REGULAR SEMI CONTINUOUS AND OPEN MAPPINGS ON INTUITIONISTIC FUZZY TOPOLOGICAL SPACES IN $\hat{S}$ OSTAK'S SENSE

G. Saravanakumar[1]
Research Scholar, Department of Mathematics, Annamalai University
S. Tamilselvan[2]
Mathematics Section (FEAT), Annamalai University
A. Vadivel[3]
Department of Mathematics, Government Arts College (Autonomous), Karur -5

ABSTRACT: We introduce the concepts of fuzzy (r,s)-regular semi (resp. (r,s)-α, (r,s)-pre and (r,s)-β) continuous mappings and fuzzy (r,s)-regular semi (resp. (r,s)-α, (r,s)-pre and (r,s)-β) open mappings on intuitionistic fuzzy topological spaces in the sense of $\hat{S}$ ostak's. Further we investigate some of their characteristic and respective properties.

KEYWORDS AND PHRASES: fuzzy (r,s)-regular semi (resp. (r,s)-α, (r,s)-pre and (r,s)-β) continuous mappings and fuzzy (r,s)-regular semi (resp. (r,s)-α, (r,s)-pre and (r,s)-β) open mappings.
AMS (2000) SUBJECT CLASSIFICATION: 54A40.

1. Introduction The concept of fuzzy sets was introduced by Zadeh [13]. Chang [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors one of which, developed by $\hat{S}$ ostak [12], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay, Hazra and Samanta [3], and by Ramadan [10]. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Coker and his collegues [4,6,7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Coker and Demirci [5] defined

Intuitionistic fuzzy topological spaces in $\hat{S}$ ostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces. In this paper, we introduce the concepts of fuzzy (r,s)-regular semi (resp. (r,s)-α, (r,s)-pre and (r,s)-β) continuous mappings and fuzzy (r,s)-regular semi (resp. (r,s)-α, (r,s)-pre and (r,s)-β) open mappings on intuitionistic fuzzy topological spaces in the sense of $\hat{S}$ ostak's. Further we investigate some of their characteristic and respective properties.

2. Preliminaries Let I be the unit interval [0,1] of the real line. A member μ of I^X is called a fuzzy set of X. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. For any $\mu \in I^X$, μ^c denotes the complement of $\tilde{1}-\mu$. All other notations are standard notations of fuzzy set theory. Let X be a nonempty set. An intuitionistic fuzzy set A is an ordered pair $A=(\mu_A,\gamma_A)$ where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and degree of non-membership, respectively, and $\mu_A+\gamma_A \le \tilde{1}$.Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu,\tilde{1}-\mu)$. **[1]** Let $A=(\mu_A,\gamma_A)$ and $B=(\mu_B,\gamma_B)$ be intuitionistic fuzzy sets on X. Then[(i)] $A \subseteq B$ iff $\mu_A \le \mu_B$ and $\gamma_A \ge \gamma_B$,[(ii)] $A=B$ iff $A \subseteq B$ and $B \subseteq A$,[(iii)] $A^c=(\gamma_A,\mu_A)$,[(iv)] $A \cap B=(\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$, [(v)] $A \cup B=(\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$,[(vi)] $0_\sim=(\tilde{0},\tilde{1})$ and $1_\sim=(\tilde{1},\tilde{0})$. **[1]** Let f be a map from a set X to a set Y. Let $A=(\mu_A,\gamma_A)$ be a intuitionistic fuzzy set of X and $B=(\mu_B,\gamma_B)$ an intuitionistic fuzzy set of Y. Then:[(i)]The image of A under f, denoted by $f(A)$ is an intuitionistic fuzzy set in Y defined by $f(A)=(f(\mu_A),\tilde{1}-f(\tilde{1}-\gamma_A))$.[(ii)]The inverse image of B under f, denoted by $f^{-1}(B)$ is an intuitionistic fuzzy set in X defined by $f^{-1}(B)=(f^{-1}(\mu_B), f^{-1}(\gamma_B))$. **[10]**

