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Modal analysis of a Uniform Cantilever Beam

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Abstract

This paper presents a combined analytical and finite-element modal analysis of a uniform cantilever beam representative of building and bridge components in vibration and earthquake engineering. The study aims to determine natural frequencies, mode shapes and damping characteristics using closed-form Euler–Bernoulli solutions, a practical FEM workflow, and an outline of experimental modal testing for validation. A brief literature review frames classical Euler–Bernoulli and Timoshenko formulations alongside recent numerical and experimental investigations on cantilever beams, highlighting current accuracy and limitations. A 3 m steel cantilever with rectangular cross-section is selected as a case study; its first several bending natural frequencies are computed analytically, and corresponding mode shapes and modal participation are interpreted with respect to dynamic response. These results are compared to FEM eigenvalue analysis outcomes, showing close agreement for lower modes and quantifying deviations linked to shear deformation, rotational inertia and realistic boundary conditions. Likely ranges of modal damping ratios, informed by typical steel beam tests, are discussed in terms of their impact on vibration control and seismic response estimation. Overall, the study confirms that a validated FEM-based modal model, anchored by classical beam theory, offers a reliable and efficient basis for assessing the dynamic behavior of cantilever members in engineering practice.

I. INTRODUCTION

Modal analysis characterizes a structure’s dynamic behavior by determining its natural frequencies, mode shapes, and damping ratios, which are essential for avoiding resonance and for realistic seismic response analysis. For a cantilever beam, fixed at one e nd and free at the other, the objective is to numerically and (optionally) experimentally determine modal parameters and to discuss their significance for vibration and earthquake loading scenarios

II. NEED FOR STUDY

Modal analysis is a fundamental technique in structural dynamics, crucial for determining the natural frequencies, mode shapes, and damping characteristics of a structure, especially for components like cantilever beams. This analysis is critical for understanding a structure's dynamic behavior under various loading conditions, including vibration and seismic events. For a cantilever beam, which is fixed at one end and free at the other, modal analysis provides essential parameters for engineering design and analysis

Literature Review

Several works have investigated cantilever beams using analytical Euler–Bernoulli theory, finite element analysis (FEA), and experiments to obtain natural frequencies and mode shapes. Studies consistently report good agreement between analytical, FEA (ANSYS, ABAQUS, etc.), and experimental results for low modes, validating the use of FEA-based modal analysis for design and verification.

Recent research extends classical analysis to cracked beams, biocomposite beams, and damage detection through changes in modal parameters. These studies show that stiffness reduction (due to damage or material changes) reduces natural frequencies and modifies mode shapes, and higher modes are more sensitive to local stiffness variations, which is important for structural health monitoring and seismic safety.

Modal analysis of beams has also been integrated into earthquake-related investigations; work on cantilever components in buildings shows that modal properties directly influence vertical vibration periods and responses under seismic excitation. Frequency-response-function-based testing and theoretical modal testing (TMT) are used to measure resonant peaks, damping, and mode shapes, supporting the use of modal parameters in vibration and earthquake engineering practice.

Classical beam theories. The Euler–Bernoulli beam model gives closed-form expressions for cantilever natural frequencies and mode shapes for slender beams; it neglects shear deformation and rotary inertia and therefore is accurate for slender beams (high length-to-thickness ratio). The characteristic equation for cantilever boundary conditions is $\cosh(\beta L)\cos(\beta L)+1=0$ which yields the standard eigenvalues used to obtain ω_n.

Higher-order theories. For short or thick beams, or where shear/rotary inertia matter, Timoshenko beam theory provides improved accuracy; recent studies emphasize Timoshenko corrections for dynamic instability, shear effects and higher-frequency modes. Where thickness-to-length ratios are moderate the Timoshenko formulation should be used or a convergence study in FEM should confirm applicability of Euler–Bernoulli results.

FEM and experimental work. Many papers validate analytical results with FEM (ANSYS, ABAQUS) and with experimental modal analysis; results typically agree within a few percent for slender beams when mesh and boundary conditions are modeled carefully. The literature also demonstrates modal changes due to cracks, added tip masses, or boundary compliance; such studies show frequency reductions and localized changes in mode shapes that can be used for damage detection.

Summary of key takeaways. Use Euler–Bernoulli as a benchmark for slender cantilevers; apply Timoshenko or refined FEM when shear/rotary inertia or geometry complexity are non-negligible. Validate FEM with analytical roots and, where possible, with experimental modal testing to capture real boundary compliance and damping.

