



Transfer of heat and mass of a Jeffrey Fluid over a linearly stretching sheet with chemical reaction: Numerical study

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Abstract

This paper numerically explores the effect of heat and mass transfer on Jeffrey fluid flow over a horizontal stretching sheet in the absence of magnetic field and under chemical reaction. The governing coupled non linear momentum, thermal, concentration boundary layer equations are rendered into a system of coupled nonlinear ordinary differential equations through similarity transformation with suitable boundary conditions. The obtained fourth order and second order differential equations are reduced to first order ordinary differential equations using shooting method Then it is numerically solved using bvp4c in MATLAB. This present investigation is of great interest relevant to colling of metallic plates, polishing of artificial heart valves and separation processes in chemical industries.

Introduction

Heat and mass transfer over a stretching sheet is important because the rate of cooling has great effect on the quality of the product. Boundary layer flow has extensive application in industry, Engineering, aerospace manufacturing, and medical Industries, numerous researchers and scientists achieved results in the fluid flow and heat transfer [1]-[4], flow over a flat with uniform free stream has been examined by Blasius [5], mass transport [6]-[7], several investigations have been carried on Jeffery fluid few of them are hereby cited. Hayat and et al. have traced out Jeffrey fluid for unsteady mixed convection flow under the existence of thermal radiation over a stretching sheet (Hayat and Mustafa, (2010) [8]-[11], In the present attempt we explore the flow of a Jeffrey fluid [12]-[16], heat and mass transfer by . S. Srinivas and M. Kothandapani, the effect of temperature dependent viscosity in momentum and thermal transport processes [18-22]. Malik et al. [23] presented the Jeffrey fluid flow with a pressure -dependent viscosity. Raju et al. [24] have discussed MHD chemically reacting boundary layer flow of Jeffrey nanofluid over a permeable cone in a porous medium with the effect of thermophoresis, Brownian motion, and thermal radiation. P.V Satya Narayana,D Harish Babu [25] have discussed the MHD heat transfer of Jeffrey fluid with chemical reaction and thermal radiation.

In view of all the mentioned above research the main objective of the present article is to explore the heat and mass transfer on the Jeffrey fluid in the absence of magnetic field over a linearly stretching sheet. Effect of non -dimensional governing parameters such as Prandtl number, the ratio of relaxation to retardation times. Here we also describe the numerical method, we present result and discuss. Finally we summarize our result and present our conclusion.

Nomenclature

$A_1 A_2$ constants

C concentration[kmol/m³]

c_p specific heat at constant pressure

C_∞ species concentration far away from wall

C_w species concentration at the wall

D diffusion coefficient[m²/s]

E extra stress tensor

R radiation parameter

R_1 Rivlin-Erickse tensor

U_w shrinking velocity [m/s]

u, v velocity components in the x,y directions,resp.[m/s]

λ_1 ratio of relaxation and retardation time

λ_2 relaxation time

τ Cauchy stress tensor

ν kinematic viscosity [m²/s]

ρ fluid density[Kg/m]

T fluid temperature

T_∞ temperature far away from the wall[K]

K fluid thermal conductivity [W/m/K]

K_s Rosseland mean absorption coefficient

Kr^* chemical reaction parameter

σ^* Stefan-Boltzmann constant

θ non dimensional temperature

ϕ non dimensional concentration

q_r radiative heat flux

Pr Prandtl number

β Deborah number

Sc Schmidt number

γ heat source parameter

x distance along the wall[m]

y distance normal to the wall [m]

ξ similarity variable

l characteristic length

m surface temperature parameter

Subscripts

W sheet surface

∞ infinity

Superscript

' differentiation with respect to ξ

Mathematical formulations

The essential equations for Jeffrey fluid can be written as

$$\tau = -pl + E \quad (1)$$

$$E = \frac{\mu}{1+\lambda_1} \left[R_1 + \lambda_2 \left(\frac{\partial R_1}{\partial t} + V \cdot \Delta \right) R_1 \right] \quad (2)$$