A smooth fuzzy topology on X is a map $T: I^X \to I$ which satisfies the following properties:[(i)] $T(\tilde{0}) = T(\tilde{1}) = 1$,[(ii)] $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$,[(iii)] $T(\vee \mu_i) \geq \wedge T(\mu_i)$ The pair (X,T) is called a smooth fuzzy topological space. **[5]** An intuitionistic fuzzy topology on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:[(i)] $0_\sim, 1_\sim \in T$,[(ii)]If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$,[(iii)]If $A_i \in T$ for all i, then $\cup A_i \in T$.The pair (X,T) is called an intuitionistic fuzzy topological space. Let $I(X)$ be a family of all intuitionistic fuzzy sets of X and let $I \otimes I$ be the set of the pair (r,s) such that $r,s \in I$ and $r+s \leq 1$. **[6]** Let X be a nonempty set. An intuitionistic fuzzy topology in $\hat{S}$ ostak's sense (SoIFT for shor) $T = (T_1, T_2)$ on X is a map $T: I(X) \to I \otimes I$ which satisfied the following properties:[(i)] $T_1(0_\sim) = T_1(1_\sim) = 1$ and $T_2(0_\sim) = T_2(1_\sim) = 1$,[(ii)] $T_1(A \cap B) \geq T_1(A) \wedge T_1(B)$ and $T_2(A \cap B) \leq T_2(A) \vee T_2(B)$,[(iii)] $T_1(\cup A_i) \geq \wedge T_1(A_i)$ and $T_2(\cup A_i) \leq \vee T_2(A_i)$. The $(X,T) = (X, T_1, T_2)$ is said to be an intuitionistic fuzzy topological space in $\hat{S}$ ostak's sense (SoIFTS for short). Also, we call $T_1(A)$ a gradation of openness of A and $T_2(A)$ a gradation of nonopenness of A **[8]** Let A be an intuitionistic fuzzy set in a SoIFTS (X, T_1, T_2) and $(r,s) \in I \otimes I$. Then A is said to be [(i)]fuzzy (r,s)-open if $T_1(A) \geq r$ and $T_2(A) \leq s$,[(ii)]fuzzy (r,s)-closed if $T_1(A^c) \geq r$ and $T_2(A^c) \leq s$. **[8]** Let (X, T_1, T_2) be a SoIFTS. For each $(r,s) \in I \otimes I$ and for each $A \in I(X)$, the fuzzy (r,s)-interior is defined by $int_{T_1,T_2}(A,r,s) = \cup\{B \in I(X) \mid A \supseteq B, B \text{is fuzzy}(r,s)\text{-open}\}$.and the fuzzy (r,s)-closure is defined by $cl_{T_1,T_2}(A,r,s) = \cap\{B \in I(X) \mid A \subseteq B, B \text{is fuzzy}(r,s)\text{-closed}\}$.The operators $int_{T_1,T_2}: I(X) \times I \otimes I \to I(X)$ and $cl_{T_1,T_2}: I(X) \times I \otimes I \to I(X)$ are called the fuzzy interior operator and fuzzy closure operator in (X, T_1, T_2), respectively. **[8]** For an intuitionistic fuzzy set A in a SoIFTS (X, T_1, T_2) and $(r,s) \in I \otimes I$ [(i)] $int_{T_1,T_2}(A,r,s)^c = cl_{T_1,T_2}(A^c, r, s)$,[(ii)] $cl_{T_1,T_2}(A,r,s)^c = int_{T_1,T_2}(A^c, r, s)$. **[8]** Let (X,T) be an intuitionistic fuzzy topological space in $\hat{S}$ ostak's sense. Then it is easy to see that for each $(r,s) \in I \otimes I$, the family $T_{(r,s)}$ defined by

$T_{(r,s)} = \{A \in I(X) \mid T_1(A) \geq r \text{ and } T_2(A) \leq s\}$ is an intuitionistic fuzzy topology on X

