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# EXPLORING MARDER'S SPACE-TIME IN FRACTAL COSMOLGY

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*Abstract:* In this investigation, the anisotropic and homogeneous Marder's spacetime is analyzed within the framework of fractal gravity theory, incorporating the properties of a perfect fluid. The field equations are solved using Hubble's Law Proposed by Berman and Gomide [37] and assuming the relation $P_1 = P_2{}^n$ , $n \neq 1$. We have discussed, cosmological parameters such as, pressure, density, Hubble's parameter, spatial volume and equation of state parameter.

## I. Introduction

In recent years, contemporary astrophysical investigations have highlighted the observation that the current universe is undergoing not only expansion but also an acceleration in its expansion rate. This phenomenon has been corroborated by various cosmological experiments, including the analysis of Type Ia supernovae (SNe Ia) [1]-[3], Cosmic Microwave Background (CMB) [4, 5] & Studies of large-scale structure [6, 7]and determined that dark energy constitutes the primary factor driving the observed accelerated expansion of the universe.

Dark energy accounts for approximately two-thirds of the total energy density of the universe. Modified theories of gravity offer a more promising approach to elucidating the mechanisms responsible for the accelerated expansion of the universe. These theories aim to account for the observed late-time cosmic dynamics without relying on dark energy components in the field equations. In recent times, a number of modified theories of gravity have been proposed. Some of these are, f(R) gravity, where R is Ricci Scalar [8, 9]. f(G) gravity [10, 11], Modified Gauss-Bonnet gravity [12, 13], Fractal Gravity [14], f(τ) theory [15, 16], f (R, T) gravity [17]. Fractal cosmology refers to the application of fractal geometry to the study of the large-scale structure of the universe. Numerous cosmological theories [18–20] propose that the structure of the universe exhibits fractal characteristics. Calcagni [14] developed a quantum field theory applicable to a fractal universe that is power-counting renormalizable, Lorentz invariant, and free from ultraviolet divergences. Fractal Cosmology refers to a framework for describing quantum gravity in which the coupling constants exhibit running scaling dimensions. This characteristic enables the system to transition from an effective two-dimensional phase in the ultraviolet regime to a conventional field theory in the infrared limit. Karami et al. [21] investigated the flat Friedmann-Robertson-Walker (FRW) model, incorporating both matter and dark energy within the framework of fractal cosmology. Their study revealed that this model predicts an expanding universe characterized by super-accelerated expansion. Pawar et al. [22] investigated a flat fractal FRW cosmological model incorporating a domain wall. Their analysis indicated that domain walls could be present in the early universe but are expected to dissipate or vanish in the distant future. Mamon [23] investigated the Tsallis holographic dark energy model within the framework of fractal cosmology and observed that the universe experiences a smooth transition from a decelerated to an accelerated phase of expansion in the recent past. flat Friedmann-Robertson-Walker (FRW) model incorporating quark and strange quark matter within the context of fractal gravity was investigated by Pawar et al. [24]. Das et al. [25] examined the evolution of a fractal model of the universe within the context of a homogeneous and isotropic FLRW spacetime geometry. Cosmai et al. [26] explored cosmic acceleration and the fractal nature of the universe within the framework of a Lemaˆıtre-Tolman-Bondi (LTB) scenario. Pawar et al..[27] investigated a cosmological model featuring a perfect fluid with anisotropic plane symmetry using the principles of fractal cosmology. Chattopadhyay etal.[28] investigated various holographic Ricci dark energy cosmological models within the framework of fractal gravity and observed that these models predict an accelerated expansion of the universe. Pawar et al. [29] studied dark energy cosmological model using specific Hubble parameter in fractal gravity. Recently in fractal gravity Pawar et al. [30] investigated observational constraints on wet dark fluid.

