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The solution of Fractional Kinetic Equation involving $p-k$ Generalised C- Series and $p-k$ Multi-Index Mittag Leffler Function

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Abstract

In this paper, we further generalise the fractional kinetic equation by way of associating it with $p-k$ generalised C- series and $p-k$ multi-index Mittag Leffler function. The solution of the generalised fractional kinetic equation is derived by applying a general Integral transform. Some particular cases of fractional kinetic equation are also discussed.

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1 Introduction

The main aim of this paper is to generalise fractional kinetic equation(FKE) by way of associating it with "$p-k$ generalised C series and $p-k$ multi-index Mittag Leffler function and to find solution of FKE by applying a general integral transform.

Section 2 describes definitions and equation of fractional kinetic equation and defines $p-k$ generalised C-series, $p-k$ multi-index Mittag Leffler function and general integral transform.

Section 3 finds solution of fractional kinetic equation associating it with "$p-k$ generalised C series and $p-k$ multi-index Mittag Leffler function using general integral transform and also discusses some particular cases of it.

2 Preliminaries

2.1 Fractional Kinetic Equation:

The interest in study of fractional kinetic equations has recently increased due to its application in science and engineering.These equations have been studied to describe non standard kinetic behaviours whether it is in astrophysics or statistical physics or to describe anomalous diffusion in porous media and in cells of living organism.

In statistical physics, they help describe systems with memory effects. They are also used in chaotic dynamics where they can model the complex behaviour of systems with fractal properties. Fractional kinetic equations (FKE) are used to model the dynamics of particles in astrophysical plasmas and to describe the evolution of the stellar systems.

H.J.Haubold and A.M.Mathai [9] in the year 2000, used kinetic equation and later fractional kinetic equation to describe the change of chemical composition in star like sun.
They used the kinetic equation to describe the rate of change of chemical composition of the star for species in terms of the reaction rates for destruction and production of that species (Kourganoff,1973,Perdang,,1976,Clayton,1983).
The production and destruction of species is described by kinetic equations governing the change of number density N_i of species i over time, that is,

$$\frac{dN_i(t)}{dt} = -(a_i - b_i)N_i(t), \tag{2.1}$$

where a_i is the statistically expected number of reaction per unit volume per unit time destroying the species i.
b_i is the statistically expected number of reaction per unit volume per unit time producing the species i.
There are three distinct cases $A_i = a_i - b_i$, $A_i < 0$ and $A_i = 0$. Where $A_i = 0$ means species i are in equilibrium, $A_i > 0$ means there is destruction of species i and $A_i < 0$ means there is production of species i.
For the case $A_i > 0$, we have

$$\frac{dN_i(t)}{dt} = -A_i N_i(t), \tag{2.2}$$

The rate of destruction is proportional to the value of the variable and it does not exhibit oscillations, instabilities, or chaotic dynamics.
For the sake of brevity, the index i in the equation (2.2) is dropped to generalise it. The equation (2.2) can be integrated as

$$N(t) - N_0 = -A \quad {}_0D_t^{-1} N(t), \tag{2.3}$$

where ${}_0D_t^{-1}$ in the standard Riemann integral operator and $N_0 = N_i(t)$ is the number of density of species i at time $t = 0$.
By replacing the Riemann integral operator by fractional Riemann Liouville operator ${}_0D_t^{-\nu}$ and coefficient A by A^ν accordingly for dimension reason in equation (2.3),Haubold and Mathai obtained a fractional integral equation corresponding to equation (2.2)

$$N(t) - N_0 = -A^\nu {}_0D_t^{-\nu} N(t), \qquad \nu > 0 \tag{2.4}$$

where,

$${}_0D_t^{-\nu} f(t) = \frac{1}{\Gamma\nu} \int_0^t (t-x)^{\nu-1} f(x)\ dx. \tag{2.5}$$

The solution of the equation (2.4) is

$$N(t) = N_0 \sum_{n=0}^{\infty} \frac{(-1)^n (At)^{\nu n}}{\Gamma(\nu n + 1)}. \tag{2.6}$$

R.K. Saxena and S.L.Kalla [15] introduced the following fractional kinetic equation in the year 2008,

$$N(t) - N_0 f(t) = -A^\nu {}_0D_t^{-\nu} N(t), \qquad (Re(\nu) > 0 \tag{2.7}$$

where $f(t)$ is an integral function of t.

2.2 p-k generalised C-series

The $p - k$ generalised C-series is defined by Gehlot and Devra [7] as

$${}_t^p\left[\ {}_{r,m}C_{q,d}^{(\alpha,\beta)l}(a_1, \cdots a_r; b_1, \cdots b_q; (\alpha_1, \beta_1) \cdots (\alpha_l, \beta_l); z)\right]_s^k = {}_t^p\left[\ {}_{r,m}C_{q,d}^{(\alpha,\beta)l}(z)\right]_s^k$$

$$= \sum_{n=0}^{\infty} \frac{p_1(a_1)_{nm,k_1} \cdots p_r(a_r)_{nm,k_r}}{t_1(b_1)_{nd,s_1} \cdots t_q(b_q)_{nd,s_q}} \frac{z^n}{\prod_{\delta=1}^{l} \Gamma(\alpha_\delta n+\beta_\delta)},$$

$$\frac{p}{t}\left[{}_{r,m}C_{q,d}^{(\alpha,\beta)l}(z)\right]_{s}^{k} = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} {}_{p_i}(a_i)_{nm,k_i}}{\prod_{j=1}^{q} {}_{t_j}(b_j)_{nd,s_j}} \frac{z^n}{\prod_{\delta=1}^{l} \Gamma(\alpha_\delta n+\beta_\delta)}, \tag{2.8}$$

where $p_i, k_i, t_j, s_j, \alpha_\delta \in \mathbb{R}^+ - (0)$, $z, a_i, b_j, \beta_\delta \in \mathbb{C}$, $(i = 1,2 \cdots r; j = 1,2 \cdots q; \delta = 1,2 \cdots l)$, $Re(a_i) > 0$, $Re(b_j) > 0$, m, d are non negative integers.