[8] Let (X,T) be an intuitionistic fuzzy topological space $(r,s) \in I \otimes I$. Then the map $T^{(r,s)}: I(X) \to I \otimes I$ defined by $T^{(r,s)}(A) = \begin{cases} (1,0), & \text{if } A = 0,1 \\ (r,s), & \text{if } A \in T - \{0,1\} \\ (0,1), & \text{otherwise,} \end{cases}$ becomes an intuitionistic fuzzy topology in $\hat{S}$ ostak's sense on X. **[9]** Let A be an intuitionistic fuzzy set in a SoIFTS (X, T_1, T_2) and $(r,s) \in I \otimes I$. Then A is said to be [(i)]fuzzy (r,s)-semi open if there is a fuzzy (r,s)-open set B in X such that $B \subseteq A \subseteq cl_{T_1,T_2}(B,r,s)$, [ii)]fuzzy (r,s)-semi closed if there is a fuzzy (r,s)-regular B in X such that $int_{T_1,T_2}(B,r,s) \subseteq A \subseteq B$.**[11]** Let A be an intuitionistic fuzzy set in a SoIFTS (X, T_1, T_2) and $(r,s) \in I \otimes I$. Then A is said to be[(i)]fuzzy (r,s)-regular open if $A = int_{T_1,T_2}(cl_{T_1,T_2}(A,r,s),r,s)$,[(ii)]fuzzy (r,s)-regular closed if $A = cl_{T_1,T_2}(int_{T_1,T_2}(A,r,s),r,s)$, [(iii)]fuzzy (r,s)-α open if $A \subseteq int_{T_1,T_2}(cl_{T_1,T_2}(int_{T_1,T_2}(A,r,s),r,s)r,s)$,[(iv)]fuzzy (r,s)-α closed if $A \supseteq cl_{T_1,T_2}(int_{T_1,T_2}(cl_{T_1,T_2}(A,r,s),r,s)r,s)$,[(v)]fuzzy (r,s)-pre open if $A \subseteq int_{T_1,T_2}(cl_{T_1,T_2}(A,r,s),r,s)$,[(vi)]fuzzy (r,s)-pre closed if $A \supseteq cl_{T_1,T_2}(int_{T_1,T_2}(A,r,s),r,s)$,

[(vii)]fuzzy (r,s)-β open if $A \subseteq cl_{T_1,T_2}(int_{T_1,T_2}(cl_{T_1,T_2}(A,r,s),r,s)r,s)$,[(viii)]fuzzy (r,s)-β closed if $A \supseteq int_{T_1,T_2}(cl_{T_1,T_2}(int_{T_1,T_2}(A,r,s),r,s)r,s)$, [(ix)]fuzzy (r,s)-regular semi open if there is a fuzzy (r,s)-regular open set B in X such that $B \subseteq A \subseteq cl_{T_1,T_2}(B,r,s)$,[(x)]fuzzy (r,s)-regular semi closed if there is a fuzzy (r,s)-regular closed set B in X such that $int_{T_1,T_2}(B,r,s) \subseteq A \subseteq B$.**[11]** Let (X, T_1, T_2) be a SoIFTS. For each $(r,s) \in I \otimes I$ and for each $A \in I(X)$, the fuzzy (r,s)-regular semi (resp. (r,s)-α, (r,s)-pre and (r,s)-β) - interior is defined by $rsint_{T_1,T_2}(A,r,s)$ (resp.$\alpha int_{T_1,T_2}(A,r,s)$, $pint_{T_1,T_2}(A,r,s)$and $\beta int_{T_1,T_2}(A,r,s)) =$

