JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

mg*s-IRRESOLUTE MAPS AND mg*s-HOMEOMORPHISM IN MINIMAL STRUCTURE

[1]K. Anitha, [2]B. Sujatha

[1]Assistant Professor, [2]Assistant Professor
[1,2] Department of Science and Humanities
[1,2] Hindusthan College of Engineering and Technology, Coimbatore, India.

Abstract: Perachi Sundari and Latha Martin introduced properties of g*s-irresolute and g*s-homeomorphism in topological spaces. In this paper, we introduced mg*s-irresolute and mg*s-homeomorphism and study their basic properties in minimal structure.

***Index Terms* - mg*s-closed sets, mg*s-open sets, mg*s-continuous map, mg*s-irresolute map and mg*s-homeomorphism.**

I. INTRODUCTION

In 1970, Levine [2] introduced the notion of generalized closed (briefly g-closed) sets in topological spaces and showed that compactness, locally compactness, countably compactness and normality etc are all g-closed hereditary. Recently many modifications were defined and investigated. In 2006 [8] Takashi Noiri introduced the concept is of mg-closed sets on minimal spaces. We introduced the properties of g*s-closed sets in topological space, mg*s-closed sets [5,4] and mg*s-continuous functions in minimal structures. M.Perachi Sundari and Latha Martin [3] introduced g*s-irresolute maps and g*s-homeomorphism in topological spaces. The notion homeomorphism plays a very important role in topology. In this paper, we introduce a new class of irresolute map called mg*s-irresolute map and then we study mg*s-homeomorphism and mg*sc-homeomorphism.

II. PRELIMINARIES

Definition: 2.1 [5] A subset of a topological space (X,τ) is called g*s-closed set [5] if scl(A)⊆U Whenever A⊆U,U is gs-open in X.

Definition: 2.2 [10] A function f: $(X, m_X) \rightarrow (Y, m_Y)$ is said to be mg*s-continuous if the inverse image of every m_Y -closed set in (Y, m_Y) is mg*s-closed in (X, m_X).

Definition: 2.3 [7] A map f: X→Y from a topological space X into a topological space Y is called g*s-continuous if the inverse image of every closed set in Y is g*s-closed in X.

Remark: 2.4 [7] Every continuous map is g*s- continuous and g*s-continuous map is gs-continuous.

Definition: 2.5 [7] A map f: X→Y is said to be strongly g*s-continuous if the inverse image of every g*s-open set in Y is open in X.

Definition: 2.6 [7] A topological space X is g*s-compact if every g*s-open cover of X has a finite sub cover of X.

Definition: 2.7 [6] A topological space X is called a g*s-connected if X cannot be written as a disjoint union of two non-empty g*s-open sets.

Definition: 2.8 [1] A bijection f: $(X, \tau) \rightarrow (Y, \sigma)$ is called a homeomorphism if f is both continuous and open.

Definition: 2.9 [1] A bijection f: $(X, \tau) \rightarrow (Y, \sigma)$ is called a gs- homeomorphism if f is both gs-continuous and gs-open.

Definition: 2.10 [7] A subset B of a topological space X is called g*s-compact relative to X, if for every collection $\{A_i;\ i \in I\}$ of g*s-open subsets of X such that $B \subseteq \cup\, i \in A_i$.

Definition: 2.11[3] A map f: $(X, \tau) \rightarrow (Y, \sigma)$ is called g*s-irresolute if the inverse image of every g*s-closed set in Y is g*s-closed in X.

Definition: 2.12 [3] A bijection f: $(X,\tau) \rightarrow (Y,\sigma)$ is called a g*s-homeomorphism if f is both g*s-open and g*s-continuous.

Definition: 2.13[3] A bijection f: $(X, \tau) \rightarrow (Y, \sigma)$ is called a g*sc-homeomorphism if both f and f^{-1} are g*s-irresolute.

Definition: 2.14[8] A sub family m_X of the power set P(X) of a non-empty set X is called a minimal structure (briefly m-structure) on X if $\emptyset \in m_X$ and $X \in m_X$. By (X, m_X), we denote a non-empty set X with a minimal structure m_X on X and call it an m-space. Each a member of m_X is said to be m_X-open and the complement of a m_X-open set is said to be m_X-closed.