The total action of Einstein's gravitational theory in a fractal space time framework can be expressed as:

$$S=\frac{1}{16\pi G}\int\sqrt{-g}\left(R-2\Lambda-\omega\partial_\mu\tau\partial^\mu\tau\right)d\xi(x)+\int\sqrt{-g}\,\mathcal{L}_m d\xi(x) \tag{1}$$

where $g$ denotes the determinant of the metric tensor, $R$ represents the Ricci scalar, and $\Lambda$ is the cosmological constant. The matter component of the Lagrangian density is denoted as $\mathcal{L}_\mathrm{m}$. Additionally, $\tau$ represents the fractal function, and $\omega$ denotes the fractal parameter, respectively. The standard measure $d^4(x)$ is reformulated using the Lebesgue–Stieltjes measure $d\xi(x)$. The scaling dimension of $\xi$ is given by $[\xi] = -B\eta$, where $B$ represents the dimensionality of the underlying space-time and $\eta$ be the positive parameter. In the present study, we will assume $B = 4$. Inspired by the above work in the present research, we investigate Marder's space time with perfect fluid in the framework of fractal gravity. This paper is structured as follows: Section 2 addresses the metric and field equations pertinent to fractal cosmology. In Section 3 solutions to the field equations are derived and the associated physical and kinematical parameters of the model are discussed. Section 4 is dedicated to the conclusions of the study.

## 2. Metric and Field Equations

We consider Marder's space time, characterized by the line element given by

$$ds^2={P_1}^2(dx^2-dt^2)+{P_2}^2dy^2+{P_3}^2dz^2 \tag{2}$$

Where $P_1$, $P_2$ and $P_3$ are functions of t only.

In the investigation of the early stages of the universe, the anisotropic and homogenous Marder's space- time plays a crucial role. With the help of Marder's Metric Singh and Abdussattar [31], Prakash [32] derived plane symmetric cosmological models. Aygun et.al. [33] have studied energy momentum localization in Marder's space-time. Pawar et.al. [34] have discussed power law and exponential expansion model in Marder's space-time. Dhore and Ugale [35] have investigated Tsallis holographic dark energy cosmological models based on Marder's space-time within the framework of *f(R, T)* theory of gravity. Santhi and Chinappalanaidu [36] have studied Marder's space-time with Tsallis holograpgic dark energy.

By varying the total action (1) with respective $g_{ij}$, the field equations in fractal cosmology are given by

$$R_{ij}-\frac{1}{2}g_{ij}(R-2\Lambda)+\ g_{ij}\frac{\Box\upsilon}{\upsilon}-\frac{\nabla_i\nabla_i\upsilon}{\upsilon}\ +\ \omega\left(\frac{1}{2}g_{ij}\partial_\mu\partial^\mu\upsilon-\partial_i\partial_j\right)=k^2T_{ij} \tag{3}$$

The field equations (3) for metric (2) using perfect fluid reduces to

$$\frac{1}{{P_1}^2}\left[\frac{\ddot{P_2}}{P_2}+\frac{\ddot{P_3}}{P_3}-\frac{\dot{P_1}\dot{P_2}}{P_1P_2}+\frac{\dot{P_2}\dot{P_3}}{P_2P_3}-\frac{\dot{P_1}\dot{P_3}}{P_1P_3}-\left(\frac{\dot{P_1}}{P_1}+\frac{\dot{P_2}}{P_2}+\frac{\dot{P_3}}{P_3}\right)\frac{\dot{\upsilon}}{\upsilon}+2\Lambda-\frac{\ddot{\upsilon}}{\upsilon}+\frac{1}{2}\omega\dot{\upsilon}^2\right]=K^2p \tag{4}$$

$$\frac{1}{{P_1}^2}\left[-\left(\frac{\dot{P_1}}{P_1}\right)^2+\frac{\ddot{P_1}}{P_1}-\frac{\ddot{P_3}}{P_3}-\left(\frac{2\dot{P_1}}{P_1}+\frac{\dot{P_2}}{P_2}+\frac{\dot{P_3}}{P_3}\right)\frac{\dot{\upsilon}}{\upsilon}+\Lambda-\frac{\ddot{\upsilon}}{\upsilon}-\frac{1}{2}\omega\dot{\upsilon}^2\right]=K^2p \tag{5}$$

