in \Lambda\}$  of fuzzy mg-super open sets in a fuzzy topological space  $(Y, \mathfrak{T})$  is called a fuzzy rh-super open cover of a fuzzy set  $P$  of  $Y$  if  $P \leq \cup\{F_\beta: \beta \in \Lambda\}$ . [16]

**Definition 1.6:** A fuzzy topological space  $(Y, \mathfrak{T})$  is said to be fuzzy rh - super compact if every fuzzy rh-super open cover of  $Y$  has a finite subcover. [16]

**Definition 1.7:** A fuzzy set  $A$  of a fuzzy topological space  $(Y, \mathfrak{T})$  is said to be fuzzy rh- super compact relative to  $Y$  if every collection  $\{F_\beta: \beta \in \Lambda\}$  of fuzzy rh-super open subsets of  $Y$  such that  $P \leq \cup\{F_\beta: \beta \in \Lambda\}$  there exists a finite subset  $\Lambda_0$  such that  $A \leq \cup\{F_\beta: \beta \in \Lambda_0\}$ . [16]

**Definition 1.8:** A fuzzy topological space  $Y$  is fuzzy rh-connected if there is no proper fuzzy set of  $Y$  which is both fuzzy rh-super open and fuzzy mg-closed. [17]

## II. FUZZY RH -SUPER IRRESOLUTE MAPPINGS

**Definition 2.1:** A mapping  $f$  from a fuzzy topological space  $(Y, \mathfrak{T})$  to another fuzzy topological space  $(X, \mu)$  is said to be fuzzy rh -super irresolute if the pre image of every fuzzy rh-super closed set of  $X$  is fuzzy rh -super closed in  $Y$ .

**Theorem 2.1:** A mapping  $f : (Y, \mathfrak{T}) \rightarrow (X, \mu)$  is fuzzy rh -super irresolute if and only if the pre image of every fuzzy rh -super open set in  $X$  is fuzzy rh -super open in  $Y$ .

**Proof:** It is obvious because  $f^{-1}(U^c) = (f^{-1}(U))^c$ , for every fuzzy set  $U$  of  $Y$ .

**Remark 2.1:** Every fuzzy h-super closed set is fuzzy rh -super closed it is clear that every fuzzy rh -super irresolute mapping is fuzzy rh -super continuous but the converse may not be true.

**Definition 2.2:** A mapping  $f : (Y, \mathfrak{T}) \rightarrow (X, \mu)$  is said to be fuzzy regular super open if the image of every fuzzy regular super open set of  $X$  is fuzzy regular super open set in  $Y$ .

**Theorem 2.2:** Let  $f : (Y, \mathfrak{T}) \rightarrow (X, \mu)$  be bijective fuzzy regular super open and fuzzy rh-super continuous then  $f$  is fuzzy rh -super irresolute.

**Proof:** Let  $P$  be a fuzzy mg-super closed set in  $X$  and let  $f^{-1}(P) \leq F$  where  $F$  is fuzzy regular super open set in  $Y$ . Then  $P \leq f(F)$ . Since  $f$  is fuzzy regular super open and  $P$  is fuzzy rh-super closed in  $X$ ,  $\text{cl}(P) \leq f(F)$  and  $f^{-1}(\text{cl}(P)) \leq F$ . Since  $f$  is fuzzy mg-super continuous and  $\text{cl}(P)$  is fuzzy super closed in  $X$ ,  $\text{cl}(f^{-1}(\text{cl}(P))) \leq F$ . And so  $\text{cl}(f^{-1}(P)) \leq F$ . Therefore  $f^{-1}(P)$  is fuzzy rh-super closed in  $Y$ . Hence  $f$  is fuzzy rh -irresolute.

**Theorem 2.3:** Let  $f : (Y, \mathfrak{T}) \rightarrow (X, \mu)$  and  $g : (X, \mu) \rightarrow (Z, \eta)$  be two fuzzy rh -super irresolute mappings, then  $g \circ f : (Y, \mathfrak{T}) \rightarrow (Z, \eta)$  is fuzzy rh -super irresolute.

**Proof:** Obvious.

**Theorem 2.4:** Let  $f : (Y, \mathfrak{T}) \rightarrow (X, \mu)$  is fuzzy rh -super irresolute mapping, and if  $Q$  is fuzzy rh- super compact relative to  $Y$ , then the image  $f(Q)$  is fuzzy rh -super compact relative to  $X$ .

**Proof:** Let  $\{P_i: i \in \Lambda\}$  be any collection of fuzzy rh-super open set of  $X$  such that  $f(Q) \leq \cup\{P_i: i \in \Lambda\}$ . Then  $Q \leq \cup\{f^{-1}(P_i): i \in \Lambda\}$ . By using the assumption, there exists a finite subset  $\Lambda_0$  of  $\Lambda$  such that  $Q \leq \cup\{f^{-1}(P_i): i \in \Lambda_0\}$ . Therefore,  $f(Q) \leq \cup\{P_i: i \in \Lambda_0\}$ . Which shows that  $f(Q)$  is fuzzy rh- super compact relative to  $X$ .

**Theorem 2.5:** A fuzzy rh-super irresolute image of a fuzzy rh- super compact space is fuzzy rh-compact.

**Proof:** Obvious.

**Theorem 2.6:** If the product space  $(Y \times X, \mathfrak{T} \times \mu)$  of two non- empty fuzzy topological spaces  $(Y, \mathfrak{T})$  and  $(X, \mu)$  is fuzzy rh -super compact, then each factor space is fuzzy rh- super compact.

**Proof:** Obvious.

**Theorem 2.7:** Let  $f : (Y, \mathfrak{T}) \rightarrow (X, \mu)$  is a fuzzy rh -super irresolute surjection and  $(Y, \mathfrak{T})$  is fuzzy rh -super connected, then  $(X, \mu)$  is fuzzy rh -super connected.

**Proof:** Suppose  $Y$  is not fuzzy rh -connected then there exists a proper fuzzy set  $G$  of  $Y$  which is both fuzzy rh -super open and fuzzy rh -closed, therefore  $f^{-1}(G)$  is a proper fuzzy set of  $X$ , which is both fuzzy rh -super open and fuzzy rh -closed, because  $f$  is fuzzy rh -super continuous surjection. Therefore  $X$  is not fuzzy rh -connected, which is a contradiction. Hence  $Y$  is fuzzy rh -super connected.

## III. REFERENCES

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