Where E is the extra stress tensor, τ is the Cauchy stress tensor, λ_1 and λ_2 are the material parameters of Jeffrey fluid and R_1 is the Rivlin-Ericksen tensor defined by

$$R_1 = (\nabla V) + (\nabla V)'$$

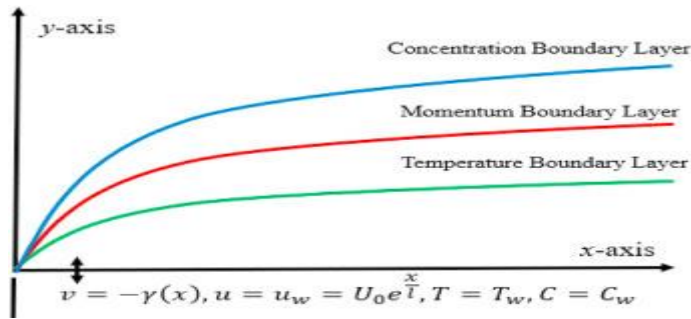


fig 1 The physical model of the flow problem and coordinate system

A steady two-dimensional incompressible, electrically conducting Jeffrey fluid over a linear stretching sheet in the presence of chemical reaction, thermal radiation and heat source flow is generated, due to linear stretching of the sheet, caused by simultaneous application of two equal and opposite forces along the x -axis and y -axis is taken normal to it. The origin is fixed as shown in Fig. 1.

The temperature and the species concentration have power index m variations with the distance from the origin. At $t = 0$, the sheet is impetuously stretched with the variable velocity $U_w(x)$.

Under these assumptions the governing equation of continuity and momentum take the following form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\nu}{1+\lambda_1} \left[\frac{\partial^2 u}{\partial y^2} + \lambda_2 \left(u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right) \right] \quad (4)$$

Where u, v are the velocity components in the x and y direction, respectively, ν is the kinematic viscosity, λ_1 is the ratio of relaxation and retardation time, λ_2 is the relaxation time.

The equation of heat transfer and thermal radiation is given as

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} - Q(T - T_\infty) \quad (5)$$

Where c_p is the specific heat and k is the thermal conductivity, T is the temperature of the fluid. T_∞ is the constant temperature of the fluid far away from the sheet

By using Rosseland diffusion approximation the radiative heat flux q_r is given by

$$q_r = -\frac{4\sigma^*}{3K_s} \frac{\partial T^4}{\partial y} \quad (6)$$

Where K_s and σ^* are the Rosseland mean absorption coefficient and the Stefan-Boltzmann constant, resp. the temperature within the fluid flow is considered sufficiently small such that T^4 can be expressed as linear function of temperature.

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \quad (7)$$

On solving (6) (7) and (5) we get

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3K_s} \frac{\partial^2 T}{\partial y^2} \quad (8)$$

We introduce a dimensionless temperature variable $\theta(\xi)$ of the form

$$\theta(\xi) = \frac{T - T_\infty}{T_w - T_\infty} \quad (9)$$

The first order chemical reaction with concentration diffusion of the laminar boundary layer flow is given as

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - Kr^*(C - C_\infty) \quad (10)$$

Where D is the diffusion coefficient Kr^* is the chemical reaction parameter

We introduce a dimensionless temperature variable $\phi(\xi)$ of the form

$$\phi(\xi) = \frac{C - C_\infty}{C_w - C_\infty} \quad (11)$$

Boundary Conditions:

The following boundary conditions on velocity, temperature and concentration are appropriate in order to employ the effect of stretching of the boundary surface causing flow in x -direction as

$$u = U_w(x) = cx, v = 0 \text{ at } y = 0$$

$$u \rightarrow 0, u' \rightarrow 0 \text{ as } y \rightarrow \infty$$

$$T = T_w = T_\infty + A_1 \left(\frac{x}{l}\right)^m \text{ at } y=0$$

$$T \rightarrow T_\infty \text{ as } y \rightarrow \infty$$

$$C = C_w = C_\infty + A_2 \left(\frac{x}{l}\right)^m \text{ at } y=0$$

$$C \rightarrow C_\infty \text{ as } y \rightarrow \infty \tag{12}$$

Where A_1, A_2 are constants, l is the characteristic length, m is the surface temperature parameter, T_w is the stretching sheet temperature, C_w and C_∞ are the concentration at the wall and far away from the wall, resp.