The series (2.8) is **not** defined when either of the parameters b_j or s_j $(j = 1,2 \cdots q)$ is negative integer or zero. The series terminates into polynomial in z if any parameter of either a_i or k_i, $(i = 1,2 \cdots r)$ is negative integer or zero.

Convergence criteria of $p - k$ generalised C-series when applying ratio test is

(i) if $qd + \sum_{i=1}^{l} \alpha_i > mr$, then the series is absolutely convergent for all values of $z \in \mathbb{C}$,

(ii) if $qd + \sum_{i=1}^{l} \alpha_i = mr$, then the series is absolutely convergent for all values of $|z| < \frac{(t_1 \cdots t_q)^d}{(p_1 \cdots p_r)^m} d^{dq} m^{-mr} \prod_{\delta=1}^{l} (\alpha_\delta)^{\alpha_\delta}$,

(iii)if $qd + \sum_{i=1}^{l} \alpha_i = mr$ and $|z| = |\frac{(t_1 \cdots t_q)^d}{(p_1 \cdots p_r)^m} d^{dq} m^{-mr} \prod_{\delta=1}^{l} (\alpha_\delta)^{\alpha_\delta}|$,then series is absolutely convergent when $\sum_{j=1}^{q} \frac{b_j}{s_j} + \sum_{\delta=1}^{l} \beta_\delta - \sum_{i=1}^{r} \frac{a_i}{k_i} > \frac{2+q+l-r}{2}$.

Particular Cases of p-k generalised C-series When some particular values given to parameters of equation (2.8)

(i) Putting $p = k = t = s = 1$ and $m = 1 = d$ in the equation (2.8), the equation reduces to the K-series defined by Kuldeep Singh Gehlot [3],

$$\frac{1}{1}\left[{}_{r,1}C_{q,1}^{(\alpha,\beta)l}(z)\right]_{1}^{1} = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} (a_i)_n}{\prod_{j=1}^{q} (b_j)_n} \frac{z^n}{\prod_{\delta=1}^{l} \Gamma(\alpha_\delta n+\beta_\delta)} = {}_r K_q^{(\alpha,\beta)_l}(z). \tag{2.9}$$

(ii) Putting $p = k = t = s = 1$, $m = 1 = d$ and $l = 1$ in the equation (2.8), it reduces to M-series introduced by Sharma [18],

$$\frac{1}{1}\left[{}_{r,1}C_{q,1}^{(\alpha,\beta)1}(z)\right]_{1}^{1} = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} (a_i)_n}{\prod_{j=1}^{q} (b_j)_n} \frac{z^n}{\Gamma(\alpha_\delta n+\beta_\delta)} = {}_r M_q^{(\alpha,\beta)}(z). \tag{2.10}$$

(iii) Putting $p = k = t = s = 1$, $m = 1 = d$, $l = 2$ and $\alpha_1 = 1, \alpha_2 = \alpha, \beta_1 = 1, \beta_2 = \beta$ in the equation (2.8), the equation reduces to the R-series defined by M.F.Ali et.al [1],

$$\frac{1}{1}\left[{}_{r,1}C_{q,1}^{(\alpha,\beta)2}(z)\right]_{1}^{1} = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} (a_i)_n}{\prod_{j=1}^{q} (b_j)_n} \frac{z^n}{\Gamma(\alpha n+\beta)\Gamma(n+1)} = {}_r R_q^{(\alpha,\beta)}(z). \tag{2.11}$$

(iv) Putting $p = k = t = s = 1$ in the equation (2.8), the equation reduces to the C series defined by Mohd.Farman Ali et al.[2],

$$\frac{1}{1}\left[{}_{r,m}C_{q,d}^{(\alpha,\beta)l}(z)\right]_{1}^{1} = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} (a_i)_{nm}}{\prod_{j=1}^{q} (b_j)_{nd}} \frac{z^n}{\prod_{\delta=1}^{l} \Gamma(\alpha_\delta n+\beta_\delta)} = {}_{r,m} C_{q,d}^{(\alpha,\beta)_l}(z). \tag{2.12}$$

(v) Putting $p = k = t = s = 1$, $m = 1 = d$, $l = 2$, $\alpha_1 = 1, \alpha_2 = \alpha, \beta_1 = 1, \beta_2 = \beta$, $r = 1$ and $q = 0$ i.e no lower parameter q in the equation (2.8), the equation reduces to the generalised Mittag-Leffler function introduced by Prabhakar [14] in (1971),

$$\frac{1}{1}\left[{}_{1,1}C_{0,1}^{(\alpha,\beta)2}(z)\right]_{1}^{1} = \sum_{n=0}^{\infty} \frac{(a)_n z^n}{\Gamma(\alpha n+\beta)\Gamma(n+1)} = E_{\alpha,\beta}^{a}(z). \tag{2.13}$$

(vi) If there is no upper and lower parameter i.e $r = 0 = q$, $m = 1 = d$, $p = k = t = s = 1$ and $l = 1$ in the equation (2.8), the equation reduces to Mittag-Leffler function by Wiman [20] in 1905,

$$\frac{1}{1}\left[{}_{0,1}C_{0,1}^{(\alpha,\beta)1}(z)\right]_{1}^{1} = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha_1 n+\beta_1)} = E_{\alpha_1,\beta_1}(z). \tag{2.14}$$

(vii) Putting $r = 0 = q, m = 1 = d, l = 1, \alpha_1 = \alpha, \beta_1 = 1$ and $p = k = t = s = 1$ in in the equation (2.8), the equation reduces to Mittag-Leffler function defined by Gosta Mittag-Leffler [13] in 1903,

$$\frac{1}{1}\left[{}_{0,1}C_{0,1}^{(\alpha,1)1}(z)\right]_{1}^{1} = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n+1)} = E_{\alpha}(z). \tag{2.15}$$

