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Fuzzy Jump-Diffusion Model for Pricing of Equity of Banking Corporations in Emerging Economies

Ephraim Matanda (DPhil Candidate in the Department of Banking and Finance, Midlands State University, Gweru, Zimbabwe)

Eriyoti Chikodza (Department of Mathematics and Computer Science, Great Zimbabwe University,Masvingo, Zimbabwe)

Farai Kwenda (Department of Finance, University of Swaziland, Manzini, Eswatini)

Corresponding Author: Ephraim Matanda (MSc Finance and Investment)(Department of Banking and Finance, Midlands State University, Gweru, Zimbabwe)

Permanent Address: Midlands State University, P. Bag 9055

ABSTRACT

Contrary to the non-variable nature of most assumptions on which structural models are based, financial markets are characterized by variables that are imprecisely defined. The research incorporates such imprecise variables in the valuation of banking corporations operating under uncertain and frictional financial markets. Fluctuating financial markets are defined by variables such as interest rates and volatilities of assets that are not precisely expressed as given under structural models. By assuming that risk free rates, asset volatilities and geometric Brownian motion (BM) intensities are fuzzy numbers, the paper proposes a jump-diffusion model to estimation of the values of banking corporations. We validate the model using financial data drawn from two banks conveniently drawn from countries in Southern Africa.

Keywords: structural models, imprecise variables, fluctuating financial markets, asset volatilities, fuzzy numbers, jump-diffusion model

1.0 Introduction

It is the option pricing problem that is pivotal in all issues relating to contemporary financial market management with specific reference to capitalization, investment and firm valuation. European options for instance are valued using structural asset valuation models (AVM) whose applicability is centred on the requirements of classical probability theory. However the conditions for the application of probability theory such as precision or crispness are said to be far from meeting the linguistic expectations of investors when it comes to valuation of firms. It is on the basis of the absence of market friction and human perceptions in structural models that this study sought to introduce the jump-diffusion variable to the valuation of firms.

Currently a new stock valuation model for uncertain financial markets has been developed based on an acceptable uncertain process (Karaman, 2018). Xu, Peng and Xiao (2013) state that under fluctuating financial markets, parameters such as interest rates and volatilities of assets cannot be measured or described precisely. On the other hand, Qin and Li (2008) introduce that fuzziness to modelling of finance after noting that investors' decisions in financial markets were influenced by variablles such as vagueness, ambiguity and imprecision. In this study we first introduce Liu's stock valuation model and its European option pricing formulae. This stage will be followed by an examination of the general stock model for use in uncertain financial markets, which is founded on the basis of uncertain differential equations. The discussion is extended to the framework on general stock models for valuation of American calls and puts that are not valued through most structural models such as the Black-Scholes (1973) and Merton (1974) models. It is on the basis of the option valuation models covered above that the study will proceed and present jump-diffusion models used in estimation of values of firms.

Bachelier (1964) came up with the concept of Brownian motion (BM) to financial management. According to Samuelson (1980) BM is a significant tool for determination of stock prices. On the other hand Merton (1974) and Black-Scholes (1973) use the Geometric BM to construct the theory for valuation of stock-option prices. The two models are based on the assumptions that financial markets were characterized by fluctuating interest rates and asset volatilities that were precisely defined. By assuming that the risk free rate, asset volatility and average jump interest rates are fuzzy numbers, this study presents a fuzzy jump-diffusion model to the valuation of firms in fuzzy financial markets. Wang and Tian (2013) came up with a jump-stock model for uncertain financial markets centred on **canonical** uncertain processes. They start by introducing Liu's stock model for European option pricing before extending the framework to cover uncertain differential equations for valuation of American calls and puts.

The study was motivated by the need to include fuzzy factors and market friction in existing jump-diffusion models to pricing of assets in fuzzy financial markets (See Xu, Peng and Xiao, 2013 as well as Li and Qin, 2008). The firm investigated in the study was drawn from a Southern African country using a non-probability sampling method, specifically convenience sampling method. One of the reasons for use of the convenience sampling method was the availability of its published and audited financial data for the study to be able to generate findings, conclusions and inferences. The study therefore proposed and validated a fuzzy jump-diffusion model using the financial data of the firm alluded to above. This was necessitated by the fact that most investors' financial decisions in emerging markets are based on the unpredictable nature of asset prices and human judgments more than the requirements of classical probability requirements used worldwide. The results generated by the paper revealed that market friction and human perceptions had a strong inverse bearing on market prices of equity and returns to market investments in fuzzy financial environments.

The last sections of the research paper are organised in such a way that the concept of fuzzy events is introduced in the second section. Section 2 of the paper goes on to discuss fuzzy theory and its link to jump-diffusion applications in fuzzy financial environments. The third section of the paper examines the derivation of the fuzzy jump-diffusion model based on structural models in explaining estimation of market prices of assets and equities of firms in a fuzzy financial environment. Section 4 assesses the accuracy of the validation to the fuzzy jump-diffusion model with an illustrative example based on financial data drawn from a bank in one of the countries of Southern Africa. Last but not least conclusions and recommendations of the study are presented in section 5 of the paper.

2.0 Background to the Study

The literature reviewed in this paper is based on two main areas namely fuzzy theory and jump-diffusion models that is how the two theories can be combined in valuation of assets and equities of firms.

2.1 Theory of Fuzzy Sets

According to Zadeh (1965) fuzzy theory is essential for use in the investigation of the jump-diffusion model. This is because real situations in financial markets are not crisp and deterministic and therefore cannot be precisely described at all times. Zimmermann (1980) arguments Zadeh (1965) by stating that fuzzy set theory can be applied to solving realistic financial problems. Fuzzy sets which the study prefers to mere jump-diffusion models are based on the knowledge of crisp or classical sets but add one more element and range from 0 to 1 in values that they assume. In other words all it requires is for one element in a set, for instance in A should not be in set B for inclusion to fail.