Definition: 2.15 [8] Let X be a non-empty set and m_X an m-structure on X. For a subset A of X, the m_X-closure of A and the m_X -interior of A are defined in [8] as follows:

(i) m_X-cl(A) = $\cap\{ F ; A \subset F, X - F \in m_X\}$
(ii) m_X-int(A) = $\cup\{ F ; \cup \subset A, X - \cup \in m_X\}$

Definition: 2.16 [8] A minimal structure m_X on a non-empty set X is said to have property B if the union of any family of subsets belong to m_X

Lemma: 2.17 [8] Let X be non-empty set and m_X a minimal structure on X satisfying property B. For a subset A of X, the following properties hold:

(i) $A \in m_X$ if and only if m_X-int (A) = A

(ii) A is m_X-closed if and only if m_X-cl(A) = A

(iii) m_X-int (A) $\in m_X$ and m_X-cl(A) is m_X-closed.

III. mg*s - IRRESOLUTE MAPS IN MINIMAL STRUCTURES

In this section, we introduce the concepts of mg*s-irresolute maps in minimal structures.

Definition: 3.1 A map f: $(X, m_X) \rightarrow (Y, m_Y)$ is said to be mg*s-irresolute if the inverse image of every mg*s-closed set in Y is mg*s-closed set in X.

Theorem: 3.2 A map f: $(X, m_X) \rightarrow (Y, m_Y)$ is mg*s-irresolute if and only if for every mg*s-open A of Y, f^{-1}(A) is mg*s-open in X.

Proof: Necessity: If f: $(X, m_X) \rightarrow (Y, m_Y)$ is irresolute, then for every mg*s-closed B of (Y, m_Y) , f^{-1}(B) is mg*s-closed in X. If A is any mg*s-open subset of Y, then A^c is mg*s-closed. Thus $f^{-1}(A^c)$ is mg*s-closed, but $f^{-1}(A^c) = f^{-1}(A^c)^c$. So that f^{-1}(A) is mg*s-open in X.

Sufficiency: If for all mg*s-open subsets A of (Y, m_Y), f^{-1}(A) is mg*s-open in (X, m_X) and if B is any mg*s-closed subset of (Y, m_Y) then B^c is mg*s-open. Also $f^{-1}(B^c) = (f^{-1}(B))^c$ is mg*s-open in X. Thus f^{-1}(B) is mg*s-closed in (X,m_X). Hence f is mg*s-irresolute.

Theorem: 3.3 If a map f: $(X, m_X) \rightarrow (Y, m_Y)$ is mg*s-irresolute, then it is mg*s-continuous.

Proof: Let A be a m_Y-closed in (Y, m_Y). Since every m_Y-closed set is mg*s-closed, A is mg*s-closed in (Y, m_Y). Since f is mg*s-irresolute, $f^{-1}(A)$ is mg*s-closed in (X, m_X). Hence f is mg*s-continuous.

Remark: 3.4 The converse need not be true as seen from the following example.

Example: 3.5 Let X = Y = {a, b, c}, $m_X = \{\emptyset, X, \{b\}, \{c\},\{b, c\}\}$ and $\sigma = \{\emptyset, X, \{b\}\}$. Let f: $(X, m_X) \rightarrow (Y, m_Y)$ be defined by f(b) = f(c) = a and f(a) = c. Then f is mg*s- continuous, {a} is g*s-closed in (Y, m_Y) but $\{f^{-1}$(b)}={a} is not mg*s-closed in (X, m_X). Therefore f is not mg*s-irresolute.

Theorem: 3.6 If f : $(X, m_X) \rightarrow (Y, m_Y)$ and g : $(Y, m_Y) \rightarrow (Z, m_Z)$ are both mg*s-irresolute then g∘f : $(X, m_X) \rightarrow (Z, m_Z)$ is irresolute.