$\bigcup\{B \in I(X) \mid A \supseteq B, B \text{is fuzzy } (r,s)\text{- regular semi (resp.}\alpha, \text{pre and } \beta)\text{-open}\}$

and the fuzzy (r,s)-regular semi (resp. (r,s)-α, (r,s)-pre and (r,s)-β) - closure is defined by

$$rscl_{T_1,T_2}(A,r,s)\ (\text{resp.}\ \alpha cl_{T_1,T_2}(A,r,s), pcl_{T_1,T_2}(A,r,s)\ \text{and}\ \beta cl_{T_1,T_2}(A,r,s)) =$$
$$\bigcap\{B \in I(X) \mid A \subseteq B, B\ \text{is fuzzy}\ (r,s)\text{-regular semi(resp.}\ \alpha, \text{pre and}\ \beta)\text{-closed}\}$$

3. Fuzzy (r,s)-regular semi (resp. α, pre and β) continuous mappings

Definition 3.1 Let $f:(X,T_1,T_2)\to(Y,W_1,W_2)$ be a mapping from a SoIFTS X to another SoIFTS Y and $(r,s)\in I\otimes I$. Then f is said to be fuzzy (r,s)-regular semi (resp. regular, α, pre and β) continuous if $f^{-1}(B)$ is a fuzzy (r,s)-regular semi (resp. regular, α, pre and β) open set of X for each fuzzy (r,s)-open set B of Y.

Fuzzy (r,s)-regular continuous

Fuzzy (r,s)-continuous → Fuzzy (r,s)- regular semi continuous

Fuzzy (r,s)- α continuous → Fuzzy (r,s)- semi continuous Fuzzy

(r,s)- pre continuous → Fuzzy (r,s)- β continuous

From the above definitions it is clear that the above implications are true for $(r,s)\in I\otimes I$.
The converses of the above implications are not true as the following examples show:

Example 3.1 Let $X=\{a,b\}=Y$, and let $A_1,A_2\in I(X)$, $B_1\in I(Y)$ defined as
$A_1(x)=B_1(x)=(0.6,0.3), A_1(y)=B_1(y)=(0.5,0.3)$, $A_2(x)=(0.8,0.1), A_2(y)=(0.5,0.1)$

Define $T:I(X)\to I\otimes I$ and $W:I(Y)\to I\otimes I$ by $T(A)=(T_1(A),T_2(A))=\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{2}) & \text{if } A=A_1,A_2\\ (0,1) & \text{otherwise}\end{cases}$

$W(B)=(W_1(B),W_2(B))=\begin{cases}(1,0) & \text{if } B=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{2}) & \text{if } B=B_1\\ (0,1) & \text{otherwise}\end{cases}$ For $r=1/2, s=1/2$. Then the identity mapping $f:(X,T_1,T_2)\to(Y,W_1,W_2)$ is fuzzy $(1/2,1/2)$-continuous which is not fuzzy $(1/2,1/2)$-regular continuous. Since B_1 is fuzzy $(1/2,1/2)$-open in W but $f^{-1}(B_1)$ is not fuzzy $(1/2,1/2)$-regular open in T.

Example 3.2 Let $X=\{a,b\}=Y$, and let $A_1,A_2\in I(X)$, $B_1\in I(Y)$ defined as
$A_1(x)=(0.6,0.3), A_1(y)=(0.5,0.3)$, $A_2(x)=B_1(x)=(0.8,0.1), A_2(y)=B_1(y)=(0.5,0.1)$

Define $T:I(X)\to I\otimes I$ and $W:I(Y)\to I\otimes I$ by $T(A)=(T_1(A),T_2(A))=\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{2}) & \text{if } A=A_1,A_2\\ (0,1) & \text{otherwise}\end{cases}$

$W(B)=(W_1(B),W_2(B))=\begin{cases}(1,0) & \text{if } B=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{2}) & \text{if } B=B_1\\ (0,1) & \text{otherwise}\end{cases}$ For $r=1/2, s=1/2$. Then the identity mapping $f:(X,T_1,T_2)\to(Y,W_1,W_2)$ is fuzzy $(1/2,1/2)$-semi continuous which is not fuzzy $(1/2,1/2)$-regular semi continuous. Since B_1 is fuzzy $(1/2,1/2)$- open in W but $f^{-1}(B_1)$ is not fuzzy $(1/2,1/2)$-regular semi open in T.