The following similarity transformations are introduced to solve equation (4) (5) and (10)

$$u = cx f'(\xi), v = -\sqrt{cv} f(\xi) \text{ where } \xi = \sqrt{\frac{c}{v}} y \tag{13}$$

Where ξ is the similarity variable and $f(\xi)$ is the dimensionless stream function

Substituting eq (13) in eq (4) (5) and (10) we obtain second and fourth order ordinary differential equations as follows

$$f''' + (1 + \lambda_1)(f f'' - f'^2) + \beta(f''^2 - f f^{iv}) = 0 \tag{14}$$

$$\left(1 + \frac{4R}{3}\right) \theta'' + Pr(f\theta' - mf'\theta + \gamma\theta) = 0 \tag{15}$$

$$\phi'' + Sc(f\phi' - mf'\phi - Kr\phi) = 0 \tag{16}$$

With boundary conditions (12) takes the form:

$$\begin{aligned} f(\xi) = s, f'(\xi) = 1 \text{ at } \xi = 0; f'(\xi) = 0, f''(\xi) = 0 \text{ as } \xi \rightarrow \infty \\ \theta(\xi) = 1 \text{ at } \xi = 0; \theta(\xi) = 0 \text{ as } \xi \rightarrow \infty \\ \phi(\xi) = 1 \text{ at } \xi = 0; \phi(\xi) = 0 \text{ as } \xi \rightarrow \infty \end{aligned} \tag{17}$$

Where $\beta = \lambda_2 c$ is the Deborah number, $R = \frac{4\sigma^* T_\infty^3}{K_s}$ the radiation parameter,

$Pr = \frac{\rho c_p}{k}$ the Prandtl number, $\gamma = \frac{Qv}{\rho c_p}$ is a heat source parameter, $Sc = \frac{v}{D}$ is the Schmidt number and $Kr = \frac{Kr^* \delta^2}{v}$ is the chemical reaction parameter, $s = \frac{-v_w}{\sqrt{cv}}$ is the parameter with $s > 0$.

The system of non-linear ordinary differential equations (14) (15) (16) with the boundary conditions (17) are converted to ordinary differential equations using shooting method and using MATLAB bvp4c the numerical solution is obtained, thus the fourth order and second order equations are reduced to system of simultaneous equations of order one.

$$\begin{aligned} f = y(1), f' = y(2), f'' = y(3), f''' = y(4) \\ \theta = y(5), \theta' = y(6) \\ \phi = y(7), \phi' = y(8) \end{aligned} \tag{18}$$

Substituting these in (14)(15)(16) and (17) we have

$$y(4) + (1 + \lambda)(y(1)y(3) - y(2)^2) + \beta(y(3)^2 - y(1)y(4)) = 0 \tag{19}$$

$$\theta'' \left(1 + \frac{4}{3R}\right) - Pr(y(1)y(6) - my(2)y(5) + \gamma y(5)) = 0 \tag{20}$$

$$\phi'' - Sc(my(2)y(7) + Kr y(7) - y(1)y(8)) = 0 \tag{21}$$

With boundary conditions

$$y_0(1) = 1, y_0(2) = 1; y_\infty(2) = 0, y_\infty(3) = 0; y_0(5) = 1, y_\infty(5) = 0; y_0(7) = 1, y_\infty(7) = 0;$$

Equations (19)(20)(21) are reduced to eight simultaneous equations of first order as follows

$$y'(1) = y(2)$$

$$y'(2) = y(3)$$

$$y'(3) = y(4)$$

$$y'(4) = \frac{1}{\beta\gamma_1} (y_4 + (1 + \lambda)(y_1y_3 - y_2^2) + \beta y_3^2) \tag{22}$$

$$y'(5) = y(6)$$

$$y'(6) = -\frac{Pr}{(1+\frac{4}{3R})} (y_1y_6 - my_2y_5 + \gamma y_5) \tag{23}$$

$$y'(7) = y(8)$$

$$y'(8) = Sc(my_2y_7 + Kry_7 - y_1y_8) \tag{24}$$

The governed equations are solved numerically using MATLAB using bvp4c

Results and discursion

The Following Graphs Gives the variation of velocity ,temperature and concentration

The fig 2 shows the effect of Sc Schmidth number on concentration ,the increase of Sc means decrease of molecular diffusion ,hence the concentration is higher for small Sc and lower for high Sc.