Proof: Let A be a mg*s-open subset of (Z, m_Z). Since g is mg*s-irresolute, g^{-1}(A) is mg*s-open in (Y, m_Y). Since f is mg*s-irresolute, $f^{-1}(g^{-1}(A))$ is mg*s-open in (X, m_X). Thus $(g∘f)^{-1}$ is mg*s-open in (X, m_X). Hence g∘f is mg*s-irresolute.

Theorem: 3.7 Let (X, m_X), (Y, m_Y) and (Z, m_Z) be any minimal spaces. For any mg*s-irresolute map f:$(X,m_X) \rightarrow (Y,m_Y)$ and any mg*s-continuous map g:$(Y,m_Y) \rightarrow (Z,m_Z)$. Then the composition, g∘f : $(X, m_X) \rightarrow (Z, m_Z)$ is mg*s-continuous.

Proof: Let F be a m_Z-closed set in (Z, m_Z). Since g is mg*s-continuous, g^{-1}(F) is mg*s-closed in (Y, m_Y). Since f is mg*s-irresolute $f^{-1}(g^{-1}(F))$ is mg*s-closed in (X, m_X). Thus $(gof)^{-1}(F) = f^{-1}(F)$ is mg*s-closed in (X,m_X). Hence g∘f is mg*s-continuous.

Theorem: 3.8 If a map f: $(X, m_X) \rightarrow (Y, m_Y)$ is mg*s-irresolute and a subset B of X is mg*s-compact relative to (X, m_X), then the image f(B) is mg*s-compact relative to (Y, m_Y).

Proof: Let $\{A_i;\ i \epsilon I\}$ be any collection of mg*s-open subsets of (Y,m_Y) such that $f(B) \subset \cup\{A_i;\ i\epsilon I\}$. Then $B \subset \cup\{f^{-1}(A); i \in I\}$ holds. By hypothesis, there exists a finite subset I_o of I such that $B \subset \cup \{f^{-1}(A); i \in I\}$. Therefore we have $f(B) \subset \cup\{A_i;\ i\epsilon I\}$ which shows that f(B) is g*s-compact relative to (Y,m_Y).

Theorem: 3.9 If f: $(X, m_X) \rightarrow (Y, m_Y)$ is mg*s-irresolute surjection and (X, m_X) is mg*s-connected, then (Y, m_Y) is mg*s-connected.

Proof: Suppose (Y, m_Y) is mg*s-connected. Let Y = A ∪ B where A and B are disjoint non-empty mg*s-open set in Y. Since f is mg*s-irresolute and onto, X = f^{-1}(A) ∪ f^{-1}(B) when f^{-1}(A) and f^{-1}(B) are disjoint non-empty and mg*s-open in (X, m_X). This contradicts the fact that X is mg*s-connected. Hence (Y, m_Y) is mg*s-connected.

IV. mg*s- HOMEOMORPHISM IN MINIMAL SPACES

In this section, we introduce the new homeomorphisms namely mg*s-homeomorphism mg*sc-homeomorphism and study some of their properties.

Definition: 4.1 A bijection f: (X, m_X)→(Y, m_Y) is called a mg*s-homeomorphism if f is both mg*s-open and mg*s-continuous. We denote the family of all mg*s- homeomorphisms of (X, m_X) onto itself by mgs*-h (X, m_X).

Theorem: 4.2 Every homeomorphism is a mg*s- homeomorphism
Proof: Let f: (X, m_X) →(Y, m_Y) be a homeomorphism. To prove that f is mg*s-homeomorphism. Since f is homeomorphism, f is bijection and also f is but m-open and m-continuous. Since every m-open map is mg*s-open and every m-continuous map is mg*s-continuous, f is bijection, mg*s-open and mg*s-continuous. Hence f is mg*s-homeomorphism.

Remark: 4.3 The converse of the above theorem 4.2 need not be true as seen from the following example.

Example: 4.4 Consider X = Y = {a, b, c}, m_X = {∅, x, {b, c}}, m_Y = {∅, x,{b}}. Let f: (X, m_X)→(Y, m_Y) be an identity map. Then f is mg*s-homeomorphism but not homeomorphism. Since {b, c} is m-open in (X, m_X) but the image is not m_Y-open in (Y, m_Y).