$$\frac{1}{{P_1}^2}\left[-\left(\frac{\dot{P_1}}{P_1}\right)^2+\frac{\ddot{P_1}}{P_1}-\frac{\ddot{P_2}}{P_2}-\left(\frac{2\dot{P_1}}{P_1}+\frac{\dot{P_2}}{P_2}+\frac{\dot{P_3}}{P_3}\right)\frac{\dot{\upsilon}}{\upsilon}+\Lambda-\frac{\ddot{\upsilon}}{\upsilon}-\frac{1}{2}\omega\dot{\upsilon}^2\right]=K^2p \tag{6}$$

$$-\frac{\dot{P_1}\dot{P_2}}{P_1P_2}-\frac{\dot{P_2}\dot{P_3}}{P_2P_3}-\frac{\dot{P_1}\dot{P_3}}{P_1P_3}-\left(\frac{3\dot{P_1}}{P_1}+\frac{\dot{P_2}}{P_2}+\frac{\dot{P_3}}{P_3}\right)\frac{\dot{\upsilon}}{\upsilon}+2\Lambda-\frac{\ddot{\upsilon}}{\upsilon}+\frac{1}{2}\omega\dot{\upsilon}^2=K^2\left(p\left(1-{P_1}^2\right)+\rho\right) \tag{7}$$

For K=1, without cosmological constant and assuming $\upsilon = t^{-\alpha}$ field equations (4)-(5), reduces to

$$\frac{1}{{P_1}^2}\left[\frac{\ddot{P_2}}{P_2}+\frac{\ddot{P_3}}{P_3}-\frac{\dot{P_1}\dot{P_2}}{P_1P_2}+\frac{\dot{P_2}\dot{P_3}}{P_2P_3}-\frac{\dot{P_1}\dot{P_3}}{P_1P_3}-\left(\frac{\dot{P_1}}{P_1}+\frac{\dot{P_2}}{P_2}+\frac{\dot{P_3}}{P_3}\right)\frac{\alpha}{t}-\frac{\alpha(1+\alpha)}{t^2}+\frac{\omega\alpha^2}{2t^{2(1+\alpha)}}\right]=p \tag{8}$$

$$\frac{1}{P_1{}^2}\left[-\left(\frac{\dot{P}_1}{P_1}\right)^2+\frac{\ddot{P}_1}{P_1}-\frac{\ddot{P}_3}{P_3}-\left(\frac{2\dot{P}_1}{P_1}+\frac{\dot{P}_2}{P_2}+\frac{\dot{P}_3}{P_3}\right)\frac{\alpha}{t}-\frac{\alpha(1+\alpha)}{t^2}-\frac{\omega\alpha^2}{2t^{2(1+\alpha)}}\right]=p \tag{9}$$

$$\frac{1}{P_1{}^2}\left[-\left(\frac{\dot{P}_1}{P_1}\right)^2+\frac{\ddot{P}_1}{P_1}-\frac{\ddot{P}_2}{P_2}-\left(\frac{2\dot{P}_1}{P_1}+\frac{\dot{P}_2}{P_2}+\frac{\dot{P}_3}{P_3}\right)\frac{\alpha}{t}-\frac{\alpha(1+\alpha)}{t^2}-\frac{\omega\alpha^2}{2t^{2(1+\alpha)}}\right]=p \tag{10}$$

$$-\frac{\dot{P}_1\dot{P}_2}{P_1P_2}-\frac{\dot{P}_2\dot{P}_3}{P_2P_3}-\frac{\dot{P}_1\dot{P}_3}{P_1P_3}-\left(\frac{3\dot{P}_1}{P_1}+\frac{\dot{P}_2}{P_2}+\frac{\dot{P}_3}{P_3}\right)\frac{\alpha}{t}-\frac{\omega\alpha^2}{2t^{2(1+\alpha)}}=\left(p\left(1-P_1{}^2\right)+\rho\right) \tag{11}$$