The membership function, $\mathrm{m_A}$ of a fuzzy set = Mapping to the unit interval (0;1) that is $\mathrm{m_A(X)}$: Universal set $(0;1)$ represents the index of the set-hood that measures the degree to which an object X is a member of a particular set. According to Cao and Chen (1983) and Blockley (1980) reality in financial markets creates uncertainty or vagueness stochastic uncertainty in contrast to vagueness from semantic meaning of events called fuzziness, found in many areas of daily life. In other words, fuzzy sets are designed to be used in handling particular kinds of uncertainty, which is the degree of vagueness for an exposure that can be possessed by objects to varying degrees, for example the volatility of stock returns in financial markets. Moller et al (2002) researched on fuzzy randomness and uncertainty modelling. The following critical terms are defined for purposes of improving the conceptualization of fuzzy sets and financial modelling based on linguistic variables used in planning and decision making processes by investors in financial markets.

2.1.1 Fuzziness

The concept of fuzziness is intimately related to vagueness, generality and ambiguity. Fuzziness does not have a well-defined set of bounds and is not resolvable with specific reference to context as opposed to the other terms (Qin and Li, 2008). The other terms above can be contextually eliminated and conclusions that are closely linked to investors' language judgements can be made. It is a fact that integral applications that combine semantics, linguistic variables and pragmatism are more powerful and beneficial to individual investors and firms in a given financial system.

2.1.2 Vagueness

A word, phrase or sentence is said to be vague, when it refers unclear or imprecise circumstances. Vague statements often call for follow-up questions. For example, if we say that, "The son did not live up to our expectations" this is a vague statement that may call for another question such as, "What were our expectations?" On the other hand if were to say, "The son was prone to worse forms of behavior than that," the likely follow-up question would be, "Like what?" Therefore it is critical in researches that we review our words, phrases and sentences in order to make them as clear and precise as possible (See, Zhang, 1998). This can be achieved by adding follow-up sentences to make the meanings more explicit if it aids to do so. Readers or listeners should not be put in the position of assuming our intentions through asking vague questions to them.

2.1.3 Ambiguity

It should be noted from the onset that words, phrases and sentences that are ambiguous, carry more than one possible meaning, which scenario may cause confusion for the reader or listener. For example, a statement like, "We know more gifted teachers than Charles Johns," this could be taken to mean that we know teachers who are more talented than Charles Johns or that we know a greater number of gifted teachers than Charles Johns does. In some cases, ambiguous phrases and statements can be unintentionally humorous, but this quality does not exonerate the confusion they cause in real life situations. On the other hand if we were to write, "Tabeth likes her friend, and so did Elizabeth," This could be taken to mean that Tabeth also liked Elizabeth's friend or that she liked her own friend (See Zhang, 1998). Therefore ambiguous words, phrases and sentences can also be vague. Hence the best way to be able to identify ambiguity in words, phrases or sentences is to ask ourselves whether these carry double meanings or not. In other words if the answer to such questions is "yes," then that simply means the words, phrases or sentences are ambiguous.

2.1.4 Uncertainty

The concept of uncertainty is defined from two perspectives namely the state of being unsettled, in doubt or dependent on chance and that of being unsure of an event or something happening.

The following are some of the words often used in combination with uncertainty (Cambridge English Corpus).

.Conditional uncertainty

This means that rules used must account for each and every condition of uncertainty.

Considerable uncertainty

This refers to considerable uncertainty about the future of informal care given to an event or situation.

Degree of uncertainty

As is usually expected, the values drawn from events are subject to a high degrees of uncertainty.

2.2 Fuzzy Theory and Financial Models

Fuzzy models are efficient in determination of approximate solutions to financial problems compared to systems of structural differential equations (Bardossy, 1996). The solutions to systems of equations by Bardossy were found to be almost the same for all practical purposes, given the inaccuracies and uncertainties in the input data drawn into the model applied. Precise or crisp sets are fuzzy sets that restrict own membership values to (0;1) as end points of the unitary interval as is the case in probability theory. Therefore fuzzy set theory is able to model ambiguous or vague phenomena arising from human behaviours by assigning weights to objects based on the values of the membership function. The theory goes further to evaluate the extent to which an object in a given set is judged to be true or false.

2.3 Application of Fuzzy Calculus to Finance

Stochastic calculus is an indispensable tool of classical probability theory. Therefore to be able to deal with fuzzy financial world, investors need some understanding of fuzzy calculus that is introduction of fuzzy theory to the application of finance in the real world. Liu (2007) came up with a fuzzy process, diffusion formula and fuzzy integral, which were later renamed, Liu process, formula and integral centred on their importance and usefulness. An understanding of these three concepts is pertinent before we move on to proposing a fuzzy jump-diffusion model for firm valuation in emerging markets. The following terms are defined and illustrated to understand the Liu process, formula and integral before we propose the new model and validate it.

2.3.1 Membership Function

We first assume that $f:R^n \rightarrow R$ be a function and E_1,……………,E_n be fuzzy variables on some credibility space (θ; P; Cr). Therefore if U ={θ; P; Cr}is a universal set, then its membership function is defined from u(x) =2Cr (E =x) -1, x∈ R.