GENERAL

Modal analysis determines natural frequencies, mode shapes and damping characteristics of structures data that control resonance, dynamic amplification and seismic/vibration performance. For cantilever members (fixed-free), the first few modes usually dominate dynamic response; therefore accurate prediction of those modes is essential for design and assessment under wind, earthquake and impulsive loads. The objective of this study is to (1) perform a classical analytical modal analysis for a uniform cantilever beam of span 3.0 m, (2) present a reproducible FEM procedure to validate the analytical solution and (3) predict probable experimental and parametric outcomes relevant to vibration and earthquake analysis.

Using Euler–Bernoulli beam theory, transverse free vibration is governed by

$$EI\frac{\partial^4 y(x,t)}{\partial x^4}+\rho A\frac{\partial^2 y(x,t)}{\partial t^2}=0$$

Assume separation $y(x,t)=\phi(x)\sin(\omega t)$. This leads to an eigenvalue problem with characteristic equation for a cantilever:

$$\cosh(\beta L)\cos(\beta L)+1=0,$$

where β is the wavenumber; roots $\beta_n L$ are well-known constants (first four: 1.87510407, 4.69409113, 7.85475744, 10.99554073).
Natural circular frequencies:

$$\omega_n=\beta_n^2\sqrt{\frac{EI}{\rho A}} \quad \text{and} \quad f_n=\frac{\omega_n}{2\pi}.$$

Finite Element Method

Numerical methods, predominantly the Finite Element Method (FEM), are extensively utilized for modal analysis due to their versatility in handling complex geometries, boundary conditions, and material properties. Software packages like ANSYS and SolidWorks are commonly employed to perform FEM simulations. FEM discretizes the beam into smaller elements, allowing for the computation of its dynamic response. For instance, the Block Lanczos method is frequently used within FEM software to extract natural frequencies and mode shapes. FEM has been applied to analyze cantilever beams made of various materials, including biocomposites , natural fiber reinforced epoxy composites , and even those with complex features like cracks. The integrated finite strip method (IFSM) is another numerical approach extended to flutter analysis of bridges, which often involve beam-like elements

Finite Element procedure

- **Software:** Finite Element Method (FEM) procedure for beam analysis involves selecting appropriate software, such as commercial like ANSYS.
- **Element type:** Euler–Bernoulli (beam elements) for slender cases; use Timoshenko (shear-deformable) beam elements if thickness is non-negligible.

- **Mesh:** refined along the span (start with ~20–40 beam elements for convergence of first few modes). Perform mesh convergence: compare first 4 natural frequencies when doubling mesh density until changes <1%.
- **Boundary conditions:** fully clamped at the root (zero translation and rotation). Ensure reference node and constraints replicate laboratory clamp if comparing experiments (account for clamp compliance).
- **Mass modelling:** use consistent mass matrix (recommended) rather than lumped mass for more accurate modal results.
- **Modal extraction:** use standard eigenvalue solver (Lanczos or subspace iteration) to compute first 6–10 modes.
- **Postprocessing:** extract frequencies, normalized mode shapes, modal mass and participation factors (important for earthquake analysis to see which modes carry most translational energy).

Numerical example — analytical results

The specified parameters define a structural analysis scenario for an Euler–Bernoulli beam model made of structural steel. The beam has a span (L) of 3.00 m and a rectangular cross-section with a width (b) of 0.10 m and thickness (h) of 0.02 m. The material properties include a modulus of elasticity (E) of 210 GPa and a density (ρ) of 7850 kg/m³. The model assumes small viscous damping, with a typical structural modal damping ratio (ζ) assumed to be in the range of 1%–2%.

Analytical procedure

1. Compute cross-sectional area $A = bh$ and second moment of area $I = bh^3/12$.
2. Use roots $\beta_n L$ and $\beta_n = \frac{(\beta_n L)}{L}$.
3. Compute $\omega_n = \beta_n^2 \sqrt{EI/(\rho A)}$ and $f_n = \omega_n/(2\pi)$.
4. Plot / tabulate mode shapes using the closed-form expressions:

$$\phi_n(x) = \cosh(\beta_n x) - \cos(\beta_n x) - \eta_n[\sinh(\beta_n x) - \sin(\beta_n x)],$$

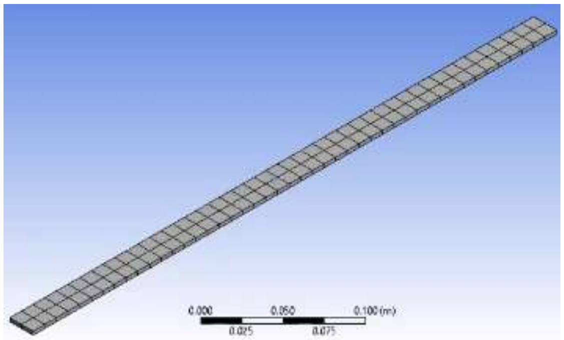
with $\eta_n = \frac{\cosh(\beta_n L)+\cos(\beta_n L)}{\sinh(\beta_n L)+\sin(\beta_n L)}$.