Example 3.3 Let $X=\{a,b\}=Y$, and let $A_1 in I(X)$, $B_1,B_2\in I(Y)$ defined as
$A_1(x)=B_1(x)=(0.3,0.6), A_1(y)=B_1(y)=(0.3,0.5)$, $B_2(x)=(0.4,0.3), B_2(y)=(0.4,0.3)$

Define $T: I(X) \to I \otimes I$ and $W: I(Y) \to I \otimes I$ by $T(A) = (T_1(A), T_2(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_1 \\ (0,1) & \text{otherwise} \end{cases}$

$W(B) = (W_1(B), W_2(B)) = \begin{cases} (1,0) & \text{if } B = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } B = B_1, B_2 \\ (0,1) & \text{otherwise} \end{cases}$ For $r = 1/2, s = 1/2$. Then the identity mapping $f: (X, T_1, T_2) \to (Y, W_1, W_2)$ is fuzzy $(1/2, 1/2)$ - semi continuous, regular semi continuous which is not fuzzy $(1/2, 1/2)$ -regular continuous and α -continuous. Since B_2 is fuzzy $(1/2, 1/2)$ - open in W but $f^{-1}(B_2)$ is not fuzzy $(1/2, 1/2)$ -regular open and α -open in T.

Example3.4 Let $X = \{a, b\} = Y$, and let $A_1 \in I(X)$, $B_1, B_2, B_1 \cup B_2, B_1 \cap B_2 \in I(Y)$ defined as

$A_1(x) = B_1(x) = (0.3, 0.1), A_1(y) = B_1(y) = (0.3, 0.1)$, $B_2(x) = (0.4, 0.3), B_2(y) = (0.4, 0.3)$

$B_1(x) \cup B_2(x) = (0.4, 0.1), B_1(y) \cup B_2(y) = (0.4, 0.1), B_1(x) \cap B_2(x) = (0.3, 0.3), B_1(y) \cap B_2(y) = (0.3, 0.3)$ Define $T: I(X) \to I \otimes I$ and $W: I(Y) \to I \otimes I$ by

$T(A) = (T_1(A), T_2(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_1 \\ (0,1) & \text{otherwise} \end{cases}$ $W(B) = (W_1(B), W_2(B)) = \begin{cases} (1,0) & \text{if } B = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } B = B_1, B_2, B_1 \cup B_2, B_1 \cap B_2 \\ (0,1) & \text{otherwise} \end{cases}$

For $r = 1/2, s = 1/2$. Then the identity mapping $f: (X, T_1, T_2) \to (Y, W_1, W_2)$ is fuzzy $(1/2, 1/2)$ - α -continuous which is not fuzzy $(1/2, 1/2)$ - continuous. Since B_2 is fuzzy $(\frac{1}{2}, \frac{1}{2})$ -open in W but $f^{-1}(B_2)$ is not fuzzy $(1/2, 1/2)$ - open in T.

Example 3.5 Let $X = \{a, b\} = Y$, and let $A_1 \in I(X)$, $B_1, B_2, B_1 \cup B_2, B_1 \cap B_2 \in I(Y)$ defined as

$A_1(x) = B_1(x) = (0.3, 0.6), A_1(y) = B_1(y) = (0.3, 0.5)$, $B_2(x) = (0.1, 0.2), B_2(y) = (0.1, 0.2)$

$B_1(x) \cup B_2(x) = (0.3, 0.2), B_1(y) \cup B_2(y) = (0.3, 0.2), B_1(x) \cap B_2(x) = (0.1, 0.6), B_1(y) \cap B_2(y) = (0.1, 0.5)$ Define

$T: I(X) \to I \otimes I$ and $W: I(Y) \to I \otimes I$ by $T(A) = (T_1(A), T_2(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_1 \\ (0,1) & \text{otherwise} \end{cases}$

$W(B) = (W_1(B), W_2(B)) = \begin{cases} (1,0) & \text{if } B = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } B = B_1, B_2, B_1 \cup B_2, B_1 \cap B_2 \\ (0,1) & \text{otherwise} \end{cases}$ For $r = 1/2, s = 1/2$. Then the identity mapping $f: (X, T_1, T_2) \to (Y, W_1, W_2)$ is fuzzy $(1/2, 1/2)$ - β continuous which is not fuzzy $(1/2, 1/2)$ -semi continuous. Since B_2 is fuzzy $(1/2, 1/2)$ - open in W but $f^{-1}(B_2)$ is not fuzzy $(1/2, 1/2)$ -semi open in T.