2.3.2 Equi-possible Fuzzy Variable

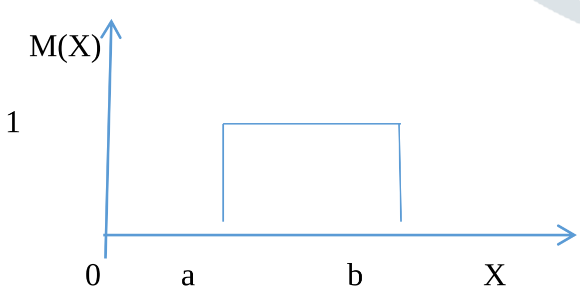

Fig. 2.3.1

It refers to a fuzzy variable, f that is determined by a pair of values (a;b) of crisp numbers such that a< b with M function

$$\mu(x)=\begin{pmatrix} 1, \text{if } a \leq x \leq b \\ 0, \text{Otherwise} \end{pmatrix}.$$

This type of fuzzy variable is as demonstrated graphically above.

2.3.3 Triangular Fuzzy Variable

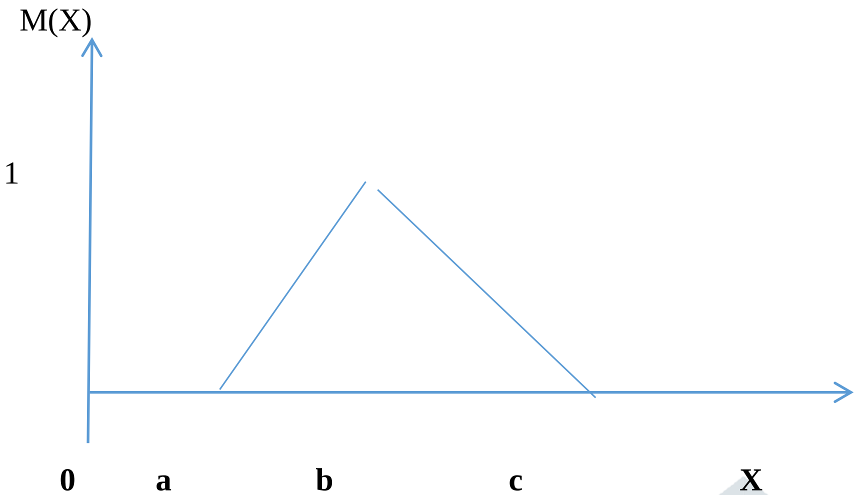

Fig. 2.3.2

It is a fuzzy variable, f that is determined by a triplet of values (a;b;c) of crisp numbers such that $a < b < c$ with M function $\mu(x) = \begin{pmatrix} \frac{x-a}{b-a} \text{ for } a \le x \le b \\ \frac{x-b}{c-b} \text{ for } b \le x \le c \\ 0, \text{Otherwise} \end{pmatrix}$. This kind of fuzzy variable is as demonstrated graphically above.

2.3.4 Trapezoidal Variable

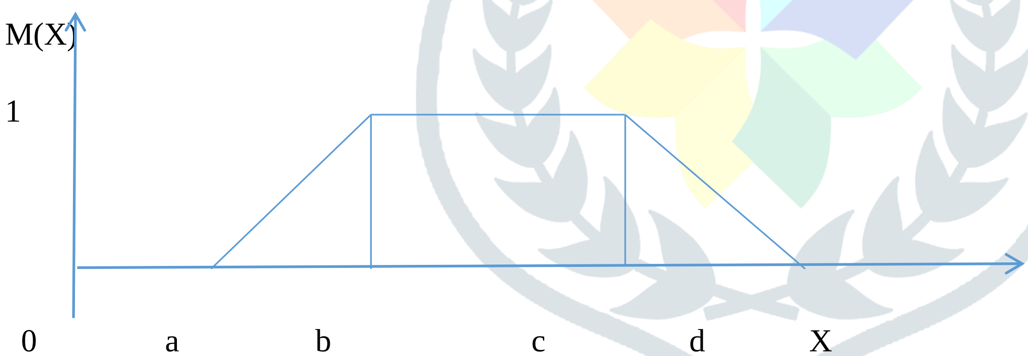

Fig 2.3.3

This refers to a fuzzy variable, f that is determined by a set of quadruplet values (a; b; c; d) of crisp numbers such that $\begin{pmatrix} \frac{x-a}{b-a} \text{ for } a \le x \le b \\ \frac{x-b}{c-b} \text{ for } b \le x \le c \\ \frac{x-c}{d-c} \text{ for } c \le x \le d, \\ 0, \text{Otherwise} \end{pmatrix}$.

This type of fuzzy variable is as demonstrated graphically above.

2.4 The Theory of Jump-Diffusion Processes

By jump diffusion we mean a stochastic process that is a random process that involves jumps and diffusion variables. The concept of jump diffusion has important applications in magnetic reconnections, economics and finance particularly in derivative markets when it comes to option pricing techniques. Jump-diffusion processes were first introduced by Liu and Liu (2010) as forms of random sampling algorithms which mixed "focus" like motions, the diffusion processes and with motions such as "**saccades"** that is quick, smooth and simultaneous motions via jump processes. The jump diffusion approach was used in modelling of sciences of electron-micrographs as containing multiple shapes, each with some fixed dimensional representation. The collections of micrographs filled out sample spaces corresponding to the unions of multiple finite-dimensional spaces. Based on techniques from pattern theory, a posterior probability model was constructed over the countable union of sample space. This is a hybrid system model that contains the discrete notions of object numbers along with the continuous notions of shape. The jump-diffusion process was constructed to have ergodic properties so that after flowing away from the initial condition it would generate statistical samples from the posterior probability model.