(viii) Putting $p = k = t = s = 1,\ m = 1 = d, l = 1, \alpha_1 = 1, \beta_1 = 1$ in the equation (2.8), the equation reduces to generalised hypergeometric function [12],

$${}_1^1\left[\ {}_{r,1}C_{q,1}^{(1,1)1}(z)\right]_1^1 = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r}(a_i)_n}{\prod_{j=1}^{q}(b_j)_n}\frac{z^n}{\Gamma(n+1)} = {}_r F_q(a_1, a_2 \cdots a_r; b_1, b_2 \cdots b_q; z). \quad (2.16)$$

(ix) Putting $p = k = t = s = 1,\ m = 1 = d, l = 1, \alpha_1 = 1, \beta_1 = 1$ and no upper and lower parameter i.e $r = 0 = q$ in the equation (2.8), the equation reduces to an exponential function,

$${}_1^1\left[\ {}_{0,1}C_{0,1}^{(1,1)1}(z)\right]_1^1 = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n+1)} = e^z. \quad (2.17)$$

(x) Putting $p = k = t = s = 1,\ l = 1, \alpha_1 = 1, \beta_1 = 1$ in the equation (2.8), the equation reduces to the general Wright function [12],

$${}_1^1\left[\ {}_{r,m}C_{q,d}^{(1,1)1}(z)\right]_1^1 = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r}(a_i)_{nm}}{\prod_{j=1}^{q}(b_j)_{nd}}\frac{z^n}{\Gamma(n+1)} = \tau \qquad {}_r\psi_q\left[\begin{matrix}(a_1, m)\cdots(a_r, m);\\(b_1, d)\cdots(b_q, d)\end{matrix}\middle|z\right], \quad (2.18)$$

where $\tau = \frac{\prod_{j=1}^{q}\Gamma b_j}{\prod_{i=1}^{r}\Gamma a_i}$.

2.3 p-k Multi-Index Mittag Leffler Function

The $p - k$ multi-index Mittag Leffler function is defined by Gehlot and Devra [8] as

$${}_pE_k^{\gamma,q}[(\mu_1, \tfrac{1}{\rho_1})(\mu_2, \tfrac{1}{\rho_2})\cdots(\mu_m, \tfrac{1}{\rho_m}); z] = \sum_{n=0}^{\infty} \frac{{}_p(\gamma)_{nq,k}}{{}_p\Gamma_k(\mu_1 + \frac{n}{\rho_1})\cdots{}_p\Gamma_k(\mu_m + \frac{n}{\rho_m})}\frac{z^n}{n!}. \quad (2.19)$$

Where z is any complex variable, $m > 1$ and $m \in \mathbb{Z}$ and $\mu_1, \mu_2, \cdots \mu_m$ are arbitrary real (complex) numbers and $Re(\mu_i) > 0$ $\forall i = 1,2 \cdots m;\ q \in \mathbb{N};\ k, p \in \mathbb{R}^+ - (0),\ \gamma \in C/Z^-,\ \rho_1, \rho_2, \cdots \rho_m > 0$ and $\frac{1}{\rho_1}, \frac{1}{\rho_2}, \cdots \frac{1}{\rho_m} \in \mathbb{R}$

The convergence criteria of $p - k$ multi-index Mittag Leffler function is

(i) the function converges absolutely for all $z \in \mathbb{C}$ if $(\frac{1}{k\rho_1} + \frac{1}{k\rho_2} + \cdots + \frac{1}{k\rho_m} + 1) > q$.

(ii)if $(\frac{1}{k\rho_1} + \frac{1}{k\rho_2} + \cdots + \frac{1}{k\rho_m}) + 1 = q$ then the series is absolutely convergent for $|z| < \frac{\prod_{i=1}^{m}(\frac{1}{k\rho_i})^{\frac{1}{k\rho_i}}}{q^q p^{(q - \frac{1}{\rho_1 k} - \frac{1}{\rho_2 k} \cdots - \frac{1}{\rho_m k})}}$,

(iii) if $(\frac{1}{k\rho_1} + \frac{1}{k\rho_2} + \cdots + \frac{1}{k\rho_m}) + 1 = q$ and $|z| = \frac{\prod_{i=1}^{m}(\frac{1}{k\rho_i})^{\frac{1}{k\rho_i}}}{q^q p^{(q - \frac{1}{\rho_1 k} - \frac{1}{\rho_2 k} \cdots - \frac{1}{\rho_m k})}}$, then the function is convergent when $\sum_{j=1}^{m}(\frac{\mu_j}{k}) + \frac{\gamma}{k} > \frac{m}{2}$

Particular cases: for some particular values of parameters of equation(2.19)

(a) For $m = 1,\ \mu_1 = \beta,\ \rho_1 = \frac{1}{\alpha}$ equation (2.19) is reduced to $p - k$ Mittag Leffler function defined by Kuldeep Singh Gehlot [6]

$${}_pE_k^{\gamma,q}[(\beta, \alpha); (z)] = \sum_{n=0}^{\infty} \frac{{}_p(\gamma)_{nq,k}}{{}_p\Gamma_k(n\alpha + \beta)}\frac{z^n}{n!} = {}_p\ E_{k,\alpha,\beta}^{\gamma,q}(z). \quad (2.20)$$

(b) For $p = k,\ m = 1,\ \mu_1 = \beta,\ \rho_1 = \frac{1}{\alpha}$ equation (2.19) is reduced to generalised k Mittag Leffler function defined by Kuldeep Singh Gehlot [4]

$${}_kE_k^{\gamma,q}[(\beta, \alpha); (z)] = \sum_{n=0}^{\infty} \frac{{}_k(\gamma)_{nq,k}}{{}_k\Gamma_k(n\alpha + \beta)}\frac{z^n}{n!} = GE_{k,\alpha,\beta}^{\gamma,q}(z). \quad (2.21)$$