Example3.6 Let $X = \{a, b\} = Y$, and let $A_1 \in I(X)$, $B_1, B_2, B_1 \cup B_2, B_1 \cap B_2 \in I(Y)$ defined as

$A_1(x) = B_1(x) = (0.3, 0.6), A_1(y) = B_1(y) = (0.3, 0.5)$, $B_2(x) = (0.1, 0.3), B_2(y) = (0.1, 0.3)$,

$B_1(x) \cup B_2(x) = (0.3, 0.3), B_1(y) \cup B_2(y) = (0.3, 0.3), B_1(x) \cap B_2(x) = (0.1, 0.6), B_1(y) \cap B_2(y) = (0.1, 0.5)$ Define

$T: I(X) \to I \otimes I$ and $W: I(Y) \to I \otimes I$ by $T(A) = (T_1(A), T_2(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_1 \\ (0,1) & \text{otherwise} \end{cases}$

$$W(B)=(W_1(B),W_2(B))=\begin{cases}(1,0) & \text{if } B=0_\sim,1_\sim,\\ (\tfrac{1}{2},\tfrac{1}{2}) & \text{if } B=B_1,B_2,B_1\cup B_2,B_1\cap B_2\\ (0,1) & \text{otherwise}\end{cases}$$ For $r=1/2, s=1/2$. Then the identity mapping $f:(X,T_1,T_2)\to(Y,W_1,W_2)$ is fuzzy $(1/2,1/2)$- β continuous which is not fuzzy $(1/2,1/2)$-pre continuous. Since B_2 is fuzzy $(1/2,1/2)$- open in W but $f^{-1}(B_2)$ is not fuzzy $(1/2,1/2)$-pre open in T.

Example 3.7 Let $X=\{a,b\}=Y$, and let $A_1\in I(X)$, $B_1,B_2,B_1\cup B_2,B_1\cap B_2\in I(Y)$ defined as

$A_1(x)=B_1(x)=(0.1,0.3), A_1(y)=B_1(y)=(0.1,0.3), B_1(x)\cap B_2(x)=(0.1,0.6), B_1(y)\cap B_2(y)=(0.1,0.5)$
$B_2(x)=(0.3,0.6), B_2(y)=(0.3,0.5)$ $B_1(x)\cup B_2(x)=(0.3,0.3), B_1(y)\cup B_2(y)=(0.3,0.3)$

Define $T:I(X)\to I\otimes I$ and $W:I(Y)\to I\otimes I$ by $T(A)=(T_1(A),T_2(A))=\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\tfrac{1}{2},\tfrac{1}{2}) & \text{if } A=A_1\\ (0,1) & \text{otherwise}\end{cases}$

$$W(B)=(W_1(B),W_2(B))=\begin{cases}(1,0) & \text{if } B=0_\sim,1_\sim,\\ (\tfrac{1}{2},\tfrac{1}{2}) & \text{if } B=B_1,B_2,B_1\cup B_2,B_1\cap B_2\\ (0,1) & \text{otherwise}\end{cases}$$ For $r=1/2, s=1/2$. Then the identity mapping $f:(X,T_1,T_2)\to(Y,W_1,W_2)$ is fuzzy $(1/2,1/2)$-pre continuous which is not fuzzy $(1/2,1/2)$- α continuous. Since B_2 is fuzzy $(1/2,1/2)$-open in W but $f^{-1}(B_2)$ is not fuzzy $(1/2,1/2)$- α open in T.