Now for the UV-regime ($\alpha = 2$), above field equations takes the form

$$\frac{1}{P_1{}^2}\left[\frac{\ddot{P}_2}{P_2}+\frac{\ddot{P}_3}{P_3}-\frac{\dot{P}_1\dot{P}_2}{P_1P_2}+\frac{\dot{P}_2\dot{P}_3}{P_2P_3}-\frac{\dot{P}_1\dot{P}_3}{P_1P_3}-\frac{2}{t}\left(\frac{\dot{P}_1}{P_1}+\frac{\dot{P}_2}{P_2}+\frac{\dot{P}_3}{P_3}\right)-\frac{6}{t^2}+\frac{2\omega}{t^6}\right]=p \tag{12}$$

$$\frac{1}{P_1{}^2}\left[-\left(\frac{\dot{P}_1}{P_1}\right)^2+\frac{\ddot{P}_1}{P_1}-\frac{\ddot{P}_3}{P_3}-\frac{2}{t}\left(\frac{2\dot{P}_1}{P_1}+\frac{\dot{P}_3}{P_3}\right)-\frac{6}{t^2}+\frac{2\omega}{t^6}\right]=p \tag{13}$$

$$\frac{1}{P_1{}^2}\left[-\left(\frac{\dot{P}_1}{P_1}\right)^2+\frac{\ddot{P}_1}{P_1}-\frac{\ddot{P}_2}{P_2}-\frac{2}{t}\left(\frac{2\dot{P}_1}{P_1}+\frac{\dot{P}_3}{P_3}\right)-\frac{6}{t^2}+\frac{2\omega}{t^6}\right]=p \tag{14}$$

$$-\frac{\dot{P}_1\dot{P}_2}{P_1P_2}+\frac{\dot{P}_2\dot{P}_3}{P_2P_3}-\frac{\dot{P}_1\dot{P}_3}{P_1P_3}-\frac{2}{t}\left(\frac{3\dot{P}_1}{P_1}+\frac{\dot{P}_2}{P_2}+\frac{\dot{P}_3}{P_3}\right)-\frac{2\omega}{t^6}=\left(p\left(1-P_1{}^2\right)+\rho\right) \tag{15}$$

## 3. Solution of Field Equations

The Field equations (12)-(15) are nonlinear differential equations in six unknowns. Hence, to find an explicit solution, we have to consider two extra conditions. First condition is

Assuming relation $P_1 = P_2{}^{n}$, where $n \neq 1$ (16)

and secondly, we use Hubble's Law Proposed by Berman and Gomide [37] given by

$$A = \frac{\dot{A}}{A} = BA^{-m} \tag{17}$$

Where A is average scale factor, B > 0 and m ≥ 0 are constants.

On comparing equations (13) and (14), we have obtained P2 = P3.

Solving eq. (17), we

$$A = (mBt + c)^{\frac{1}{m}},\ m \neq 0 \tag{18}$$

where c is constant of integration

## 4. Physical and Geometric Parameters of the Model

For metric (2) , we define the following Physical and Geometric Parameters

The average scale factor is defined as

$$A = \left(P_1P_2{}^2\right)^{\frac{1}{3}} \tag{19}$$

From eq. (16), (18) and (19), we get

$$P_1 = (mBt + c)^{\frac{3n}{m(n+2)}} \tag{20}$$

Using eqs. (16) & (20), we have

$$P_2 = P_3 = (mBt + c)^{\frac{3}{m(n+2)}} \tag{21}$$

The Spatial volume is given by

$$V = P_1P_2{}^2 = (mBt + c)^{\frac{3}{m}} \tag{22}$$

Hubble parameter is given by

$$A = \frac{\dot{A}}{A} = \frac{B}{(mBt+c)} \tag{23}$$

The expansion scalar, reads as

$$\theta = 3H = \frac{3B}{(mBt+c)} \tag{24}$$

The mean anisotropic parameter is defined as

$$A_m = \frac{1}{3}\sum_{r=3}^{3}\left(\frac{\Delta H_r}{H}\right)^2 = 2\left(\frac{n-1}{n+1}\right)^2 \quad \Delta H_r = H_r - H \tag{25}$$