(c) For $p = k$,and $k = 1\ m = 1,\ \mu_1 = \beta,\ \rho_1 = \frac{1}{\alpha}$ equation (2.19) is reduced to equation (2.6) defined by Shukla and Prajapati [19]

$${}_1E_1^{\gamma,q}[(\beta, \alpha); (z)] = \sum_{n=0}^{\infty} \frac{(\gamma)_{nq}}{\Gamma(n\alpha + \beta)}\frac{z^n}{n!} = E_{\alpha,\beta}^{\gamma,q}(z). \quad (2.22)$$

Special case of the equation (2.22) when we substitute $\gamma = \frac{\gamma}{\xi}$; $\beta = \frac{\mu_1}{\xi}, \frac{\mu_2}{\xi}, \cdots \frac{\mu_m}{\xi}$ and $\alpha = \frac{1}{\xi\rho_1}, \frac{1}{\xi\rho_2}, \cdots \frac{1}{\xi\rho_m}$, then

$$ {}_1E_1^{(\frac{\gamma}{\xi}),q}[(\frac{\mu_1}{\xi}, \frac{1}{\xi\rho_1}) \cdots (\frac{\mu_m}{\xi}, \frac{1}{\xi\rho_m}); z] = \sum_{n=0}^{\infty} \frac{(\frac{\gamma}{\xi})_{nq}}{\Gamma(\frac{\mu_1}{\xi}+\frac{n}{\xi\rho_1})\Gamma(\frac{\mu_2}{\xi}+\frac{n}{\xi\rho_2})\cdots\Gamma(\frac{\mu_m}{\xi}+\frac{n}{\xi\rho_m})} \frac{z^n}{n!} $$

$$ = E^{(\frac{\gamma}{\xi}),q}[(\frac{\mu_1}{\xi}, \frac{1}{\xi\rho_1}) \cdots (\frac{\mu_m}{\xi}, \frac{1}{\xi\rho_m}); z]. \tag{2.23} $$

(d) For $p = k$,and $k = 1, q = 1,\ \gamma = 1,\ m = 1,\ \mu_1 = 1,\ \rho_1 = \frac{1}{\alpha}$ equation (2.19) is reduced to Mittag Leffler function defined by Mittag [13]

$$ {}_1E_1^{1,1}[(1,\alpha); (z)] = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\alpha+1)} = E_\alpha(z). \tag{2.24} $$

2.4 A General Integral Transform

The general integral transform was introduced by Hossein Jafari [11] in the year 2020. Various integral transforms such as Laplace integral, α-integral Laplace, Sumudu,Elzaki,Natural,Abodh,Kamal,Sawi etc. transforms are particular cases of it.It is defined as:

Let $f(t)$ be an integral function defined for $t \geq 0$, $P(\theta) \neq 0$ and $Q(\theta)$ are positive real function, then the general integral transform of $f(t)$ is defined as

$$ T[f(t); \theta] = T(\theta) = P(\theta) \int_0^{\infty} e^{-Q(\theta)t} f(t)\ \ dt, \tag{2.25} $$

provided the integral exists for some $Q(\theta)$.

Convolution property: Let $F_1(\theta)$ and $F_2(\theta)$ are general integral transform of $f_1(t)$ and $f_2(t)$, respectively,then

$$ T\left[f_(t) * f_2(t)\right] = T\left[\int_0^t f_1(\tau)f_2(t-\tau)\ \ d\tau\right] = \frac{1}{P(\theta)} F_1(\theta)F_2(\theta). \tag{2.26} $$

3 Solution of fractional kinetic equation (FKE) by applying general integral transform and associating with it p-k generalised C-series and $p-k$ multi-index Mittag Leffler function

Theorem:3.1 Let $A > 0$, $\nu > 0$, $p_i, k_i, t_j, s_j, \alpha_\delta \in \mathbb{R}^+ - (0)$, $z, a_i, b_j, \beta_\delta \in \mathbb{C}$, $(i = 1,2 \cdots r; j = 1,2 \cdots q; \delta = 1,2 \cdots l)$, $Re(a_i) > 0$, $Re(b_j) > 0$, m, d are non negative integers, then the solution of the following FKE associating with $p-k$ generalised C-series

$$ N(z) - N_0{}_t^p\left[\ {}_{r,m}C_{q,d}^{(\alpha,\beta)l}(z)\right]_s^k = -A^\nu{}_0D_z^{-\nu}N(z), \tag{3.1} $$

is given by

$$ N(z) = N_0 \sum_{n=0}^{\infty} \sum_{\xi=0}^{\infty} \frac{\prod_{i=1}^{r} {}_{p_i}(a_i)_{nm,k_i}}{\prod_{j=1}^{q} {}_{t_j}(b_j)_{nd,s_j}} \frac{\Gamma(n+1)}{\prod_{\delta=1}^{l} \Gamma(\alpha_\delta n + \beta_\delta)} \frac{(-1)^\xi A^{\nu\xi} z^{\nu\xi+n}}{\Gamma(\nu\xi+n+1)}. \tag{3.2} $$

Where $A < |Q(\theta)|$.