Fig 3 shows the effect of Pr Prandtl number on temperature, it shows that the temperature decreases with the increase of Pr. Physically it can be stated as the reduction in temperature is due to the thermal diffusivity ,as thermal diffusivity decreases the Pr number increases and thus the temperature decreases.

Fig 4 shows the effect of chemical reaction parameter Kr on concentration, as Kr increases the concentration decreases. Physically the increase in the chemical reaction decreases the concentration and boundary layer.

Fig 5 shows the effect of ratio of relaxation and retardation time λ on velocity, increase in λ cause the reduction of boundary layer velocity of fluid.

fig 6 shows the effect of heat source parameter λ_1 on temperature.it is clear that heat source gives an increase in the temperature of the fluid ,physically the increase of heat source in the boundary layer generates energy which causes the temperature of the fluid to increase.

fig 7 and fig 8 shows the effect of Deborah number β on the fluid velocity ,as β increases the velocity increases, physically Deborah number β is proportional to the rate of stretching sheet, the increase of β results in a higher fluid motion in the boundary layer.

fig 9 the concentration profiles decreases with increase in wall concentration parameter m.

fig 10 it shows the effect of R on temperature profile, as temperature distribution increases with the increase in the value of R, this is due to fact that the thermal boundary layer thickness increases with an increase in thermal radiation ,thus to proceed cooling process faster the radiation should be minimized.

