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Behaviours of Bianchi Type-III Dark energy cosmological model universe with Polytropic Equation of state in Lyra manifold

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Abstract : In this paper, we study the dynamical aspects of the perfect fluid Dark energy (DE) cosmological model with polytropic equation of state (EoS) in an axially symmetric Bianchi type-III space- time in Lyra geometry in five dimensional space-time. Using valid conditions, an exact cosmological model is presented by solving Einstein field equations. All the dynamical and geometrical parameters of the cosmological model are discussed and their physical significance in modern cosmology are evaluated.

***Keywords*: Bianchi type-III model; Dark energy; Polytropic equation of state; Lyra Geometry.**
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I. Introduction

We considered the importance of studying the Bianchi Type-III universe with higher dimension in Lyra Manifold with perfect fluid. Assuming the relation between scalar expansion (θ) and shear scalar (σ), we have obtained the solutions of the Einstein's field equations (EFE). This perfect fluid obeys the polytropic equation of state $p = a\rho^m - \rho$. The physical and cosmological properties of the model are discussed. To incorporate certain desirable physical features into general theory of relativity (GTR) various modifications have been made in Einstein's theory and Riemannian geometry. Based on the generalization of Riemannian geometry, the first so-called unified field theory (UFT) was formulated by Weyl[1] in 1918. The UFT was never taken seriously as it is based on the concept of non-integrability of length transfer. Later, another modification of Riemannian geometry into the structure less manifold with a gauge function φ_μ, as a result of which the cosmological constant arises quite naturally from the geometry was proposed by Lyra[2] in 1951. In Lyra geometry the connection is metric preserving and length transfers are integrable, which is different from that of Weyl's geometry. Based on Lyra's geometry, a new scalar–tensor theory of gravitation was proposed by Sen[3] and Sen and Dunn[4] and developed an analog of the Einstein field equations (EFE). It is, thus, possible to construct a geometrized theory of gravitation and electromagnetism much along the lines of Weyl's "unified" field theory but without the inconvenience of non-integrability of length transfer. In the framework of Lyra Geometry, Halford[5] analysed a cosmolog- ical theory and pointed out that the constant displacement vector field φ_μ in this geometry plays the role of cosmological constant Λ in general theory of relativity (GTR) in 1970. In addition to the above, Bhamra,[6] Beesham,[7] Singh and Singh,[8,9] Rahaman and Bera,[10] Rahaman et al.,[11] Singh et al.,[12] Mohanty et al.,[13, 14] Singh and Mollah,[15] Dubey et al.[16] Baro et al.[17] constructed several cosmological models in Lyra's manifold.

Nowadays, many researchers and authors have studied, analysed and constructed cosmological universe in various dimensions. It is interesting and thrilling to study cosmological universe in higher dimensional space-time in the framework of both general relativity and Lyra geometry. Studying the cosmological models in higher dimensional space-time, it gives us a notion that due to the accelerated expansion of the universe our present universe is much more greater than the universe at the early stage of evolution. In an attempt to unify electromagnetism with gravi- tation, higher dimensional model was introduced by Kaluza[18] and Klein[19] known as Kaluza-Klein theory (KK-theory) in physics. Higher dimensional model can be considered as a tool to demonstrate the late time expedited expanding paradigm (Banik and Bhuyan[20]). Investigation of higher dimensional space-time can be in- terpreted as a task of utmost importance as the universe might have come across a higher dimensional era during the initial epoch (Singh et al.[21]). Marciano[22] asserts that the detection of a time varying fundamental constants can possibly show us the proof for extra dimensions. Extra dimensions generate huge amount of entropy which gives possible solution to flatness and horizon problem are mentioned in.[23, 24] As we are living in a four dimensional space-time, the hidden extra dimension in five dimensional is highly likely to be related with the invisible Dark Matter (DM) and Dark Energy (DE) (Chakraborty and Debnath[25]). A massive string cosmo- logical model in higher dimensional homogeneous space-time was constructed by Chatterjee.[26] Rahaman et al.[27] derived an exact solutions of the field equations for a higher dimensions in Lyra manifold when the source of gravitation are taken to be massive strings. Mohanty and Samanta[28] constructed higher dimensions string cos- mological models with massive scalar field in Lyra manifold and obtained that the models avoid initial singularity due to the presence of massive scalar field. Samanta et al.[29] formulated some five dimensional Bianchi type-III cosmological models in general theory of relativity (GTR) with massive string as a source of gravitational field. Within the framework of Lyra geometry, in 2010 Bianchi type-III anisotropic universes with cloud of strings was derived by Yadav et al.[30] In 2017, within the same framework *i.e.*, Lyra Manifold, Bianchi type-III string cosmological universe with bulk viscous fluid was investigated by Sahoo et al.[32] In different Bianchi type space-times, some researchers *viz.*, Kaiser,[33] Banerjee,[34] Bali,[35] Goswami,[36] Sahoo,[37] Reddy,[38] Khadekar,[39–41] Samanta and Debata,[42] Ladke[43] Singh,[44] studied some cosmological models with various modified theories of relativity. In addition to the above mentioned researchers, Choudhury,[45] Tripathi,[46] Tiwari et al.,[47] Ram et al.,[48] Mollah et al.,[49] Singh and Baro,[50] Baro and Singh[51] studied string cosmo- logical models in numerous contexts and in various space-times.