Where *H1, H2* and *H3* are the directional Hubble's parameter along x,y & z-axes respectively. The shear scalar is given by

$$\sigma^2 = \frac{1}{2}\sum_{r=3}^{3}\left({H_r}^2 - 3H^2\right) = 3B^2\left(\frac{n-1}{n+2}\right)^2 \ (mBt + c)^{-2} \tag{26}$$

The deceleration parameter is given by

$$q = -\frac{A\ddot{A}}{\dot{A}^2} = m - 1 \tag{27}$$

Where $m \geq 0$ is constant. Deceleration parameter (q) and the Hubble's parameter (H) are critical observational metrics in cosmology that facilitate the assessment of the universe's expansion history.A positive value of the deceleration parameter ($q > 0$) indicates a decelerating universe, while a negative value ($q < 0$) denotes an accelerating universe. In our model, we consider the range $0 \leq m < 1$, which corresponds to an accelerating universe and is indicative of conditions conducive to inflation. From eqs. (14) & (15) the pressure and density of the model are given by

$$p = (mBt + c)^{\frac{-6n}{m(n+2)}}\begin{bmatrix} \frac{3mB^2}{(n+2)^2}\left(\frac{3}{m(n+2)} - 1\right)(mBt + c)^{-2}\begin{bmatrix} 2(n+1) - 3Bn(mBt + c)^{-1} \\ +(mBt + c)^{-2} \end{bmatrix} \\ -\frac{6B(mBt+c)^{-1}}{t(n+2)}\left(n + 3mB\left(\frac{3}{m(n+2)} - 1\right)(mBt + c)^{-1}\right) - \frac{6}{t^2} + \frac{2\omega}{t^6} \end{bmatrix} \tag{28}$$

$$\rho = \frac{9B^2}{(n+2)^2}(2n + 1)(mBt + c)^{-2} - \frac{2}{t}\left[\frac{3B}{(n+2)(3n+2)(mBt+c)^{-1}}\right] - \frac{2\omega}{t^6} - p\left(1 - (mBt + c)^{\frac{-6n}{n+2}}\right) \tag{29}$$

Figure 1 illustrates the variation of the scale factor as a function of cosmic time. It is observed that at the initial time $t = 0$, the scale factor is finite and subsequently increases with the progression of cosmic time, indicating an expanding universe. From Figure 2, it can be observed that the Hubble parameter is a decreasing function of cosmic time. At $t = \frac{-c}{mB}$, the spatial volume of the model approaches zero, leading to the divergence of all kinematics and physical parameters, including the Hubble parameter, shear scalar, expansion scalar, pressure, and density. This singular behavior indicates that the universe initiates its expansion from a Big Bang singularity. As $t \rightarrow \infty$, all the physical and kinematic parameters approach zero, and the spatial volume becomes infinite. Therefore, our model predicts a vacuum spacetime for very large cosmic times.