2.4.1 Jump diffusion models

Jump-diffusion models are an essential and easy-to-learn tools for option pricing and risk management. These models provide an adequate description of stock price fluctuations and systematic risks in financial markets. By introducing several widely used jump-diffusion models, fourier or periodic transformation based methods for European option pricing, partial differential equations for barrier and American options, and existing approaches to calibration, hedging of financial exposures are examined. Merton (1976) examined various aspects of jump-diffusion models from the academic financial perspective. Research departments of major banks of the world started to accept jump-diffusions as valuable tools in their day-to-day modeling in the last ten or so years. Some of the reasons given for use of jump diffusions in financial theory are that the price process behaves like a Brownian motion (BM) and that the probability of a stock moving by a large amount over a short period of time is very limited, unless we fix an unrealistically high value of asset volatility. Therefore in jump diffusion models prices of short term out of the money options are expected to be much lower than what we observe in real financial markets. On the other hand, if we allow stock prices to jump, even when the time to maturity is very short, there is a non-negligible probability that after a sudden change in the stock price the option can move to be in the money (See Xu et al, 2013, and Chen, Zhang and Gupta, 2014)).

Alternatively, from the point of view of hedging, continuous models of stock price behaviours can lead to a complete market or one which can be made complete by adding one or several instruments for instance in the case of stochastic volatility models. In such markets every terminal payoff of an option can be exactly replicated, that is options are made redundant assets, and hence the existence of traded options becomes a puzzle. From a risk management perspective, jumps are used to quantify and take into account the risk of strong stock price movements over very short time intervals, which may not exist in the diffusion framework. By giving an example from the domain of portfolio management, the constant proportion portfolio insurance strategy will consist of holding a proportion, Xt of the risky asset in the portfolio, where Xt is given by Xt = mVt − FtVt , where Vt is the portfolio value, Ft is the 'floor value of the portfolio', that is the 'insured' lower bound on the portfolio value, and m is a constant multiplier(See Xu et al,2013). In other words as the portfolio value approaches the lower boundary, the proportion of risky asset tends to approach zero. In a continuous-path model with frequent trading, the investment portfolio will therefore never go below the barrier, Ft. By taking a large multiplier, we can then construct a portfolio with a very significant upside potential with almost no downside risk. However, this illusion will break down as soon as we take into account the jump risk level. In other word there is always a non-zero probability that due to a sudden downward jump in the risky asset price, investors will not have a chance to withdraw their funds before the portfolio value drops below Ft level.

2.4.2 The building blocks of jump-diffusion models

The two main building blocks of every jump-diffusion model are the Brownian and Poisson motions (the diffusion and jump parts respectively). The Brownian motion is a familiar object to every option trader since the introduction of the Black-Scholes (1973) model, but knowledge of a few words about the Poisson process could be in order. The Poisson process on the other hand assumes a sequence, {τi}≥1 of independent exponential random variables with parameter λ=m, that is, with cumulative distribution (See Chen, Zhang and Gupta, 2014). For example, if the waiting times between airplane at an airport are exponentially distributed, the total number of planes arriving up to time t is a Poisson process. The trajectories of a Poisson process are assumed to be constant that is right-continuous with left limits (RCLL), with jumps of magnitude 1.00 only. These jumps occur at times, T_i and the intervals between jumps called waiting times are exponentially distributed. In other words at every date, t > 0, N_t will follow the Poisson distribution with parameter λt= m, a variable that assumes integer values only and p(X =x) = $\frac{e^{-m}\times m^{x}}{x!}$, where

x = A Poisson variable under consideration;

X= A specific value given to the Poisson distribution or variable (X) and

e=A given Poisson distribution parameter.

The Poisson distribution process shares with the Brownian motion the important characteristics of independence and stationarity of increments, that is, for every t > s, the increment, Nt − Ns is independent from the history of the process up to some time, s and has the same law as Nt−s. The processes undertaken with independent and stationary increments are called Levy processes named after the French mathematician, Paul Levy. The notion of characteristic functions of a random variable plays an essential role in the study of jump-diffusion processes. We often do not know the distribution function of such a process in closed forms but the characteristic function is known to be explicit at all times. The characteristic function of a random variable X is defined by φX(u) ≡ $E[e^{-iuX}]$. For a Poisson process, this gives the equation $E[e^{-iuX}] = \exp\{\lambda t(e^{-iuX} - 1)\}$ (See Xu et al, 2013). The compound Poisson process is a generalization where the waiting times between jumps are exponentially distributed but the jump sizes can have an imaginary distribution. More specifically, if we let N be a Poisson process with parameter, λ =m and {Yi}i≥1 be a sequence of independent random variables with law, f, then the process $X_t = \sum_{i=1}^{Nt} Yt$ where Yt is called a compound Poisson process.

2.4.3 Merton jump diffusion model

In the jump diffusion model, the stock price, S_t follows the random process given by $\frac{dSt}{St} = \mu dt + \sigma dWt + (J-1)N(t)$. The first two terms in the formula are familiar and drawn from the Black-Scholes model that is the drift rate,μ, volatility,σ, and random walk (Wiener process), W_t. The last term in the model represents the jumps, J that is it is the jump size as a multiple of stock price while N(t) is the number of jump events that have occurred up to time, t. is assumed to follow the Poisson process , $p(X = x) = \frac{e^{-m} \times m^x}{x!}$, where m is the average number of jumps per unit time. The jump size may follow any distribution, but a common choice is a log-normal distribution of the form, $J \sim m\exp\left(-\frac{v^2}{2} + vN(0,1)\right)$, where N(0,1) is the standard normal distribution, m is the average jump size, and v is the volatility of the jump size. The three parameters π, m, v do characterize the jump diffusion model (See Liu's model).

For European call and put options, closed-form solutions for the price can be found within the jump diffusion model in terms of Black-Scholes prices. If we write $P_{BS}(S, K, \sigma, r, T)$ as the Black-Scholes price of a call or put option with spot price, S, strike price, K, volatility, σ , interest rate, r (assumed constant for simplicity), and time to expiry T, then the corresponding price within the jump diffusion model can be written as:$P_{JD}(S, K, \sigma, r, T, \pi, m, v) = \sum_{k=o}^{\infty} \frac{\exp(-m\pi T)(m\pi T)^k)}{k!}$ Where $\sigma_{k=} \sqrt{\frac{\sigma^2 + kv^2}{T}}$ and $r_k = r - \pi(m-1) + \frac{k\log(m)}{T}$. The k^{th} term in this series corresponds to the scenario where, k jumps occur during the life of the option under investigation.