Motivated by the above discussion, here we constructed the higher dimensional cosmological universe in Bianchi type-III space-time with polytropic equation of state (EoS) within the framework of Lyra Geometry with perfect fluid, where the survival field equations are solved by assuming some simplifying assumptions. Also, the geometrical and physical properties of some parameters of our model universe (i.e., Five Dimensional Bianchi Type-III) are discussed with the help of graphical representation. In section 2, we considered the line-element, the polytropic equation of state and field equations, the stress-energy-momentum tensor for perfect fluid of the model. Section 3 deals with the dynamical parameters such as pressure (p), density(ρ), Hubble's parameter (H), the expansion scalar (θ), shear scalar (σ), the anisotropy parameter (Δ), deceleration parameter (q) etc. are presented. Section 4 concludes with brief discussion of the dynamical parameters and also the graphical representations of variations of various physical quantities against time are analyses and discussed. In Section 5 the concluding remarks are given.

II. FIELD EQUATIONS AND SOLUTION OF THE MODEL

Let us consider a five-dimensional Bianchi type-III line-element in the form

$$ds^2 = A^2(t)(dx^2 + e^{-2\alpha x}dy^2 + dz^2) + B^2(t)d\psi^2 - dt^2, \tag{1}$$

where $A(t)$ and $B(t)$ are the scale factor and $\alpha \neq 0$ is a constant. Here, we consider the fifth coordinate ψ as space-like and the spatial curvature is taken as zero (Gron[52] and Rao et al.[53]).

Einstein's field equations, with normal guage in the framework of the Lyra geometry (Sen[3]) are

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\emptyset_i\emptyset_j - \frac{3}{4}g_{ij}\emptyset_k\emptyset^k = -T_{ij} \tag{2}$$

where $8\pi G = 1$ and $c = 1$ in geometrized unit and $\emptyset_i$ is the displacement vector field of the geometry define as

$$\emptyset_i = (0, 0, 0, 0, \beta(t)), \tag{3}$$

and T_{ij} is the stress-energy-momentum tensor for the perfect fluid given by

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij} \tag{4}$$

where energy density and fluid pressure are respectively given by ρ and p. And, $u^i = (0, 0, 0, 0, 1)$ is the five velocity vector satisfying

$$T_1^1 = T_2^2 = T_3^3 = T_4^4 = -p,\ T_5^5 = -\rho \text{ and } T_5^5 = 0, \forall\, i \neq j,$$
$$g_{ij}u^i u^j = u_i u^j = -1 \tag{5}$$

Also, we have

$$T^{ij}_{;j} = 0 \tag{6}$$

Here, ordinary and covariant derivatives are denoted by comma and semicolon respectively. Physically, eqn. (6) describes the energy conservation of the field and equation of motion. Again, the general form of equation of state is given by $p = p(\rho)$. Therefore in order to study the anisotropy problem, the choice of the equation of state is an automatic choice quite natural. Without loss of generality, we choose the polytropic equation of state (EoS) as

$$p = a\rho^m - \rho \tag{7}$$

where polytropic index and polytropic constant are a and m respectively. The polytropic constant a can have the positive value for radiation and stiff fluid, the zero value for dust and the negative value for inflationary scenario (Adhav et al.[54]). The physical quantities for the metric (1) like volume (V), average scale factor (R), expansion scalar (θ), Hubble's parameter (H), shear scalar (σ^2), anisotropic parameter (Δ), deceleration parameter (q) which are very important in order to explain the behaviour of the universe are defined as