Proof: By taking general integral transform on both sides of equation (3.1)

$$ [T[N(z); \theta] - N_0 T\left[\ {}_t^p\left[\ {}_{r,m}C_{q,d}^{(\alpha,\beta)l}(z)\right]_s^k; \theta\right] = -A^\nu T[{}_0D_z^{-\nu}N(z); \theta]. \tag{3.3} $$

By using definition of general integral transform i.e equation (2.25)

$$ T[N(z); \theta] - N_0 P(\theta) \int_0^{\infty} e^{-Q(\theta)z}\ \ {}_t^p\left[\ {}_{r,m}C_{q,d}^{(\alpha,\beta)l}(z)\right]_s^k = -A^\nu P(\theta) \int_0^{\infty} e^{-Q(\theta)z}\ {}_0D_z^{-\nu}N(z) \tag{3.4} $$

Now,

$$T[N(z);\theta] = N[Q(\theta)]. \tag{3.5}$$

By using equation (2.5),

$$T[{}_0D_z^{-\nu}N(z);\theta] = T\left[\frac{1}{\Gamma\nu}\int_0^z (z-x)^{\nu-1}N(x)\ dx;\theta\right],$$

By using equation (2.26), we get

$$T[{}_0D_z^{-\nu}N(z);\theta] = [Q(\theta)]^{-\nu}]N[Q(\theta)]. \tag{3.6}$$

And by using equations (2.8)

$$N_0T\left[\ {}_t^p\left[\ {}_{r,m}C_{q,d}^{(\alpha,\beta)l}(z)\right]_s^k;\theta\right] = N_0T\left[\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{r}{}_{p_i}(a_i)_{nm,k_i}}{\prod_{j=1}^{q}{}_{t_j}(b_j)_{nd,s_j}}\frac{z^n}{\prod_{\delta=1}^{l}\Gamma(\alpha_\delta n+\beta_\delta)}\right].$$

By using equation (2.25) and Euler's integral of second kind

$$= N_0\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{r}{}_{p_i}(a_i)_{nm,k_i}}{\prod_{j=1}^{q}{}_{t_j}(b_j)_{nd,s_j}}\frac{P(\theta)}{\prod_{\delta=1}^{l}\Gamma(\alpha_\delta n+\beta_\delta)}\frac{\Gamma(n+1)}{[Q(\theta)]^{n+1}} \tag{3.7}$$

By using equations (3.5),(3.6) and (3.7) in the equation (3.3), we get

$$N[Q(\theta)][1+[Q(\theta)]^{-\nu}A^{\nu}] = N_0\quad\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{r}{}_{p_i}(a_i)_{nm,k_i}}{\prod_{j=1}^{q}{}_{t_j}(b_j)_{nd,s_j}}\frac{P(\theta)}{\prod_{\delta=1}^{l}\Gamma(\alpha_\delta n+\beta_\delta)}\frac{\Gamma(n+1)}{[Q(\theta)]^{n+1}}$$

$$N[Q(\theta)] = N_0\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{r}{}_{p_i}(a_i)_{nm,k_i}}{\prod_{j=1}^{q}{}_{t_j}(b_j)_{nd,s_j}}\frac{P(\theta)}{\prod_{\delta=1}^{l}\Gamma(\alpha_\delta n+\beta_\delta)}\frac{\Gamma(n+1)}{[Q(\theta)]^{n+1}}\frac{1}{1+[\frac{A}{Q(\theta)}]^{\nu}}.$$

Where $A < |Q(\theta)|$

$$= N_0\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{r}{}_{p_i}(a_i)_{nm,k_i}}{\prod_{j=1}^{q}{}_{t_j}(b_j)_{nd,s_j}}\frac{\Gamma(n+1)P(\theta)}{\prod_{\delta=1}^{l}\Gamma(\alpha_\delta n+\beta_\delta)}\sum_{\xi=0}^{\infty}\frac{(-1)^{\xi}A^{\nu\xi}}{[Q(\theta)]^{\nu\xi+n+1}} \tag{3.8}$$

By taking inverse of general Integral transform and using $T^{-1}[\frac{P(\theta}{[Q(\theta)]^{\nu}};z] = \frac{z^{\nu-1}}{\Gamma\nu}$, we get

$$N(z) = N_0\sum_{n=0}^{\infty}\sum_{\xi=0}^{\infty}\frac{\prod_{i=1}^{r}{}_{p_i}(a_i)_{nm,k_i}}{\prod_{j=1}^{q}{}_{t_j}(b_j)_{nd,s_j}}\frac{\Gamma(n+1)}{\prod_{\delta=1}^{l}\Gamma(\alpha_\delta n+\beta_\delta)}\frac{(-1)^{\xi}A^{\nu\xi}z^{\nu\xi+n}}{\Gamma(\nu\xi+n+1)}. \tag{3.9}$$

Hence proved.

Particular cases: To solve the equation (3.1) by other integral transforms, we need to substitute some particular values of $P(\theta)$ and $Q(\theta)$ in the equation (3.4)

(i) If $P(\theta) = 1$ and $Q(\theta) = \theta$ is substituted in the equation (3.4),then general integral transform changed into Laplace transform. Therefore, the Laplace transform of equation (3.1), which is as under

$$N(\theta) = N_0\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{r}{}_{p_i}(a_i)_{nm,k_i}}{\prod_{j=1}^{q}{}_{t_j}(b_j)_{nd,s_j}}\frac{\Gamma(n+1)}{\prod_{\delta=1}^{l}\Gamma(\alpha_\delta n+\beta_\delta)}\sum_{\xi=0}^{\infty}\frac{(-1)^{\xi}A^{\nu\xi}}{(\theta)^{\nu\xi+n+1}}. \tag{3.10}$$

The inverse Laplace transform of the equation (3.10) is the equation (3.9), that is the same equation which we obtained from general integral transform.

(ii) Similarly, If $P(\theta) = 1$ and $Q(\theta) = \theta^{\frac{1}{\alpha}}$ is substituted in the equation (3.4), then general integral transform changed into α-integral Laplace transform of equation (3.1), which is as under

$$N(\theta)^{\frac{1}{\alpha}} = N_0\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{r}{}_{p_i}(a_i)_{nm,k_i}}{\prod_{j=1}^{q}{}_{t_j}(b_j)_{nd,s_j}}\frac{\Gamma(n+1)}{\prod_{\delta=1}^{l}\Gamma(\alpha_\delta n+\beta_\delta)}\sum_{\xi=0}^{\infty}\frac{(-1)^{\xi}A^{\nu\xi}}{(\theta)^{\frac{\nu\xi+n+1}{\alpha}}}. \tag{3.11}$$

The inverse α-Laplace transform of the equation (3.11) is the equation (3.9), that is same the equation which we obtained from general integral transform.