Theorem: 4.5 Every mg*s-homeomorphism is mgs-homeomorphism
Proof: Let f:(X, m_X)→(Y, m_Y) be a mg*s-homeomorphism. Since f is g*s-homeomorphism, f is bijection and also f is both mg*s-open and mg*s-continuous. Since every mg*s-open is mgs-open and every mg*s-open is mgs-open and every mg*s-continuous map is mgs-continuous. We have f is mgs-open, mgs-continuous and bijection. Hence f is mgs-homeomorphism.

Remark: 4.6 The converse of the above theorem need not be true as seen from the following example.

Example: 4.7 Consider X = Y = {a, b, c}, m_X={∅,x,{a}} m_Y ={∅,Y,{a},{c},{a, c}}. Let f: (X,m_X)→(Y,m_Y) be an identity map. Then f is mgs- homeomorphism but not mg*s- homeomorphism. Since {a, b} is m_Y-closed but f^{-1}(a, b) = (a, b) is not mg*s-closed in (X,m_X).

Theorem: 4.8 Let f: (X,m_X)→(Y,m_Y) be a bijective and mg*s-continuous map, then the following are equivalent.
(a) f is mg*s-open map
(b) f is mg*s- homeomorphism
(c) f is mg*s-closed map.
Proof:(a)⟹(b): Suppose that f is mg*s-open map. By hypothesis, f is bijective and mg*s-continuous map. By definition of mg*s- homeomorphism, f is mg*s- homeomorphism.

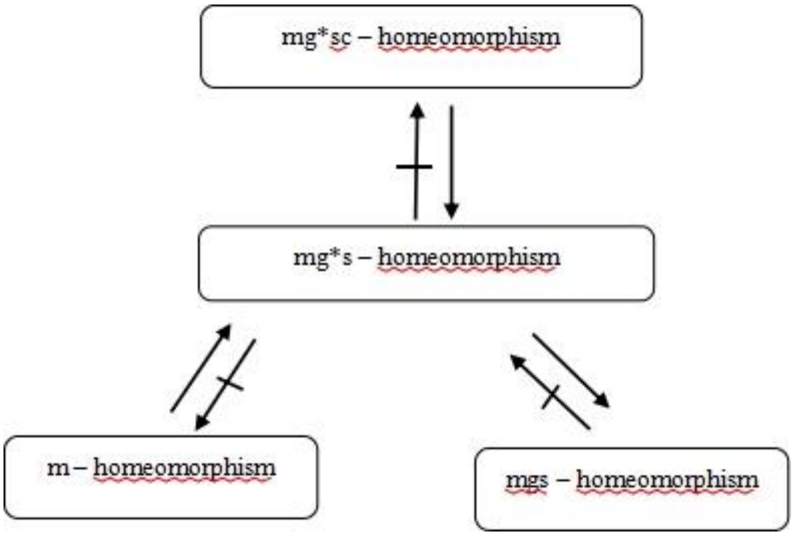
(b)⟹(c): Suppose that f is mg*s- homeomorphism. Since f is mg*s- homeomorphism, f is bijective and also f is mg*s- open and mg*s- continuous. Let F be a m_X-closed set of (X,m_X). Then F^c is m_X-open set in (X,m_X). Since f is mg*s- open map, f(F^c) is mg*s- open in (Y,m_Y). f(F^c) = $(f(F))^c$ is mg*s- open in (Y,m_Y). Thus f(F) is mg*s- closed in (Y,m_Y). Hence f is mg*s-closed map.
(c) ⟹ (a): Suppose that f is mg*s-closed map. Let A be a m_X-closed map. Let A be a m_X-closed set in (X, m_X). Since f is mg*s-closed map, f(A) is mg*s-closed set in (Y,m_Y). F(A) = $(f^{-1})^{-1}$ (A) is mg*s-closed set in (Y,m_Y), which implies f^{-1} is mg*s-continuous on (Y,m_Y).

