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Study of gso-Closed maps and gso-Homeomorphisms by gso-closed Sets

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Abstract: In this paper, first we introduce gso-closed maps between topological spaces and study their topological properties. We also introduce the gso-irresolute and investigate this with the gso-continuous maps. Also, the gso-homeomorphism between topological spaces are defined and their properties are investigated.

***Keywords* - g-closed set, gso-closed set, g-continuous map, g-irresolute map, g-homeomorphism**

1 INTRODUCTION

The generalized closed set concept was developed by Levine N. [1] in 1970. The generalized closed set has many important properties in topological spaces. In the literature, many authors introduced different types of generalized closed sets and studied their properties. The concept of gso-closed set was introduced and studied by Irshad M.I. and Elango P. [2] in 2019.

The notation of irresoluteness was introduced by Crossley S.G. and Hilderbrand S.K. [3] in 1972. Maki H. [4] introduced and studied the g-homeomorphism and gc-homeomorphism between topological spaces in 1991. Recently, many researchers have carried out research on generalized homeomorphisms and their properties. The aim of this paper is to study about gso-closed maps and gso-homeomorphism based on gso-closed sets and study the properties of gso-closed maps and gso-homeomorphism in general topological space.

2 PRELIMINARIES

In this paper, we represent X, Y and Z as the topological spaces (X,τ), (Y,σ) and (Z,γ) respectively, on which no separation axioms are assumed unless stated. For a subset of A of X, $cl(A)$ denotes the closure of A and $int(A)$ denotes the interior of A respectively.

Since we use the following definitions and some properties, we recall them in this section.

Definition 2.1. A subset A of a topological space X is called a

a) g-closed set [1] if $\text{cl}(A) \subseteq \text{U}$ whenever $A \subseteq U$ and U is open in X.
b) gs-closed set [7] if $s\text{cl}(A) \subseteq \text{U}$ whenever $A \subseteq U$ and U is open in X.
c) wg-closed set [8] if $\text{cl}(\text{int}(A)) \subseteq \text{U}$ whenever $A \subseteq U$ and U is open in X.
d) rwg-closed set [8] if $\text{cl}(\text{in}(A)) \subseteq \text{U}$ whenever $A \subseteq U$ and U is regular open in X
e) gso-closed set [2] if A is both a g-closed set and a semi-open set in X.

The complements of the above mentioned generalized closed sets are their respective generalized open sets.

Definition 2.2. Let X and Y be two topological spaces. A map $f: X \to Y$ is called a

a) g-closed [4] if for each closed set F in X, $f(F)$ is g-closed set in Y.

b) g-open [13] if for each open set U in X, $f(U)$ is g-open set in Y.
c) wg-closed [8] if for each closed set F in X, $f(F)$ is wg-closed set in Y.
d) rwg-closed [8] if for each closed set F in X, $f(F)$ is rwg-closed set in Y.
e) g-continuous [5] if $f^{-1}(V)$ is closed in X for each closed set V in Y.
f) gso-continuous [2] if $f^{-1}(V)$ is gso-closed in X for each closed set V in Y.

Definition 2.3. Let X and Y be two topological spaces. A bijective map $f: X \to Y$ is called a
a) g-homeomorphism [4] when f is both g-continuous and g-closed.
b) gs-homeomorphism [6] when f is both gs-continuous and gs-closed.
c) rwg-homeomorphism [1] when f is both rwg-continuous and rwg-closed.

3 GSO-CLOSED MAP

Definition 3.1. A map $f: X \to Y$ is said to be gso-closed map if for each closed set F in X, $f(F)$ is a gso-closed set in Y.

Definition 3.2. A map $f: X \to Y$ is said to be gso-open map if for each open set U in X, $f(U)$ is a gso-open set in Y.

Definition 3.3. A map $f: X \to Y$ is said to be gso-irresolute if $f^{-1}(V)$ is gso-closed in X for each gso-closed set V in Y.