(iii) If $P(\theta) = \frac{1}{\theta}$ and $Q(\theta) = \frac{1}{\theta}$ is substituted in the equation (3.4), then then general integral transform changed into Sumudu transform of equation (3.1), which is as under

$$N(\tfrac{1}{\theta}) = N_0 \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} {}_{p_i}(a_i)_{nm,k_i}}{\prod_{j=1}^{q} {}_{t_j}(b_j)_{nd,s_j}} \frac{\Gamma(n+1)}{\prod_{\delta=1}^{l} \Gamma(\alpha_\delta n+\beta_\delta)} \sum_{\xi=0}^{\infty} (-1)^{\xi} A^{\nu\xi} (\theta)^{\nu\xi+n}. \tag{3.12}$$

The inverse Sumudu transform of the equation (3.12) is the equation (3.9), that is the same equation which we obtained from general integral transform.

For more such particular cases of general integral transform refer [11]

Corollary:1 Let $A > 0,\ \nu > 0,\ \alpha_\delta \in \mathbb{R}^+ - (0),\ z, a_i, b_j, \beta_\delta \in \mathbb{C},\ (i = 1,2 \cdots r; j = 1,2 \cdots q; \delta = 1,2 \cdots l)$, $Re(a_i) > 0$, $Re(b_j) > 0$, then the solution of the following FKE associating with k -series (i.e equation (2.9), which is a particular case of $p-k$ generalised C-series.),

$$N(z) - N_{0\,r}K_q^{(\alpha,\beta)_l}(z) = -A^{\nu}{}_0D_z^{-\nu}N(z), \tag{3.13}$$

is given by

$$N(z) = N_0 \sum_{n=0}^{\infty} \sum_{\xi=0}^{\infty} \frac{\prod_{i=1}^{r} (a_i)_n}{\prod_{j=1}^{q} (b_j)_n} \frac{\Gamma(n+1)}{\prod_{\delta=1}^{l} \Gamma(\alpha_\delta n+\beta_\delta)} \frac{(-1)^{\xi} A^{\nu\xi} z^{\nu\xi+n}}{\Gamma(\nu\xi+n+1)}. \tag{3.14}$$

Where $A < |Q(\theta)|$.

Corollary:2 Let $A > 0,\ \nu > 0,\ \alpha \in \mathbb{R}^+ - (0),\ z, a_i, b_j, \beta \in \mathbb{C},\ (i = 1,2 \cdots r; j = 1,2 \cdots q),\ Re(a_i) > 0$, $Re(b_j) > 0$, then the solution of the following FKE associating with M-series (i.e equation (2.10), which is a particular case of $p-k$ generalised C-series.),

$$N(z) - N_0 \quad {}_rM_q^{(\alpha,\beta)}(z) = -A^{\nu}{}_0D_z^{-\nu}N(z), \tag{3.15}$$

is given by

$$N(z) = N_0 \sum_{n=0}^{\infty} \sum_{\xi=0}^{\infty} \frac{\prod_{i=1}^{r} (a_i)_n}{\prod_{j=1}^{q} (b_j)_n} \frac{\Gamma(n+1)}{\Gamma(\alpha n+\beta)} \frac{(-1)^{\xi} A^{\nu\xi} z^{\nu\xi+n}}{\Gamma(\nu\xi+n+1)}. \tag{3.16}$$

Where $A < |Q(\theta)|$.

Corollary:3 Let $A > 0,\ \nu > 0,\ z, a_i, b_j \in \mathbb{C},\ (i = 1,2 \cdots r; j = 1,2 \cdots q),\ Re(a_i) > 0,\ Re(b_j) > 0$, then the solution of the following FKE associating with generalised hyper-geometric function (i.e equation (2.16), which is a particular case of $p-k$ generalised C-series.),

$$N(z) - N_0 \quad {}_rF_q[a_1, a_2 \cdots a_r; b_1, b_2 \cdots b_q; z] = -A^{\nu}{}_0D_z^{-\nu}N(z), \tag{3.17}$$

is given by

$$N(z) = N_0 \sum_{n=0}^{\infty} \sum_{\xi=0}^{\infty} \frac{\prod_{i=1}^{r} (a_i)_n}{\prod_{j=1}^{q} (b_j)_n} \frac{(-1)^{\xi} A^{\nu\xi} z^{\nu\xi+n}}{\Gamma(\nu\xi+n+1)}. \tag{3.18}$$

Where $A < |Q(\theta)|$.

Corollary:4 Let $A > 0,\ \nu > 0,\ z, a_i, b_j \in \mathbb{C},\ (i = 1,2 \cdots r; j = 1,2 \cdots q),\ Re(a_i) > 0,\ Re(b_j) > 0, m, d$ are non negative integers, then the solution of the following FKE associating with general Wright function (i.e equation (2.18), which is a particular case of $p-k$ generalised C-series),

$$N(z) - N_0 \quad \tau \quad {}_r\psi_q\left[\begin{matrix}(a_1,m)\cdots(a_r,m); \\ (b_1,d)\cdots(b_q,d)\end{matrix} \Big| z\right] = -A^{\nu}{}_0D_z^{-\nu}N(z), \tag{3.19}$$

where $\tau = \frac{\prod_{j=1}^{q} \Gamma b_j}{\prod_{i=1}^{r} \Gamma a_i}$.

is given by

$$N(z) = N_0 \quad \tau \sum_{n=0}^{\infty} \sum_{\xi=0}^{\infty} \frac{\prod_{i=1}^{r} (a_i)_{nm}}{\prod_{j=1}^{q} (b_j)_{nd}} \frac{(-1)^{\xi} A^{\nu\xi} z^{\nu\xi+n}}{\Gamma(\nu\xi+n+1)}. \tag{3.20}$$

Where $A < |Q(\theta)|$.