It can be shown that for all derivative securities with payoffs (which include regular call and put options) the price always increases when jumps are present (that is, when, $\pi > 0$ regardless of the average jump direction. Thus, holding other parameters constant, the option price is a minimum for m=1 (that is the Black-Scholes case) and increases both for $m < 1$ and $m > 1$. Therefore this increase in price can be interpreted as compensation for the extra risk taken by the option writer due to the presence of jumps, since this risk cannot be eliminated by delta hedging (See Joshi, 2003, Section 15.5). It is therefore the purpose of this study to combine fuzzy set theory and jump-diffusion processes to come up with a model called a fuzzy jump-diffusion model for valuation of firms. The derivation of the model is as presented, discussed and justified below.

3.0 Main Focus of the Paper

This section lays emphasis on derivation of the proposed jump-diffusion model and its validation based on financial data drawn from a firm in one of the countries of Southern Africa.

3.1 Derivation of the proposed fuzzy jump-diffusion model

Xu et al (2013) came up with a fuzzy jump-diffusion model to the pricing of European vulnerable options. It is this model that the study at hand exploited in order to propose a fuzzy jump-diffusion model for valuation of assets and liabilities of a firm. The model by Xu et al (2013) is based on S_t and A_t as values of a firm's stock and assets respectively. The change in the values of the stock of a firm and assets of the counterparty firm are given by the formulae:

$$\frac{dS_t}{S_t} = (r\text{-}mJs)\ d_t + \sigma_s dW_t^s + (\varepsilon^s\text{-}1)dN_t^s \quad (1)$$

and

$$\frac{dA_t}{A_t} = (r\text{-}mJv)\ d_t + \sigma_v dW_t^v + (\varepsilon^v\text{-}1)dN_t^v \quad (2)$$

where W_t^s and W_t^v are standard Brownian Motions and co-variance $(dW_t^v;\ dW_t^v)=\rho dt$, $r=R_f$ and

N_t =Poisson process with m, ε^i ($I\in \{S;V\}$) = Percentage of jump size, which follows a sequence of log-normally, identically and independently distributed variable, $E(\ln \varepsilon^i) = a_i$, $Var(\ln \varepsilon^i) = \sigma_i$. On the other hand, $J_i = E(\varepsilon^i - 1) = \exp\{a_i + \frac{1}{2}\sigma_i^2\}-1$, where $i\in \{S;V\}$.

Therefore based on equations 1 and 2 above, and Ito's formula we obtain the following general equation for call options written on stocks of two similar firms:

$$\text{In}S_T = \text{In}S_t + (r\text{-}\frac{1}{2}\sigma_s^2 - mJ_s)(T\text{-}t) + \sigma_s dW_{T-t}^s + \sum_{i=1}^{N(T-t)} \ln \varepsilon_i^s, \quad (3)$$

$$\text{In}V_T = \text{In}\ V_t + (r\text{-}\frac{1}{2}\sigma_v^2 - mJ_v)(T\text{-}t) + \sigma_v dW_{T-t}^v + \sum_{i=1}^{N(T-t)} \ln \varepsilon_i^v \quad (4)$$

3.1.1 Partitioning of jumps of stock prices

From the model by Amin (2011) we can partition the trading interval [t; T] into n sub-intervals of length, $h = (\frac{T-t}{n})$ where n= A fixed positive integer. Therefore if financial trading takes place at equidistant time points, $t_i = t + ih$, i=0, 1, 2, …………,n($s = r\text{-}\frac{1}{2}\sigma_s^2 - mJ_s$). In this respect a discrete time stochastic process defined on a state space (R) can be used to stimulate a jump-diffusion process. In other words in equation 3 above, stock price changes due to the inclusion of the jump variable are modelled by moving up or down. In practice stock price in state, j at date, i, will move to either state j+1 or j-1 at some future date, i+1.

According to Xu et al (2013) the jump component in stock price changes is modelled by multiple ticks on the state space grid at the next date excluding all the adjacent states as illustrated below:

Table 1 Showing Modelling of Stock Price Changes Excluding Adjacent States

i=0	1	2	…………………………	n	j
T0=t	T1=t+h	T2 =t+2h		Tn =t+nh	---
$\ln\frac{S_{2n}(h)}{S_0}$	$\varepsilon h + 2n\sigma\sqrt{h}$	$\varepsilon 2h + 2n\sigma\sqrt{h}$	…………………………	$\varepsilon h + 2n\sigma\sqrt{h}$	2n
$\ln\frac{S_2(h)}{S_0}$	$\varepsilon h + 2\sigma\sqrt{h}$	$\varepsilon 2h + 2\sigma\sqrt{h}$	…………………………	$\varepsilon h + 2n\sigma\sqrt{h}$	2
$\ln\frac{S_1(h)}{S_0}$	$\varepsilon h + 1\sigma\sqrt{h}$	$\varepsilon 2h + 1\sigma\sqrt{h}$	…………………………	$\varepsilon h + 2n\sigma\sqrt{h}$	1
$\ln\frac{S_0(h)}{S_0}$	εh	$\varepsilon 2h$	…………………………	$\varepsilon h + 2n\sigma\sqrt{h}$	0
$\ln\frac{S_{-1}(h)}{S_0}$	$\varepsilon h - 1\sigma\sqrt{h}$	$\varepsilon 2h - 1\sigma\sqrt{h}$	…………………………	$\varepsilon h + 2n\sigma\sqrt{h}$	-1
$\ln\frac{S_{-2}(h)}{S_0}$	$\varepsilon h - 2\sigma\sqrt{h}$	$\varepsilon 2h - 2\sigma\sqrt{h}$	…………………………	$\varepsilon h + 2n\sigma\sqrt{h}$	-2
$\ln\frac{S_{-2n}(h)}{S_0}$	$\varepsilon h - 2n\sigma\sqrt{h}$	$\varepsilon 2h - 2n\sigma\sqrt{h}$	…………………………	$\varepsilon h + 2n\sigma\sqrt{h}$	-2n

Source (Xu et al, 2013)

The proposed fuzzy jump-diffusion model adopted the concept of partitioning of jumps in its quest to value the equity and assets of firms drawn from countries in Southern Africa for the period under review.