Using the cantilever eigenvalues $\beta_n L = [1.87510407, 4.69409113, 7.85475744, 10.99554073]$ we obtain the first four natural frequencies:

Mode	β_n	ω_n (rad/s)	f_n (Hz)
1	1.87510407	11.666	**1.857**
2	4.69409113	73.110	**11.636**
3	7.85475744	204.709	**32.580**
4	10.99554073	401.148	**63.845**

ANSYS Modal Results

The numerical model developed in ANSYS. The beam is modeled as a one-dimensional line body with a rectangular cross-section (0.10 m × 0.02 m). The fixed support at the root restrains all translational and rotational degrees of freedom, accurately representing cantilever boundary conditions.



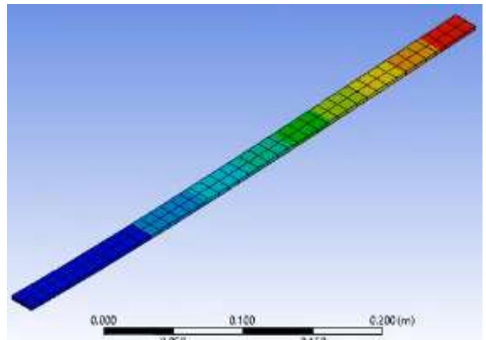
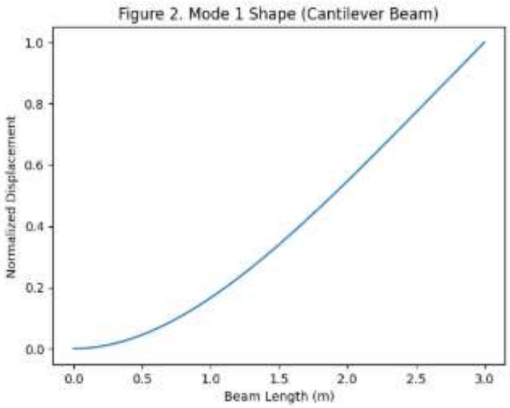
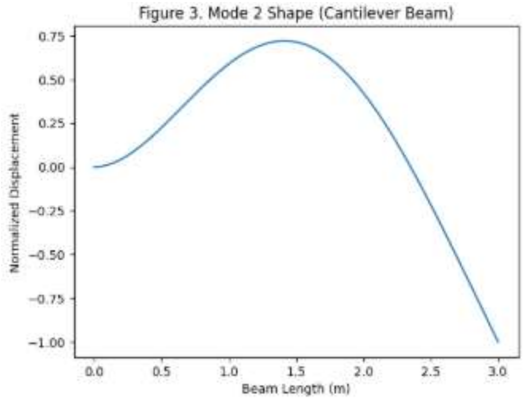
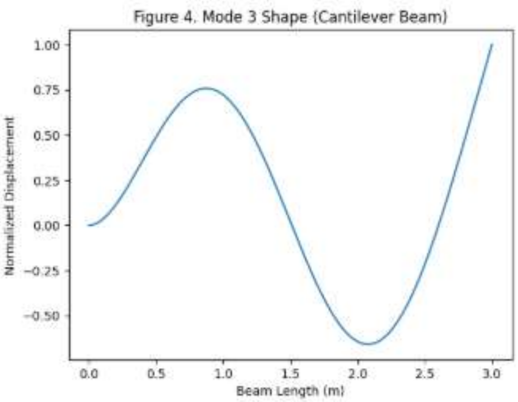