Theorem 3.2 Let $A > 0$, $\nu > 0, m \geq 1$, and $m \in z$ and $\mu_1, \mu_2, \cdots \mu_m$ are arbitrary real (complex) numbers and and $Re(\mu_i) > 0 \quad \forall i = 1,2 \cdots m$; $q \in \mathbb{N}$; $k, p \in \mathbb{R}^+ - (0)$, $\gamma \in C/Z^-$, $\rho_1, \rho_2, \cdots \rho_m > 0$ and $\frac{1}{\rho_1}, \frac{1}{\rho_2}, \cdots \frac{1}{\rho_m} \in \mathbb{R}$, then solution of the following fractional kinetic equation associating with $p - k$ multi-index Mittag Leffler

$$N(z) - N_0 \quad {}_pE_k^{\gamma,q}[(\mu_1, \frac{1}{\rho_1})(\mu_2, \frac{1}{\rho_2}) \cdots (\mu_m, \frac{1}{\rho_m}); z] = -A^\nu {}_0D_z^{-\nu} N(z), \tag{3.21}$$

is given by

$$N(z) = N_0 \sum_{n=0}^{\infty} \sum_{\xi=0}^{\infty} \frac{{}_p(\gamma)_{nq,k}}{{}_p\Gamma_k(\mu_1 + \frac{n}{\rho_1}) \cdots {}_p\Gamma_k(\mu_m + \frac{n}{\rho_m})} \frac{(-1)^\xi A^{\nu\xi} z^{\nu\xi+n}}{\Gamma(\nu\xi+n+1)}. \tag{3.22}$$

Where $A < |Q(\theta)|$.

Proof: Taking general integral transform of both side of the equation (3.21)

$$T[N(z);\theta] - N_0 T[(\quad {}_pE_k^{\gamma,q}[(\mu_1, \frac{1}{\rho_1})(\mu_2, \frac{1}{\rho_2}) \cdots (\mu_m, \frac{1}{\rho_m}); z]); \theta] = -A^\nu T[{}_0D_z^{-\nu} N(z); \theta]. \tag{3.23}$$

By using equation (2.25)

$$T[N(z);\theta] - N_0 P(\theta) \int_0^\infty e^{-Q(\theta)z} {}_pE_k^{\gamma,q}[(\mu_1, \frac{1}{\rho_1})(\mu_2, \frac{1}{\rho_2}) \cdots (\mu_m, \frac{1}{\rho_m})]. =$$

(3.24)

Now,

$$T[N(z);\theta] = N[Q(\theta)]. \tag{3.25}$$

By using equation (2.5), we get

$$T[{}_0D_z^{-\nu} N(z); \theta] = T\left[\frac{1}{\Gamma\nu} \int_0^z (z-x)^{\nu-1} N(x) \ dx; \theta\right]$$

.

By using equation (2.26), we get

$$T[{}_0D_z^{-\nu} N(z); \theta] = [Q(\theta)]^{-\nu}] N[Q(\theta)]. \tag{3.26}$$

Now,

$$N_0 T\left[\left(\quad {}_pE_k^{\gamma,q}\left[\left(\mu_1, \frac{1}{\rho_1}\right)\left(\mu_2, \frac{1}{\rho_2}\right) \cdots \left(\mu_m, \frac{1}{\rho_m}\right); z\right]\right); \theta\right]$$

$$= N_0 T\left[\sum_{n=0}^{\infty} \frac{{}_p(\gamma)_{nq,k}}{{}_p\Gamma_k(\mu_1 + \frac{n}{\rho_1}) \cdots {}_p\Gamma_k(\mu_m + \frac{n}{\rho_m})} \frac{z^n}{n!}\right],$$

By using equation (2.19) and Euler's integral of second kind, we get

$$= N_0 \quad \sum_{n=0}^{\infty} \frac{{}_p(\gamma)_{nq,k}}{{}_p\Gamma_k(\mu_1 + \frac{n}{\rho_1}) \cdots {}_p\Gamma_k(\mu_m + \frac{n}{\rho_m})} \frac{P(\theta)}{[Q(\theta)]^{n+1}}. \tag{3.27}$$

Using equation (3.25),(3.26) and (3.27) in the equation (3.23), we get

$$N[Q(\theta)][1 + [Q(\theta)]^{-\nu} A^\nu] = N_0 \sum_{n=0}^{\infty} \frac{{}_p(\gamma)_{nq,k}}{{}_p\Gamma_k(\mu_1 + \frac{n}{\rho_1}) \cdots {}_p\Gamma_k(\mu_m + \frac{n}{\rho_m})} \frac{P(\theta)}{[Q(\theta)]^{n+1}}$$

$$N[Q(\theta)] = N_0 \sum_{n=0}^{\infty} \frac{{}_p(\gamma)_{nq,k}}{{}_p\Gamma_k(\mu_1 + \frac{n}{\rho_1}) \cdots {}_p\Gamma_k(\mu_m + \frac{n}{\rho_m})} \frac{P(\theta)}{[Q(\theta)]^{n+1}} \frac{1}{1 + [\frac{A}{Q(\theta)}]^\nu}.$$

Where $A < |Q(\theta)|$

$$= N_0 \sum_{n=0}^{\infty} \frac{{}_p(\gamma)_{nq,k} P(\theta)}{{}_p\Gamma_k(\mu_1 + \frac{n}{\rho_1}) \cdots {}_p\Gamma_k(\mu_m + \frac{n}{\rho_m})} \sum_{\xi=0}^{\infty} \frac{(-1)^\xi A^{\nu\xi}}{[Q(\theta)]^{\nu\xi+n+1}} \tag{3.28}$$