3.1.2 Stock price distribution for a fixed discrete variable

The distribution of the price of a stock for a fixed variable or parameter, n is given by the equation,

$$q = \frac{\frac{\exp(rh) - mhE_{\varepsilon^S}(\varepsilon^S)}{1-nh} - \exp(Sh - \sigma_S\sqrt{h}\ )}{\exp(Sh + \sigma_S\sqrt{h}) - \exp(Sh - \sigma_S\sqrt{h}\ )} \qquad (5)$$

where q and q-1 are risk-neutral probabilities (RNP) of the up and down tick conditional movements on the diffusion component respectively and m= Probability of multiple ticks on the jump component.

In other words the option value at date,i can then be estimated from a fuzzy jump diffusion of the form given by:

$$C(I;j) = \exp(-rh)(mhE_{\varepsilon^S}[C\varepsilon^S(i+1)]\ ) + (1-mh)[qC(i+1;j+1) + (1-q)C(I+1;j-1)] \qquad (6)$$

Where $E_{\varepsilon^s}[CE_{\varepsilon^s}(i+1)]$ = The expectation operation with respect to the distribution of the variable, ε^s. Hence once the probability measure, Q is defined according to its value at date, i+1, the European option value at date, i will then be estimated using the formula,

$$Cij = \exp(-rh)E_Q[C_{i+1};.]. \quad (7)$$

Following the framework put across by Klein (2012) the time, t, price of a vulnerable option can be expressed in the form,

$$C_t = \exp[-r(T-t)]E_t^Q[(St-K)^t)(1_{VT} \geq D^{\cdot}) + 1_{VT} < D^{\cdot}\frac{(1-\theta)VT}{D}] \quad (8)$$

Using the tower property, the pricing formula of the option in (6) above can be restated to give the form;

$$C_t = \exp[-r(T-t)]E_t^Q[(St-K)^t)E_t^Q(1_{VT} \geq D^{\cdot}) + 1_{VT} < D^{\cdot}\frac{\frac{(1-\theta)VT}{D}}{S_T})]. \quad (9)$$

Finally, according to Liu and Liu (2012) and Xu et al (2012) the value of a vulnerable option at tome, T in state j, can be estimated using the formula:

$$Cnj = (St\exp\{Snh + j\sigma_s\sqrt{h}\} - K)[\emptyset n;j + \frac{1-\theta}{D}\,\omega n;j] \quad (10)$$

Where $\emptyset n;j = \emptyset(-fn;j)$ $\omega n;j =$ $gn;j\emptyset(fn;j-\sigma_T)$. It is against the above background and option valuation equations that this study proposed a jump diffusion-valuation models for the estimation of equity and assets of a firm respectively given by:

$$InS_T = InS_t + (r-\mu-\frac{1}{2}\sigma_s^2 - mJ_s)(T-t) + \sigma_s dW_{T-t}^s + \sum_{i=1}^{N(T-t)}\ln\varepsilon_i^s, \quad (11)$$

$$InA_T = In\,A_t + (r-\mu-\frac{1}{2}\sigma_v^2 - mJ_v)(T-t) + \sigma_v dW_{T-t}^v + \sum_{i=1}^{N(T-t)}\ln\varepsilon_i^v \quad (12)$$

The valuation of the assets and equity of a firm is assumed to be a function of a variety of variables as outlined above including market friction. Market friction is a variable which cannot be overlooked in all investment decisions and valuations because of its erosion of returns on investments particularly those made in financial markets of emerging economies. Section 4 below is devoted to the validation of the proposed model, results and discussions drawn from the financial data regressed using E-Views 8 package.

3.2 Model validation, results and discussion

The model proposed in the preceding section of the paper is validated using financial data drawn from a large listed banking corporation drawn from a growth promising economy in Southern Africa. The starting point in the pricing of the assets and equities of banks in emerging economies is to define the investor's belief degree, α, in order to be able to obtain the corresponding price intervals and the precise possibility mean values of vulnerable equity and asset values of firms. This section of the paper discusses the computation of corresponding belief degrees of the listed firm's assets (V_T) and equity (S_T) values from the perspective of a fuzzy financial environment, at some time interval, T-t. In this paper, the results generated are from financial theory on the use of the jump-diffusion model (JDM) by Xu et al (2013) in the valuation of options which states that

$L(\alpha) = (C(I;j))_\alpha^L$ increases with α, and decreases with $U(\alpha) == (C(I;j))_\alpha^U$. We therefore consider the following scenarios before we consider the numerical example below.

1. If $L(1) \leq c \leq U(1)$ then the corresponding investor's belief =1.00.
2. If $c \leq L(1)$. Since $c \leq L(1) \leq U(1) \leq U(O)$ then the corresponding beliefs degree is the solution to the equation, L (α)= c.
3. Iif c $\geq$U(1), since $L(0) \leq L(1) \leq U(1) \leq c$, then the corresponding belief degree of the investor is the solution to the equation, $U(\alpha)$ =c.

Hence in this numerical example, we employ the secant approach to solve the financial problem at hand using the algorithm defined as given below:

Step 1: We let ε be the risk tolerance level set x_1). $x_0 \leftarrow 0$ and $x_1 \leftarrow 1$.