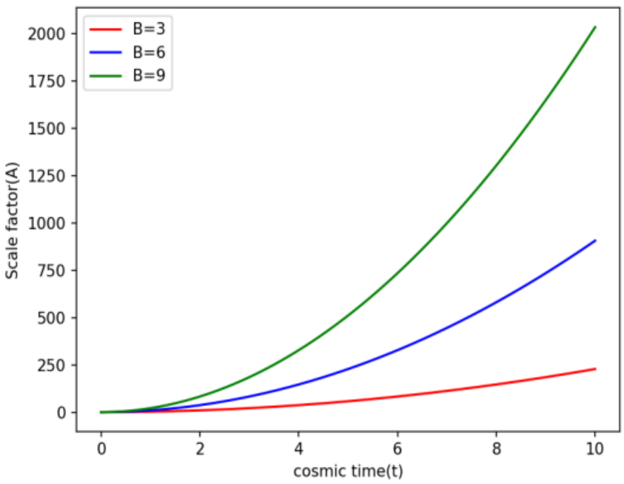


Figure1: Profile of scale against cosmic time

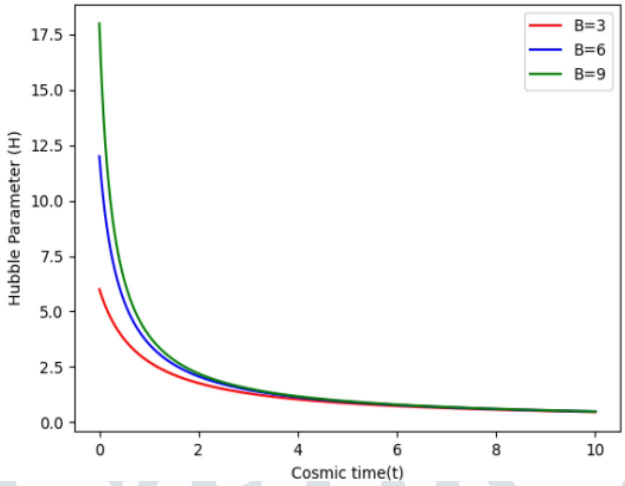


Figure 2: Profile of Hubble parameter against cosmic time

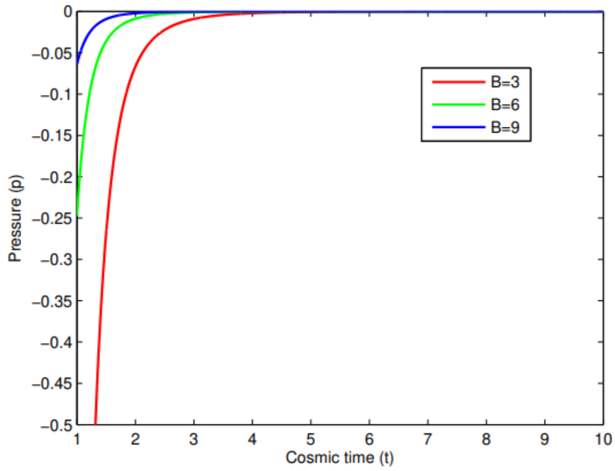


Figure 3: Profile of pressure against cosmic time

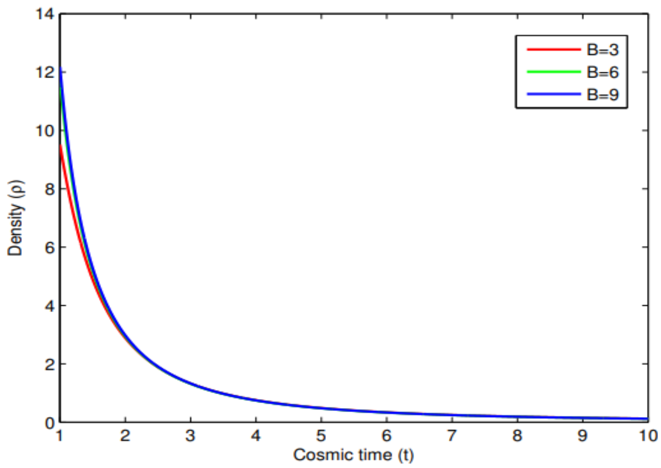


Figure 4: Profile of Density against cosmic time

Figures 3 and 4 illustrate the plots of pressure and energy density. It is observed that, at the initial conditions, pressure exhibits a significantly large negative value, while energy density demonstrates a correspondingly large positive value. The pressure is characterized as an increasing function of cosmic time, whereas energy density is identified as a decreasing function of cosmic time. As time progresses towards large values, both pressure and energy density asymptotically approach zero. The presence of negative pressure is indicative of an accelerating universe. Figure 5 shows the plot of Equation of State parameter against cosmic time. Dark energy is characterized by its equation of state (EoS) parameter, defined as ratio of pressure (p) and energy density (ρ). When EoS=0, the pressure (p) is equal to the energy density (ρ), which is referred to as stiff fluid matter. In the case where EoS=1, the model corresponds to a dust universe, characterized by non-relativistic matter. Conversely, if EoS=−1, the system is identified as the ΛCDM model, representing the standard dark energy framework. Additionally, when EoS$< -1$, the scenario is classified as a phantom model. Initially, the present model is observed to correspond to a dust universe. As cosmic time progresses, the model transitions to exhibit the characteristics of the standard ΛCDM behavior. Subsequently, it crosses the phantom divide, and eventually, a singularity is reached.