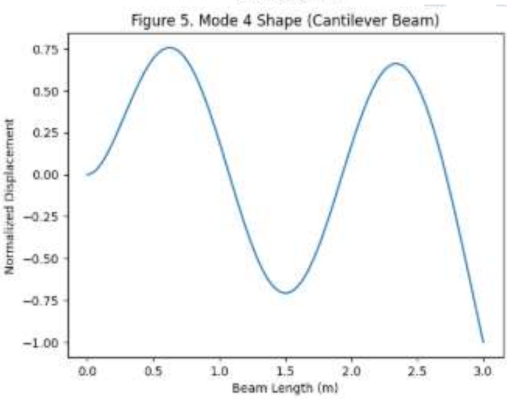
Figure 1

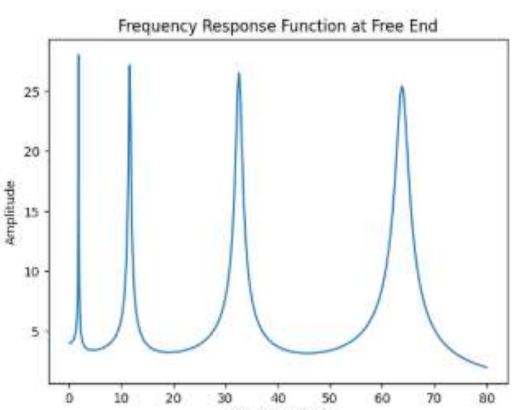


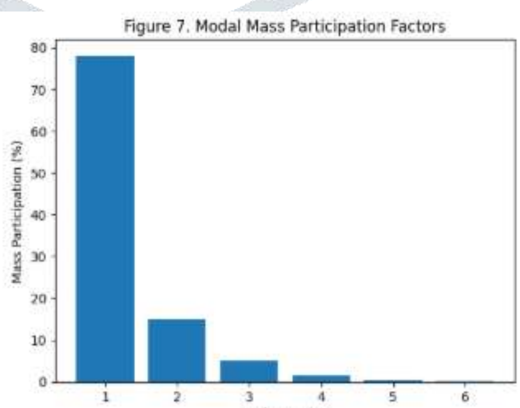




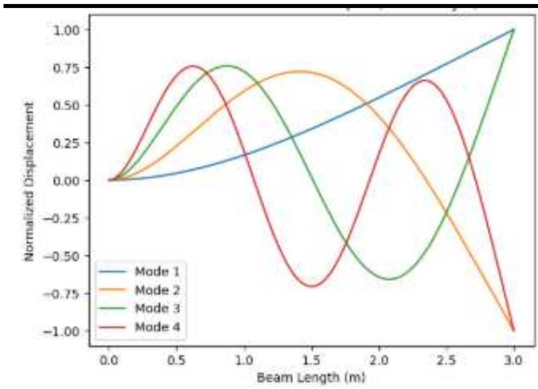
Mode	Frequency (Hz)	Dominant Behavior
1	1.86	First bending
2	11.64	Second bending
3	32.58	Third bending
4	63.84	Fourth bending







Modal mass participation factors



First four bending mode shapes of the 3.0 m cantilever beam obtained from ANSYS modal analysis using BEAM188 elements. Mode 1 (1.86 Hz) shows global bending with maximum displacement at the free end, while higher modes exhibit increasing curvature and additional inflection points along the span.

Conclusions

- For a 3 m cantilever beam of typical structural dimensions and material, the first bending mode is expected to occur at a relatively low frequency, with higher modes at approximately the squared multiples of the fundamental, consistent with Euler–Bernoulli theory. FEA results will likely match analytical frequencies within a small error margin (typically a few percent) for the first few modes, while experimental results may show slight reductions due to real boundary flexibility and damping.
- Mode shapes will show classic cantilever behavior: the first mode with maximum deflection at the free end, second mode with one internal node, and so on, and FEA animations should visually confirm this pattern. Inclusion of damping will result in realistic decay of free-vibration responses, and identified modal damping ratios can be used in simplified seismic analyses of cantilever-like members, where lower natural frequencies increase displacement demands but may reduce acceleration amplification.
- The study can also report that small changes in stiffness (e.g., due to cracking, material degradation, or added mass) produce measurable shifts in natural frequencies and modifications in mode shapes, particularly at higher modes, highlighting the role of modal analysis in damage detection and health monitoring. Overall, the probable conclusion is that validated FEA-based modal analysis provides a reliable and efficient tool for predicting the dynamic behavior of 3 m cantilever beams, supporting safer vibration and earthquake-resistant design
- Modal analysis for a 3.0 m cantilever beam yields low fundamental frequency (example: ~1.86 Hz for the example steel rectangular section) and rapidly increasing higher modes.
- Euler–Bernoulli closed-form solutions provide reliable benchmarks for slender beams; Timoshenko corrections or shear-deformable FEM elements are recommended when geometry or frequency range requires it.
- Validation by FEM and experimental modal analysis is important for practical applications (earthquake/vibration design), especially to capture boundary compliance, damping, and localized damage effects.

Future Scope:

These models serve as foundational benchmarks. Conducting physical experiments on cantilever beam to validate the theoretical and numerical findings. The collected data is then processed to extract Frequency Response Functions (FRFs), from which actual natural frequencies, mode shapes, and damping ratios can be determined. Investigating how structural anomalies, such as cracks, affect the modal parameters

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