Definition 3.4. A topological space X is a T_{gso}-space if every gso-closed set in X is a closed set in X.

Lemma 3.1. *Every gso-closed map is a wg-closed map.*

Proof. Let $f: X \to Y$ be a gso-closed map and let F be a closed set in X. Then, $f(F)$ is a gso-closed set in Y and so wg-closed set in Y. Thus, f is a wg-closed map.

Lemma 3.2. *Every gso-closed map is a gs-closed map.*

Proof. Let $f: X \to Y$ be a gso-closed map and let F be a closed set in X. Then, $f(F)$ is a gso-closed set in Y and so gs-closed set in Y. Thus, f is a gs-closed map.

Lemma 3.3. *Every gso-closed map is a rwg-closed map.*

Proof. Let $f: X \to Y$ be a gso-closed map and let F be a closed set in X. Then, $f(F)$ is a gso-closed set in Y and so rwg-closed set in Y. Thus, f is a rwg-closed map.

Lemma 3.4. *Every open map is a gso-open map.*

Proof. Let $f: X \to Y$ be an open map and let U be an open set in X. Then, $f(U)$ is an open set in Y and so gso-open set in Y. Thus, f is a gso-open map.

Lemma 3.5. *Every g-open map is a gso-open map.*

Proof. Let $f: X \to Y$ be a g-open map and let U be an open set in X. Then, $f(U)$ is a g-open set in Y and so gso-open set in Y. Thus, f is a gso-open map.

Lemma 3.6. *If $f: X \to Y$ is a gso-closed map and if $A = f^{-1}(B)$ for some closed set B in Y, then $f_A: A \to Y$ is a gso-closed map.*

Proof. Let F be a closed set in A. Then, there is a closed set H in X such that $F = A \cap H$. Then, $f_A(F) = f(A \cap H) = f(A) \cap f(H) = B \cap f(H)$. Now $f(H)$ is a gso-closed set in Y as f is a gso-closed map. Therefore, $B \cap f(H)$ is a gso-closed set in Y and so f_A is a gso-closed map.

Theorem 3.7. *Let $f: X \to Y$ and $g: Y \to Z$ be gso-closed maps. If f is a closed map, then $g \circ f: X \to Z$ is a gso-closed map.*

Proof. Let F be a closed set in X. Then, $f(F)$ is a closed set in Y as f is a closed map. Then, $(g \circ f)(F) = g(f(F))$ is a gso-closed set in Z as g is a gso-closed map. Therefore, $g \circ f$ is a gso-closed map.

Lemma 3.8. *If $f: X \to Y$ is a gso-continuous map and Y is a T_{gso}-space, then f is a gso-irresolute.*
Proof. Let F be a gso-closed set in Y. Since Y is a T_{gso}-space, F is a closed set. Then, $f^{-1}(F)$ is a gso-closed set in X. Hence, f is a gso-irresolute.

Theorem 3.9. *Let $f: X \to Y$ is a gso-irresolute and $g: Y \to Z$ is a gso-continuous map, then $g \circ f: X \to Z$ is a gso-continuous map.*

Proof. Let F be a closed set in Z. Then, $f^{-1}(F)$ is a gso-closed set in Y as g is gso-continuous. Now, $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is a gso-closed set in X as f is a gso-irresolute. Therefore, $g \circ f$ is a gso-continuous map.

Corollary 3.10. *Let $f: X \to Y$ and $g: Y \to Z$ are gso-continuous maps and Y is a T_{gso}-space, then $g \circ f$ is a gso-continuous map.*

Proof. In T_{gso}-space, each gso-closed set is a closed set, the result is directly follows from Theorem 3.9.

Theorem 3.11. *In T_{gso}-space, the finite union of gso-closed set is a gso-closed set.*

Proof. Suppose $A = \bigcup_i^n A_i$ is a finite union of gso-closed sets in T_{gso}-space. Then $A^c = (\bigcup_i^n A_i)^c = \bigcap_i^n A_i^c$. Since in T_{gso}-space, every gso-closed set is a closed set, so A_i^c open for each i and so A^c is open. Therefore, A is closed and hence gso-closed.