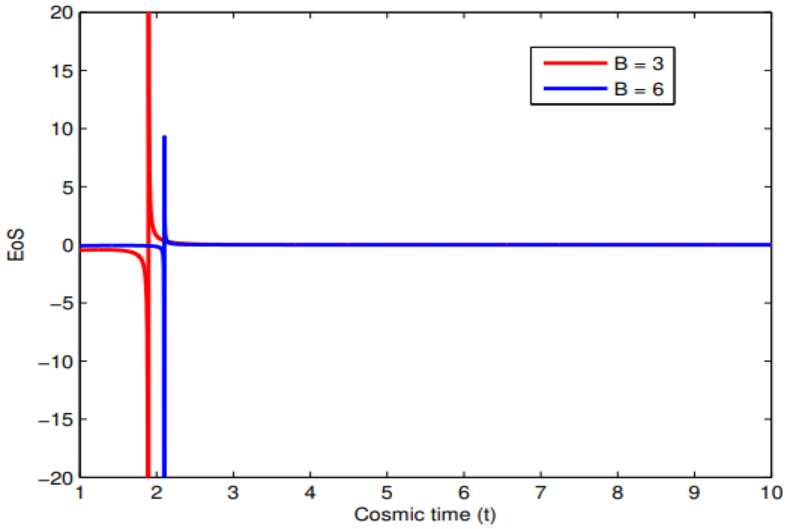


Figure 5: Profile of EoS against cosmic time

## 5. Conclusions

In the present paper, we have investigated the anisotropic and homogeneous Marder's space–time with perfect fluid in the framework of fractal gravity. To find the solutions of corresponding field equations, we have used Hubble's Law Proposed by Berman and Gomide and assuming the relation$P_1 = P_n{}^2$ , $n \neq 1$. We examine the model in the ultraviolet (UV) regime under the assumption of the absence of a cosmological constant. We found that the universe initiates its expansion from a Big Bang singularity and for large time current model essentially gives an empty space. Initially the present model behaves like dust universe and as time progresses it shows standard λCDM behaviour and crosses phantom divide line and reached a singularity. phantom dark energy model. For the range $0 \leq m < 1$, the present model exhibits both expansion and acceleration, consistent with recent observational data. For $n \neq 1$, the mean anisotropic parameter is non zero, therefore the model is anisotropic.

## Conflict of Interest

All authers declare that there are no competing interests.

## References

[1] A. G. Riess, A. V. Filippenko, P. Challis et al., The Astronomical Journal, vol. 116, no. 3, pp. 1009–1038, 1998.

[2] S. J. Perlmutter, G. Aldering, G. Goldhaber et al., The Astrophysical Journal, vol. 517, no. 2, pp. 565–586, 1999.

[3] N. A. Bahcall, J. P. Ostriker, S. Perlmutter, and P. J. Steinhardt, Science, vol. 284, no. 5419, pp. 1481–1488, 1999.

[4] C. L. Bennett, M. Halpern, G. Hinshaw et al., The Astrophysical Journal Supplement Series, vol. 148, pp. 1–27, 2003.

[5] D. N. Spergel, L. Verde, H. V. Peiris et al., The Astrophysical Journal Supplement Series, vol. 148, no. 1, pp. 175–194, 2003.