Step 2: Find $L(x_1)$. If $|L(x_1). -c| < \varepsilon$, then the belief degree is x_1, otherwise we go straight to step 3.

Step 3: We proceed and set $x_2 = \frac{x_1.-x_0.}{L(x_1).-L(x_{\backslash 0}).}$; $x_0 \leftarrow x_1$ and $x_1 \leftarrow x_2$ and go to step 2. Even for the third case above, the above algorithm still remains applicable.

In the numerical example below, we present a fuzzy JDM for assets and equity of a firm based on the works propounded by Zhang (1998). We used the fuzzy jump-diffusion (FJD) model with n=100 and n=200. However for comparison purposes we also transform Klein's option pricing model into vulnerable assets and stock pricing models with n=∞ (See,Yu, Sun and Chen, 2011). In the illustration calculations of values of vulnerable assets and stocks of firms are based on the following central statistics that is means and standard deviations of triangular fuzzy numbers respectively:

$a = (a_1; a_2; a_3)$ where $M(a) = (\frac{1}{6}(a_1 + 4a_2 + a_3)$, m =(0.04; 0.045; 0.055) or (0.035; 0.045) while T-t = (0.25; 1.00). The fuzzy parameters used in the FJD model were drawn from a large bank in one of the emerging economies in Southern Africa.

The first table below show the price intervals for the diffusion level, $\propto$ under the proposed asset and stock JDM with parameters T-t =0.25, S_t = 0.724, $A_t = 11.48$, N =100 and m = (0.035; 0.045).

Table 2 Showing a Firm's Assets and Equity Values Under a JDM (T= 0.25)

$\propto -$Level	Interval for Asset Values	Interval for Stock Values
0.80	4.64-6.32	0.38-0.96
0.90	6.58- 6.86	1.26-1.44
0.93	6.95-7.48	1.54-1.66
0.95	7.86-8.28	1.74-1.92
0.98	8.42-8.72	2.02-2.28

From a practical perspective, it is important that $\alpha -$ level for fuzzy stock and asset prices be viewed from the perspective of interval prices with investor belief degrees of α being specified for financial investment analysts. For instance at an α =0.95 shown in the table, it means that the vulnerable asset and stock prices will lie in the intervals of 7.86-8.28 and 1.74-1.92 respectively, with a belief degree of 0.95. Hence if an investor is comfortable with a belief degree of 0.95 for own stock price, one will be free to pick any value between 1.74 and 1.92 as the vulnerable stock price for use in the foreseeable future. We proceed to compare the traditional asset and stock values of the bank above with the ones below generated according to the fuzzy and FJDM vulnerable stock price models based on a T-value of 0.25.

Table 3 Showing the Traditional and FJD Model Assets and Equity Results for the Firm (T-t=0.25)

S_t	A_t	σ_S	σ_A	n	ROE	μ	mJ	Trad S(T)	Fuzzy S(T)	FJDM-S(T)	Trad A(T)	Fuzzy A(T)	FJDM-A(T)
0.724	11.48	0.20	0.07	100	0.52	0.20	0.045	0.4317	0.1813	0.62	6.85	2.88	9.84
0.724	11.48	0.20	0.07	200	0.52	0.20	0.045	0.4317	0.1813	0.63	6.88	2.89	9.86
0.921	12.45	0.27	0.08	100	0.38	0.14	0.040	0.5920	0.1510	0.52	8.00	2.04	6.98
0.921	12.45	0.27	0.08	200	0.38	0.14	0.040	0.5920	0.1510	0.53	8.02	2.06	6.99
1.159	12.23	0.26	0.02	100	0.50	0.18	0.045	0.7958	0.2126	0.74	8.40	2.24	7.66
1.159	12.23	0.26	0.02	200	0.50	0.18	0.045	0.7958	0.2126	0.74	8.41	2.25	7.67
1.148	13.16	0.10	0.08	100	0.37	0.18	0.040	0.6963	0.1685	0.58	7.98	1.93	6.60

1.148	13.16	0.10	0.08	200	0.37	0.18	0.040	0.6963	0.1685	0.58	7.99	1.95	6.62
1.532	14.21	0.34	0.08	100	0.36	0.16	0.045	1.0571	0.2818	0.96	9.81	2.62	8.88
1.532	14.21	0.34	0.08	200	0.36	0.16	0.045	1.0571	0.2818	0.96	9.82	2.64	8.89
1.929	15.81	0.26	0.11	100	8.4165	0.12	0.040	1.3432	0.2792	0.94	11.0	2.28	7.80
1.929	15.81	0.26	0.11	200	8.4165	0.12	0.040	1.3432	0.2792	0.94	11.02	2.30	7.82

Asset and stock prices from FJDM were found to be higher than those estimated using the jump model under the same conditions (See Xu et al, 2013). This finding was reasonable because uncertainty of FJDM was more than that of the jump model. The study also discovered that the firm's asset and stock prices from traditional and jump model were very different in magnitude and impact. It was found that the FJDM was based on assumptions that both values of assets and equity of the firm followed a log-normal distribution. The values from both models appeared to be high for average jump intensities mainly because of the direct relationship between average jump intensities and uncertainty. The above results of the study were also reinforced when the term to maturity was changed from (T-t) =0.25 to 1.00 as illustrated in table 3 below.