4 GSO-HOMEOMORPHISM

Definition 4.1. A bijection $f: X \to Y$ is called gso-homeomorphism when f is both gso-continuous and gso-closed map.

Lemma 4.1. *Every gso-homeomorphism is a gs-homeomorphism.*

Proof. Let $f: X \to Y$ be a gso-homeomorphism. Then, f is both gso-continuous and gso-closed. Then, clearly, f is a gs-continuous and gs-closed. So f is a gs-homeomorphism.

Lemma 4.2. *Every gso-homeomorphism is a wg-homeomorphism.*

Proof. Let $f: X \to Y$ be a gso-homeomorphism. Then, f is both gso-continuous and gso-closed. Then, clearly, f is a wg-continuous and wg-closed. So f is a wg-homeomorphism.

Lemma 4.3. *Every gso-homeomorphism is a rwg-homeomorphism.*

Proof. Let $f: X \to Y$ be a gso-homeomorphism. Then, f is both gso-continuous and gso-closed. Then, clearly, f is a rwg-continuous and rwg-closed. So f is a rwg-homeomorphism.

Theorem 4.4. *For any bijection $f: X \to Y$, the following statements are equivalent:*
  a) the inverse map $f^{-1}: Y \to X$ is a gso-continuous map,

b) f is a gso-open map,
c) f is a gso-closed map.

Proof. Let $f^{-1}: X \to Y$ be a gso-continuous map and G be any open set in X. Then, the inverse image of G under f^{-1}, $f(G)$ is gso-open in Y and so f is a gso-open map. Now, let f be a gso-open map and let F be any closed set in X. Then, F^c is open in X so $f(F^c)$ is gso-open in Y. But $f(F^c) = Y \setminus f(F)$ and so $f(F)$ is gso-closed in Y. Therefore, f is a gso-closed map. Finally, let f be a gso-closed map and let F be any closed set in X. Then, $f(F)$ is gso-closed in Y. But $f(F)$ is the inverse image of F under f^{-1}. Therefore, f^{-1} is gso-continuous map.

Theorem 4.5. *Let $f: X \to Y$ be a gso-continuous map from a space X onto a space Y. Then, the following statements are equivalent:*
a) f is a gso-open map,
b) f is a gso-homeomorphism,
c) f is a gso-closed map.

Proof. Assume that f is a gso-open map. Then, clearly f is a gso-homeomorphism. Now, if f is a gso-homeomorphism, then by definition f is a gso-closed map. Finally, if f is a gso-closed map, then by Theorem 4.4, f is a gso-open map.

Theorem 4.6. *Let X and Z be any two topological spaces and let Y be a T_{gso}-space. If $f: X \to Y$ and $g: Y \to Z$ be gso-homeomorphisms, then the composition $g \circ f: X \to Z$ is a gso-homeomorphism.*

Proof. Let F be a closed set in Z. Then, $g^{-1}(F)$ is a gso-closed set in Y as g is a gso-continuous map. Since Y is a T_{gso}-space, $g^{-1}(F)$ is a closed set in Y. Thus, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is a gso-closed set in X. Thus $g \circ f$ is a gso-continuous map.

Again, let F be a closed set in X. Then, $f(F)$ is a gso-closed set in Y as f is a gso-closed map. Since Y is a T_{gso}-space, $f(F)$ is a closed set in Y. Thus $g(f(F)) = (g \circ f)(F)$ is a gso-closed set in Z. Thus $g \circ f$ is a gso-closed map. Hence $g \circ f$ is a gso-homeomorphism.

5 CONCLUSION

In this paper, we have introduced a new kind of generalized closed map, called gso-closed map between topological spaces and investigated its properties. The gso-irresolute was also defined and its properties investigated. Finally, the gso-homeomorphisms between topological spaces were introduced and their properties were established.

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