$$V = R^4 = A^3 B, \tag{8}$$

$$\theta = u^i_{;i} = \frac{3\dot{A}}{A} + \frac{\dot{B}}{B}, \tag{9}$$

$$H = \frac{\dot{R}}{R} = \frac{1}{4}\theta = \frac{1}{4}\left(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}\right), \tag{10}$$

$$\sigma^2 = \frac{3}{8}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2, \tag{11}$$

$$\Delta = \frac{1}{4H^2}\left(3\frac{\dot{A}^2}{A^2} - \frac{\dot{B}^2}{B^2}\right) - 1, \tag{12}$$

$$q = \frac{d}{dt}\frac{1}{H} - 1, \tag{13}$$

Using co-moving coordinate system, the field equation (1) with the help of eqns (2) and (4) can be written as

$$\frac{2\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{3}{4}\beta^2 = -p, \tag{14}$$

$$\frac{2\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} + \frac{3}{4}\beta^2 = -p, \tag{15}$$

$$\frac{3\ddot{A}}{A} + \frac{3\dot{A}^2}{A^2} - \frac{\alpha^2}{A^2} + \frac{3}{4}\beta^2 = -p, \tag{16}$$

$$\frac{3\dot{A}^2}{A^2}+3\frac{\dot{A}\dot{B}}{AB}-\frac{\alpha^2}{A^2}-\frac{3}{4}\beta^2=\rho, \quad (17)$$

where an overhead dot indicates derivatives with respect to cosmic time t.

The set of field equations (14) - (17) reduces to the following system of independent equations

$$\frac{2\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}^2}{A^2}+2\frac{\dot{A}\dot{B}}{AB}+\frac{3}{4}\beta^2=-p, \quad (18)$$

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+2\frac{\dot{A}^2}{A^2}-2\frac{\dot{A}\dot{B}}{AB}=0, \quad (19)$$

$$\frac{3\dot{A}^2}{A^2}+3\frac{\dot{A}\dot{B}}{AB}-\frac{\alpha^2}{A^2}-\frac{3}{4}\beta^2=\rho, \quad (20)$$

Since the equations (18) - (20) represents a system of three-independent equations involving five unknowns A, B, β, ρ and p so in order to obtain deterministic solution two more physical conditions involving these variables are required.

The shear scalar (σ^2) given in (11) is proportional to the scalar expansion (θ), so that we may take (Collins et al.,[55] Kiran et al.[56])

$$A = B^n \quad (21)$$

where $n \neq 0$ is a constant and it will describe the anisotropy of the space-time and the equation of state (EoS) parameter in the polytropic form is given by

$$p = a\rho^m - \rho \quad (22)$$

From equation (19 and (21) we get

$$A = [(3n+1)t]^{\frac{n}{3n+1}}, \quad (23)$$

$$B = [(3n+1)t]^{\frac{1}{3n+1}}, \quad (24)$$

By the use of the equations (24) and (25), the metric can be written as

$$ds^2=[(3n+1)t]^{2n/3n+1}(dx^2+e^{-2\alpha x}dy^2+dz^2)+[(3n+1)t]^{2/3n+1}d\psi^2-dt^2 \quad (25)$$

The equation (25) is a five dimensional Bianchi type-III cosmological universe interacting with perfect fluid with polytropic equation of state (EoS).

III. DYNAMICAL PARAMETERS OF THE MODEL

Using equation (22), equation (18) and (20) we have

$$\rho = [-\frac{\alpha^2}{a}(3nt+t)^{\frac{-2n}{3n+1}}]^{\frac{1}{m}}, \quad (26)$$

where a is the arbitrary constant.
Using equation (26), equations (22) and (20) becomes

$$p = -\alpha^2(3nt+t)^{\frac{-2n}{3n+1}}-[-\frac{\alpha^2}{a}(3nt+t)^{\frac{-2n}{3n+1}}]^{\frac{1}{m}}, \quad (27)$$

$$\frac{3}{4}\beta^2 = \frac{3n(n+1)}{(3n+1)^2t^2}-\alpha^2(3nt+t)^{\frac{-2n}{3n+1}}-[-\frac{\alpha^2}{a}(3nt+t)^{\frac{-2n}{3n+1}}]^{\frac{1}{m}}, \quad (28)$$