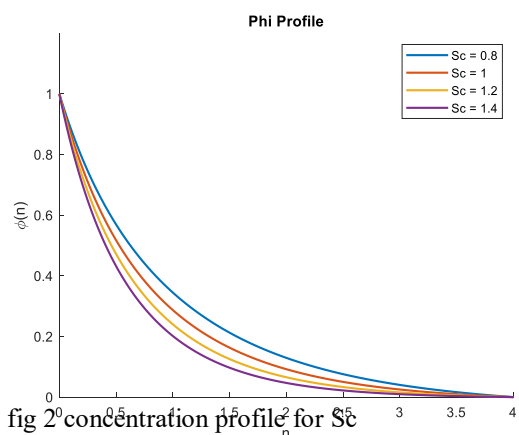


fig 2 concentration profile for Sc

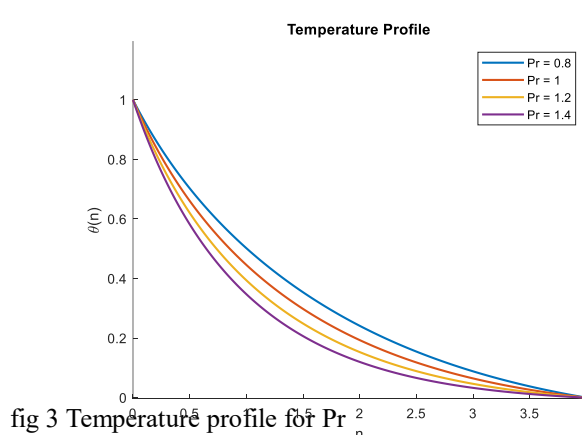


fig 3 Temperature profile for Pr

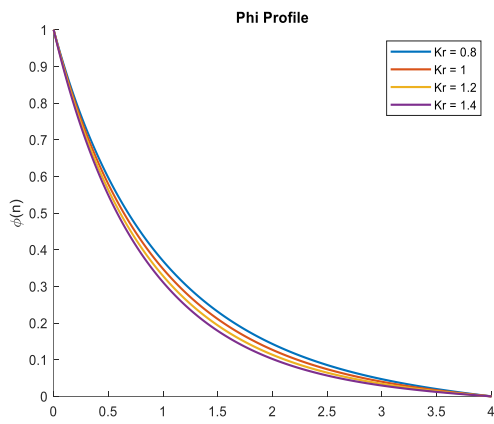


fig 4 concentration profile for Kr

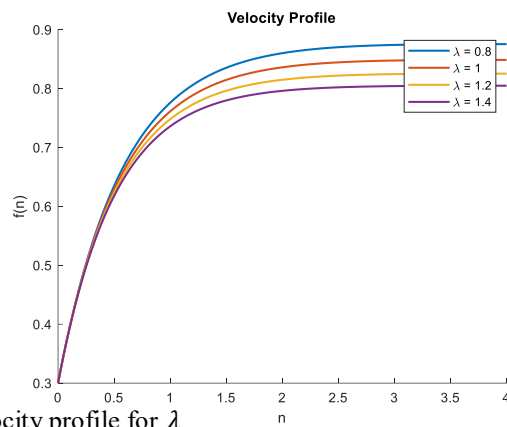


fig 5 velocity profile for λ

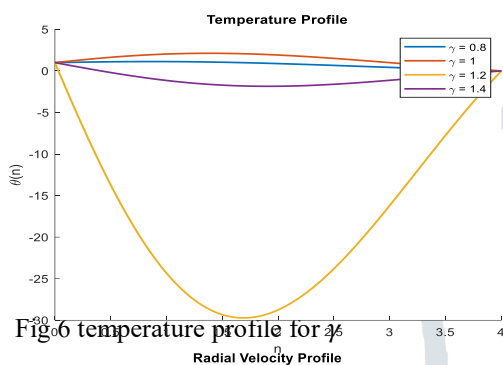


fig 6 temperature profile for γ

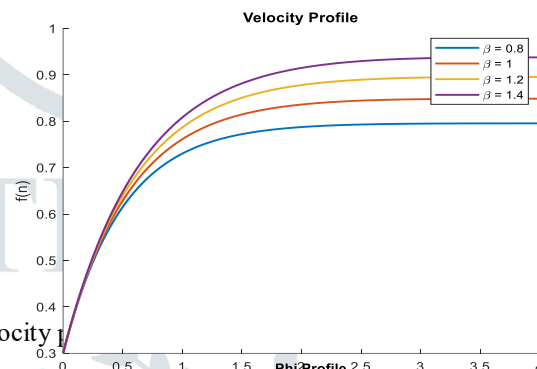


fig 7 velocity

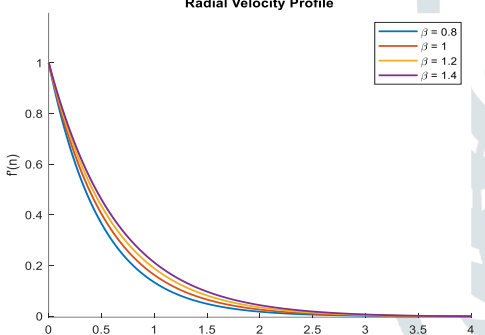


fig 8 Radial velocity profile for β

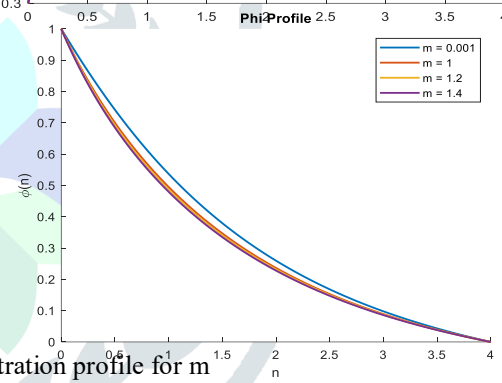


fig 9 concentration profile for m

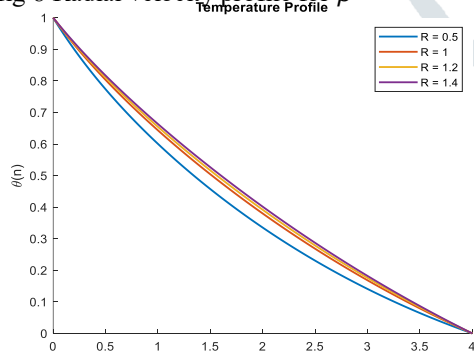


fig 10 Temperature profile for R

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