Table 4 Showing the Traditional and FJD Model Assets and Equity Results for the Bank (T-t= 1.00)

S_t	A_t	σ_S	σ_A	n	ROE	μ	m	Trad S(T)	Fuzzy-S(T)	FJDM-S(T)	Trad A(T)	Fuzzy-A(T)	JDM-A(T)
0.724	11.48	0.20	0.07	100	0.52	0.20	0.045	0.4317	0.1813	5.06	10.20	4.28	14.6
0.724	11.48	0.20	0.07	200	0.52	0.20	0.045	0.4317	0.1813	5.07	10.25	4.30	14.7
0.921	12.45	0.27	0.08	100	0.38	0.14	0.040	0.5920	0.1510	1.78	11.40	2.91	9.96
0.921	12.45	0.27	0.08	200	0.38	0.14	0.040	0.5920	0.1510	1.78	11.42	2.92	9.99
1.159	12.23	0.26	0.02	100	0.50	0.18	0.045	0.7958	0.2126	2.52	11.7	3.13	10.7
1.159	12.23	0.26	0.02	200	0.50	0.18	0.045	0.7958	0.2126	2.54	11.8	3.14	10.8
1.148	13.16	0.10	0.08	100	0.37	0.18	0.040	0.6963	0.1685	1.98	11.20	2.71	9.26
1.148	13.16	0.10	0.08	200	0.37	0.18	0.040	0.6963	0.1685	1.99	11.4	2.72	9.28
1.532	14.21	0.34	0.08	100	0.36	0.16	0.045	1.0571	0.2818	3.28	13.20	3.52	12.0
1.532	14.21	0.34	0.08	200	0.36	0.16	0.045	1.0571	0.2818	3.28	13.20	3.54	12.1
1.929	15.81	0.26	0.11	100	8.4165	0.12	0.040	1.3432	0.2792	3.20	13.30	2.76	9.44
1.929	15.81	0.26	0.11	200	8.4165	0.12	0.040	1.3432	0.2792	3.22	14.4	2.78	9.50

The changing of the time interval from T-0.25 to 1.00 improved the asset and stock values of the firm significantly. In other words the use of medium to long term capital and investment strategies had the capacity to grow the firm in terms of market share and generation of shareholders' wealth.

Table 5 Showing Belief Degrees and Prices of Vulnerable Assets and Equity of the Bank

$\propto$ **−Level**	**0.3971**	**0.5933**	**0.8268**	**0.9406**	**0.9989**	**0.9892**	**0.7457**	**0.6262**	**0.5522**
Stock Price ($)	0.64	0.72	0.98	1.22	1.38	1.62	1.38	1.16	0.96
Asset Price ($)	10.60	10.84	11.42	12.64	13.28	13.06	12.68	12.36	12.18

It was discovered that a stock price of $12.64 had a belief degree level of 0.9406. Hence if the financial investor is comfortable with such a belief degree then they can employ such a price in their future investment decisions. It is also realized that a stock with a price of $13.28 will have a belief degree of approximately 1.00.

4.0 Conclusions and Recommendations

It was concluded that the pricing of a firms' assets and stocks was very essential in mathematical finance in their desire to grow market shares and shareholders' wealth. The only problem found in pricing of assets and equities in financial markets was that of pricing them efficiently and accurately. From a classical perspective, the problem of estimating the values of assets and equity of a firm was examined from a stochastic environment that existed in the real world. The input parameters of all classical asset and stock pricing models are usually regarded as precise or exact real numbers. However in the real world, the unpredictable nature of financial markets and their operations makes it unrealistic to assume that risk free rates of return and average jump intensities are constant variables over time. The study transformed fuzzy patterns that were put across by Xu et al (2013) vulnerable option pricing models into asset and equity pricing formulae as proposed in section 3 of the paper. The transformed models are then used to estimate market values of vulnerable assets and equity of a firm based on crisp possibility jump values drawn from the mean jump-diffusion model (JDM).

The study went on to compare the traditional and fuzzy jump-diffusion model results. Both sets of results were found to be reasonable as they had the ability to instruct financial investors more efficiently in their planning and decision making processes. However, all the fuzzy numbers used in the study are assumed to be members of the triangular fuzzy set. The study ended by recommending that future researches in asset valuations or mathematical finance should consider other types of fuzzy numbers in order to increase the estimation precisions and consistency in pricing of assets, liabilities and equity of firms. It is also recommended further that asset and stock pricing models should be based on variable average returns and standard deviations to be realistic and reflective of the actual financial market operations. This is because the assumption used in this study that a firm's assets and stocks could have the same volatility could be very misleading and unrealistic.

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APPENDIX

Appendix I Showing the Bank's Equity, Assets, Liabilities and Other Fundamentals for the Period 2008-2013 (Amounts in ZAR BLNs)

Year	2008	2009	2010	2011	2012	2013
Equity (A-L)	0.724	0.921	1.159	1.148	1.532	1.929
Assets	11.482	12.449	12.233	13.159	14.214	15.807
Liabilities	10.758	11.528	11.074	12.011	12.682	13.878
ROE (T%)	51.7	44.2	37.6	50.0	37.1	36.3
CC(%)	19.9	13.5	17.8	17.6	16.4	12.4
σ_E(%)	19.24	27.03	25.84	0.95	33.54	25.77
σ_A(%)	6.84	8.42	1.74	7.57	8.02	11.21
d_1	5.64	4.60	17.1	5.52	4.04	3.35
d_2	5.57	4.52	17.09	5.44	3.96	3.24
$N(d_1)$	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996
$N(d_2)$	1.0000	1.0000	1.0000	1.0000	1.0000	0.9994
Equity (T)	0.4317	0.5920	0.7958	0.6963	1.0571	1.3432
d_{1F}	9.36	12.4	13.0	13.4	11.8	8.01
d_{2F}	9.23	12.2	12.9	13.2	11.7	7.76
ROE (F %)	137.9	180.8	169.6	191.9	169.3	193.3
σ_AF (%)	13.4	14.2	12.5	13.8	14.0	24.6
N (d_{1F})	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
N (d_{2F})	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Equity (F)	0.1823	0.1510	0.2126	0.1685	0.2818	0.2792