From (8), we get the Spatial Volume (V) as

$$V = (3n+1)t. \quad (29)$$

The expansion scalar (θ) which determines the volume behaviour of the fluid is given by

$$\theta = \frac{1}{t}, \quad (30)$$

At the initial epoch $t \to 0$, when $\theta \to \infty$ and $\theta \to 0$ when $t \to \infty$. Hubble's parameter (H) is given by

$$H = \frac{1}{4t}, \quad (31)$$

Shear scalar (σ^2) is given by

$$\sigma^2 = \frac{3(n-1)^2}{8(3n+1)^2t^2}, \quad (32)$$

From (31) and (33), we obtain

$$\lim_{n\to\infty}\frac{\sigma^2}{\theta^2}=\frac{3(n-1)^2}{8(3n+1)^2}=constant; \ \ for\ n\neq 1\ and\ n\neq -\frac{1}{3}. \quad (33)$$

Therefore, the model does not approach isotropy for large value of t for $n \neq 1$

And $n \neq -\frac{1}{3}$ (Asgar and Ansari[57]) but it approaches to isotropy for $n = 1$.

$$\Delta = \frac{3(n-1)^2}{(3n+1)^2} = constant; \;\; for\, n \neq 1\; and\, n \neq -\frac{1}{3}. \tag{34}$$

Therefore, the model (26) has constant anisotropy parameter throughout the evolution of the universe for $n \neq 1\; and\, n \neq -\frac{1}{3}$ (Asgar and Ansari[57]) and it approaches to isotropy for $n = 1$.

The expression of deceleration parameter using (31) in (13), we get

$$q \;=\; 3. \tag{35}$$

Due to the above reason, it can be easily seen that the value of the deceleration parameter q (35) is positive constant (i.e., 3) which indicate that our model universe(25) decelerates in the standard way. However, in early stage of the evolution of the universe the Bianchi type models represents cosmos and though the universe decelerates in the standard way in early universe it will accelerate in finite time due to cosmic recollapse where the model universe in turns inflates “decelerates and then accelerates”(Kandalkar and Samdurkar[58]).

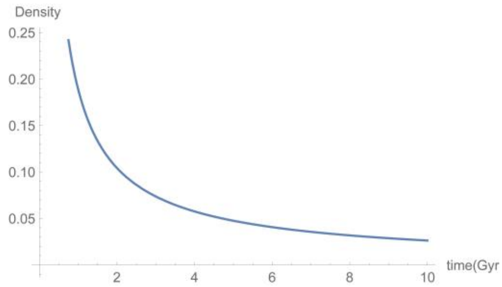

Fig. 1. Variation of ρ vs. time t.

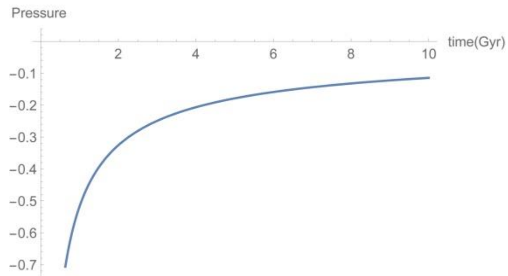

Fig. 2. Variation of p vs. time t.

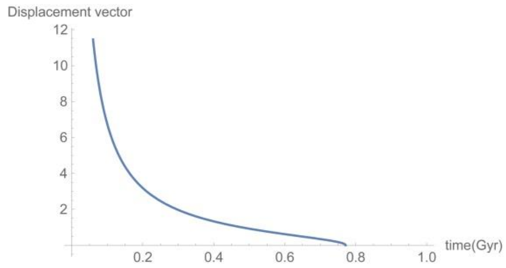

Fig. 3. Variation of Displacement vector vs. time t.

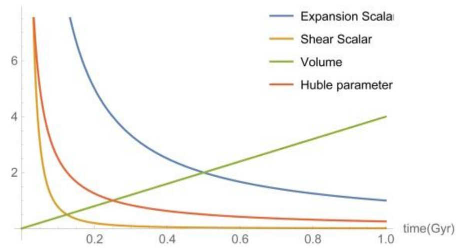

Fig. 4. Variation of θ, σ^2, V, and H vs. time t.

IV. Physical discussion

For realistic results, specific values of the arbitrary constants involved are chosen $m = \frac{2}{3}, \alpha = 1, n = 2, a = -1$. The graph of the cosmological parameters with respect to time are presented with the detailed discussion.

From Fig. (1) and Eqn. (26) we have observed that the energy density ρ is infinite at time $t = 0$ and decreases as time increases and become zero at $t \to \infty$.