Definition: 4.9 A bijection f: (X,m_X)→(Y,m_Y) is said to be mg*s-c homeomorphism if both f and $f^{-1}$are mg*s-irresolute. We denote the family of all mg*sc-homeomorphism of (X,m_X) onto itself by mg*sc-h(X,m_X).

Theorem: 4.10 Every mg*sc- homeomorphism is mg*s-homeomorphism.
Proof: Let f:(X,m_X)→(Y,m_Y) be a mg*sc-homeomorphism. Since f is mg*sc-homeomorphism, f and f^{-1} are mg*sc-homeomorphism, f and f^{-1} are mg*s-irresolute and f is bijective. Now, let V be a m_Y-closed in (Y,m_Y). Since f is mg*s-irresolute, f^{-1} (V) is mg*s-closed in (X,m_X). Thus f is mg*s-continuous. Since f^{-1} is mg*s-irresolute, $(f^{-1})^{-1}$(V) is mg*s-closed in (X,m_X). Therefore f^{-1} is mg*s-continuous. By theorem, f is mg*s-open. Hence f is mg*s-homeomorphism.

Theorem: 4.11 Let f :(X,m_X)→(Y,m_Y) and g:(Y,m_Y)→(Z,m_Z) are mg*sc- homeomorphism. Then their composition g∘f: (X,m_X)→(Z,m_Z) is also mg*sc-homeomorphism.
Proof: Let U be a mg*s-open set in (Z,m_Z). Since g is mg*s-irresolute, g^{-1}(U) is mg*s-open in (Y,m_Y). Since f is mg*s-irresolute, f^{-1} (g^{-1}(U)) = $(g∘f)^{-1}$(U) is mg*s-open in (X,m_X). Therefore, g∘f is mg*s-irresolute. Also for a mg*s-open set G in (X,m_X), we have (g∘f)(G) = g(f(G)) = g(W), where W = f(G). By hypothesis, f(G) is mg*s-open in (Y,m_Y) and so again by hypothesis, g(f(G)) is mg*s-open set in (Z,m_Z). (g∘f) (G) is mg*s-open set in (Z, m_Z) $(g∘f)^{-1}$ is mg*s-irresolute and also g∘f is bijective. Hence g∘f is mg*sc- homeomorphism.

Remark: 4.12 The following diagram shows that the relationships between mg*s–homeomorphism and other homeomorphism in minimal spaces.



REFERENCES

[1] Devi, R. Maki, H. and Balachandran, K. 1995. Semi generalized homeomorphisms and generalized semi homeomorphisms in topological spaces. Indian Journal of Pure Applied Mathematics, 26(3): 271-284.

[2] Levine, N. 1970. Generalized closed sets in topology. Rend.circ.mat.Palermo,19: 89-96.

[3] Perachi Sundari, M. and Latha Martin. 2014. g*s-irresolute maps and g*s-homeomorphism in topological spaces. International Journal of Mathematical Archive ,5(8): 15-20.

[4] Pushpalatha, A. and Anitha, K. 2012. g*s-closed sets in minimal structure. Proceedings of the International Conference on Mathematics In Enggineering and Business Management, 495-498.

[5] Pushpalatha, A. and Anitha, K. 2011. g*s-closed sets in topological spaces. Int. J. Contemp. Math. Sciences, 6(19): 917-929.

[6] Pushpalatha, A. and Anitha, K. 2012. g*s-Connectedness and g*s-locally closed sets in topological spaces. International Journal of Mathematical Archive, 3(7): 2815-2818.

[7] Pushpalatha, A. and Anitha, K. 2012. Strongly g*s-continuous maps and perfectly g*s-continuous in topological spaces. International Mathematical Forum, 7.(24): 1201-1207.

[8] Noiri, T.2006. 11th meetings on Topological spaces and its Applications. Fukuoka Univ.ser House, 1-9.

[9] Popa, V. and Noiri, T. 2002. A Unified Theory Of Weak Continuity For Functions. Ren di conti del circolo Mathematics di Palermo, 51(2): 439-464.

[10] Popa, V. and Noiri, T. 2001. On the definitions of some generalized forms of continuity under minimal conditions. Mem. Fac. Sci. Kochi Univ. Ser A Math, 22: 9-18.