[6] M. Tegmark, M. A. Strauss, M. R. Blanton et al., Physical Review D, vol. 69, no. 10, Article ID 103501, 26 pages, 2004.

[7] A. C. Pope, T. Matsubara, A. S. Szalay et al., The Astrophysical Journal, vol. 607, no. 2, pp. 655–660, 2004.

[8] S. Nojiri & S. D. Odintsov, Physical Review D, vol. 74, 2006.

[9] S. Nojiri & S. D. Odintsov, International Journal of Geometric Methods in Modern Physics, vol. 4, no. 1, pp. 115–145, 2007.

[10] S. Nojiri and S. D. Odintsov, Phys. Lett. B 631, 1, 2005.

[11] B. Li, J. D. Barrow, and D. F. Mota, Phys. Rev. D 76, 044027,2007.

[12] K. Bamba, A. N. Makarenko, A. N. Myagky, and S. D. Odintsov, Phys. Lett. B 732, 349, 2014.

[13] K. Bamba, A. N. Makarenko, A. N. Myagky, and S. D. Odintsov, J. Cosmol. Astropart. Phys. 04,001,2015.

[14] G. Calcagni, J. High Energy Phys. 03, 120,2010.

[15] E. V. Linder, Phys. Rev. D 81, 127301,2010.

[16] R. Myrzakulov, Eur. Phys. J. C 71, 1752,2011.

[17] T. Harko, F. S. N. Lobo, S. Nojiri, and S. D. Odintsov, Phys. Rev. D 84, 024020,2011.

[18] J. Ambj¨orn, J. Jurkiewicz and R. Loll, Phys. Rev. Lett. 95, 171301,2005, arXiv:hep-th/0505113. 10

[19] O. Lauscher and M. Reuter, J. High Energy Phys. 10, 050,2005, arXiv:hep-th/ 0508202v1.

[20] L. Modesto, Class. Quantum Grav. 26, 24002 (2009), arXiv:0812.2214v1 [gr-ac].

[21] K. Karami, et al., Can. J. Phys. 91, 770–776, 2013.

[22] D. D. Pawar, D. K. Raut and W. D. Patil, Int. J. Mod. Phys. A 35, 2050072, 2020.

[23] Mamon, Abdulla.Al., Mod. Phys. Letts. A 2050251, 2020.

[24] D. D. Pawar, D. K. Raut & W. D. Patil, Mod. Phys. Lett. A. 37, 19, 2022a.

[25] Das, et al., Eur. Phys. J. C 78, 849, 2018.

[26] L. Cosmai, G. Fanizza, et al., Classical Quantum Gravity 36, 045007, 2019.

[27] D. D. Pawar, D. K. Raut & W. D. Patil, New Astron. 100, 101999,2022b.

[28] S. Chattopadhyay, A. Pasqua and S. Roy, ISRN High Energy Phys, 251498,2013.

[29] D.D. Pawar, D.K. Raut, A.P. Kale, New Astronomy 110, 102214,2024a.

[30] D.D. Pawar, D.K. Raut, A.P. Nirwal , Shaily & J.K. Singh,Astronomy and Computing 48, 100848, 2024b.

[31] K. P. Singh, J. Abdussattar, J. Phys. A 6 (7), 1090, 1973.

[32] S. Prakash, Astrophys. Space Sci. 111 (2), 383–388, 1985.

[33] S. Ayg¨un, M.Ayg¨un & S. Tripathy, Pramana Journal of Physics, 68(1), 21-30,2007.

[34] D.D. Pawar, S. P. Shahare, V. J. Dagwal, New Astronomy, 87,101599, 2021.

[35] A. O. Dhore & M. R. Ugale, East European Journal of Physics, 2,36-74, 2024.

[36] M. Vijya Santhi & T. Chinnappalanaidu,Int. J. Geom.Methds. Mod. Phys. , 19(13), 2250211, 2022.

[37] M. S. Berman & de Mello Gomide , Gen. Relativ. Gravit. 20 (2), 191–198, 1998.