While from the equation (27) and Fig. (2) shows that the pressure approaches negative infinity as $t \to 0$ and the pressure $p = 0\; at\; t \to \infty$. This strong negative pressure indicates the presence of DE in our universe.

The gauge function (β) is infinite at the beginning and gradually decrease as time increases and ultimately tend to zero at finite time as shown in the Fig. 3 and therefore it behaves like cosmological constant Λ is shown in equation (28) and fig. (3)

The spatial volume (V) of the model (29) is 0 at initial epoch $t = 0$, and it increases w.r.t time and becomes $\infty\; when\; t \to \infty$ which shows that with the evolution of time, our model universe is expanding, which is clearly shown in Fig. (4).

The expansion scalar $\theta \to \infty$ and the Hubble Parameter $H \to \infty$ at initial epoch $t = 0$, and as the time progresses gradually both of them decreases and finally they become 0 when $t \to \infty$ (as shown in the Fig. (4)). Hence the model shows that the universe is expanding with the increase of time but the rate of expansion slower as time increases and the expansion stops at $t \to \infty$.

It is observed that the value of deceleration parameter (q) is positive constant which indicates that our model universe (25) decelerates in the standard way, which is not according to the present day observational scenario of accelerating universe. On the other hand, it may be noted that Bianchi type models represent cosmos in its initial stage of evolution. However, though the universe, decelerates in the standard way it will accelerate in finite time due to the cosmic recollapse where the universe in turns inflates "decelerates and then accelerates"(Kandalkar and Samdurkar[58]). The decelerating behavior of the expansion in early stage and the accelerating behavior of the expansion of present universe has been indicated by many cosmological observations such as cosmic microwave background (CMB), clusters of galaxies and type Ia supernovae etc. They suggest that the reason of this transition from decelerating to acceleration may be due to the presence of an anti-self-attraction of matter. This shows that the universe attains isotropy at late times and transits to the accelerating universe which is consistent with the present day observational data such as cosmic microwave background (CMB) and type Ia supernovae etc. The transition from decelerating to accelerating model of the universe. Thus, our model describes a transition from decelerating to accelerating in an evolving universe which is in agreement with the present observational data.

In equation (32) and Fig. (4), the value of the shear scalar $\sigma \to \infty$ at initial epoch and it decreases as the time increases and become zero at late universe showing that the universe shown is shearing in early epoch and it is shear free in the present epoch as well as late time universe. The model is shear free throughout the evolution of the universe for $n = 1$.

The mean anisotropy parameter (Δ) is constant $for\ n \neq 1\ and\ n \neq -\frac{1}{3}$ which shows that the model was anisotropic in earlier epoch and becomes isotropic in the late time for $n = 1$. Also at $t \to \infty$ the value of $\frac{\sigma^2}{\theta^2} = constant\ \ for\ n \neq 1\ and\ n \neq -\frac{1}{3}$ and $\frac{\sigma^2}{\theta^2} = 0$ when $n = 1$. From which also we can concluded that this model is isotropic for large value of t for $n = 1$.

V. CONCLUSION

We have presented a five dimensional Bianchi Type-III cosmological model with polytropic equation of state in Lyra geometry. In our work the anisotropic dark energy models in Lyra geometry play a vital role in the discussion of the accelerated expansion of the universe which is the crux of the problem in the present scenario. It is seen that the dynamical parameters $\theta, \sigma^2, \beta, and\ H$ are infinite initially and diverges as $t \to 0$. The model (25) is anisotropic throughout the evolution of the universe. However, it is isotropic at $n = 1$ shown in (33) and (34). The model represents an expanding universe that start at the time $t = 0$ with volume $V = 0$, it has initial singularity and expands with acceleration after an epoch of deceleration. The deceleration parameter (q) is decelerating at initial stage of the evolution of the universe and then accelerate after some finite time because of the cosmic recollapse, indicating inflation in the model after an epoch of deceleration which is in agreement with the present day observational scenario of the accelerated expansion of our universe as claimed by type Ia supernovae (Riess et al.[59] and Perlmutter et al.[60]). The model was anisotropic in earlier epoch and becomes isotropic in the late time for $n = 1$. It is observed that our model universe is shearing in early epoch and is shear free throughout the evolution for $n = 1$.

VI. Data Availability Statement:

No new data were analysed in this study.

VII. Conflict of Interest:

There is no conflict of interests in this paper.

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