By taking inverse of general Integral transform and using $T^{-1}[\frac{P(\theta}{[Q(\theta)]^\nu}; z] = \frac{z^{\nu-1}}{\Gamma\nu}$, we get

$$N(z) = N_0 \sum_{n=0}^{\infty} \sum_{\xi=0}^{\infty} \frac{{}_p(\Upsilon)_{nq,k}}{{}_p\Gamma_k(\mu_1+\frac{n}{\rho_1})\cdots{}_p\Gamma_k(\mu_m+\frac{n}{\rho_m})} \frac{(-1)^{\xi} A^{\nu\xi} z^{\nu\xi+n}}{\Gamma(\nu\xi+n+1)}. \quad (3.29)$$

Particular cases: To solve the equation (3.21) by other integral transforms, we need to substitute some particular values of $P(\theta)$ and $Q(\theta)$ in the equation (3.24)

(i) If $P(\theta) = 1$ and $Q(\theta) = \theta$ is substituted in the equation (3.24),then general integral transform changed into Laplace transform.Therefore, the Laplace transform of equation (3.21) is as under

$$N(\theta) = N_0 \sum_{n=0}^{\infty} \frac{{}_p(\Upsilon)_{nq,k}}{{}_p\Gamma_k(\mu_1+\frac{n}{\rho_1})\cdots{}_p\Gamma_k(\mu_m+\frac{n}{\rho_m})} \sum_{\xi=0}^{\infty} \frac{(-1)^{\xi} A^{\nu\xi}}{(\theta)^{\nu\xi+n+1}}. \quad (3.30)$$

The inverse Laplace transform of the equation (3.30) is the equation (3.29), that is the same equation which we obtained from general integral transform.

(ii) Similarly, If $P(\theta) = 1$ and $Q(\theta) = \theta^{\frac{1}{\alpha}}$ is substituted in the equation (3.24), then general integral transform changed into α-integral Laplace transform. Therefore, α-integral Laplace transform of the equation (3.21)is as under

$$N(\theta^{\frac{1}{\alpha}}) = N_0 \sum_{n=0}^{\infty} \frac{{}_p(\Upsilon)_{nq,k}}{{}_p\Gamma_k(\mu_1+\frac{n}{\rho_1})\cdots{}_p\Gamma_k(\mu_m+\frac{n}{\rho_m})} \sum_{\xi=0}^{\infty} \frac{(-1)^{\xi} A^{\nu\xi}}{(\theta)^{\frac{\nu\xi+n+1}{\alpha}}}. \quad (3.31)$$

The inverse α-Laplace transform of the equation (3.31) is the equation (3.29), that is the same equation which we obtained from general integral transform.

(iii) If $P(\theta) = \frac{1}{\theta}$ and $Q(\theta) = \frac{1}{\theta}$ is substituted in the equation (3.24), then general integral transform changed into Sumudu transform. Therefore, Sumudu transform of the equation (3.21) is as under

$$N(\tfrac{1}{\theta}) = N_0 \sum_{n=0}^{\infty} \frac{{}_p(\Upsilon)_{nq,k}}{{}_p\Gamma_k(\mu_1+\frac{n}{\rho_1})\cdots{}_p\Gamma_k(\mu_m+\frac{n}{\rho_m})} \sum_{\xi=0}^{\infty} (-1)^{\xi} A^{\nu\xi} (\theta)^{\nu\xi+n}. \quad (3.32)$$

The inverse of Sumudu transform of the equation (3.32) is the equation (3.29), that is the same equation which we obtained from general integral transform.

For more such particular cases of general integral transform refer [11]

Corollary:1 Let $A > 0$, $\nu > 0$, $\gamma \in C/Z^-$, $z, \beta \in \mathbb{C}$ $Re(\beta) > 0$, $\alpha \in \mathbb{R}$, $\alpha > 0$, then the solution of the following FKE associating with $p-k$ Mittag-Lefler function (i.e equation (2.20),which is a particular case of $p-k$ multi index Mittag-Leffler function),

$$N(z) - N_0 \quad {}_pE_k^{\gamma,q}[(\beta,\alpha); z] = -A^{\nu}{}_0D_z^{-\nu}N(z), \quad (3.33)$$

is given by

$$N(z) = N_0 \sum_{n=0}^{\infty} \sum_{\xi=0}^{\infty} \frac{{}_p(\Upsilon)_{nq,k}}{{}_p\Gamma_k(n\alpha+\beta)} \frac{(-1)^{\xi} A^{\nu\xi} z^{\nu\xi+n}}{\Gamma(\nu\xi+n+1)}. \quad (3.34)$$

Where $A < |Q(\theta)|$.

Corollary:2 Let $A > 0$, $\nu > 0$, $z \in \mathbb{C}$, $\alpha \in \mathbb{R}$, $\alpha > 0$, then the solution of the following FKE associating with Mittag Leffler function (i.e equation (2.24), which is a particular case of $p-k$ multi index Mittag-Leffler function),

$$N(z) - N_0 \quad E_{\alpha}(z) = -A^{\nu}{}_0D_z^{-\nu}N(z), \quad (3.35)$$

is given by

$$N(z) = N_0 \sum_{n=0}^{\infty} \sum_{\xi=0}^{\infty} \frac{(-1)^{\xi}}{\Gamma(n\alpha+1)} \frac{A^{\nu\xi} z^{\nu\xi+n}\Gamma(n+1)}{\Gamma(\nu\xi+n+1)}. \quad (3.36)$$

Where $A < |Q(\theta)|$.

4 Conclusion:

In this paper, The equation of FKE relating to describe the change of chemical composition in star is further associated with $p-k$ generalised C-series and $p-k$ multi-index Mittag Leffler function. These equations and their particular cases are solved by using general integral transform.The association of FKE with $p-k$ generalised C-series and $p-k$ multi-index Mittag Leffler function and their solution have widened FKE scope in the field of astrophysics specially to find solution of non-standard kinetic behaviour in astrophysics.

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