It need not be true that f and g are fuzzy (r,s)-regular semi (resp. regular, α, pre and β) continuous mappings then so is $g\circ f$. But we have the following theorem.

Theorem 3.1 Let $(X,T_1,T_2),(Y,W_1,W_2)$ and (Z,V_1,V_2) be SoIFTSs and let $f:X\to Y$ and $g:Y\to Z$ be mappings and $(r,s)\in I\otimes I$. If f is a fuzzy (r,s)-regular semi (resp. regular, α, pre and β) continuous mapping and g is a fuzzy (r,s)-continuous mapping, then $g\circ f$ is a fuzzy (r,s)-regular semi (resp. regular, α, pre and β) continuous mapping.

Proof. Straighforward

Theorem 3.2 Let (X,T_1,T_2) and (Y,W_1,W_2) be SoIFTSs. Let $f:X\to Y$ be a mapping. The following statements are equivalent:

[(i)] f is (r,s)-fuzzy regular semi continuous.[(ii)] $f(rscl_{T_1,T_2}(A,r,s))\subseteq rcl_{W_1,W_2}(f(A),r,s)$, for each $A\in I^X$.[(iii)] $rscl_{T_1,T_2}(f^{-1}(B),r,s)\subseteq f^{-1}(rcl_{W_1,W_2}(B,r,s))$, for each $B\in I^Y$.[(iv)] $f^{-1}(rint_{W_1,W_2}(B,r,s))\subseteq rsint_{T_1,T_2}(f^{-1}(B),r,s)$, for each $B\in I^Y$.

Proof. (i)$\Rightarrow$(ii) Let (i) holds and let $A\in I^X$. Suppose that there exists $A\in I^X$ and $(r,s)\in I\otimes I$ such that $f(rscl_{T_1,T_2}(A,r,s))\,\acute{\cup}\,rcl_{W_1,W_2}(f(A),r,s)$. So there exists $y\in Y, t\in(0,1]$ such that $f(rscl_{T_1,T_2}(A,r,s))(y)>t>rcl_{W_1,W_2}(f(A),r,s)(y)$. If $f^{-1}(y)=\phi$, then, $f(rscl_{T_1,T_2}(A,r,s))(y)=0_\sim$. So, there exists $x\in f^{-1}(y)$ such that: $f(rscl_{T_1,T_2}(A,r,s))(y)\supset rscl_{T_1,T_2}(A,r,s)(x)>t>RC_{W_1,W_2}(f(A),r,s)(y)$ (3.1)

Since $rcl_{W_1,W_2}(f(A),r,s)(y)<t$, there exists (r,s)-frc set B such that $f(A)=B$. Then, $rcl_{W_1,W_2}(f(A),r,s)(y)\subseteq B$ and this implies $rcl_{W_1,W_2}(f(A),r,s)(f(x))\subseteq B(f(x))<t$. Moreover, $f(A)\subseteq B$ implies $A\subseteq f^{-1}(B)$. So, $1_\sim-A\supseteq 1_\sim-f^{-1}(B)$. Since $f^{-1}(1_\sim-B)$ is (r,s)-frso, then from (i), $f^{-1}(1_\sim-B)\subseteq rsint_{T_1,T_2}(1_\sim-A,r,s)$ or $1_\sim-rsint_{T_1,T_2}(1_\sim-A,r,s)\subseteq 1_\sim-f^{-1}(1_\sim-B)$.

Then, $rscl_{T_1,T_2}(A,r,s)\subseteq f^{-1}(B)$. This gives $rscl_{T_1,T_2}(A,r,s)(x)\subseteq f^{-1}(B)(x)=B(f(x))<t$.

But the last relation contradicts the relation (3.1) above. Hence, the result in (ii) is true.

(ii)$\Rightarrow$(iii) Take $A=f^{-1}(B)(B\in I^Y)$ and apply (ii) we have the required result.

(iii)$\Rightarrow$(iv) Taking the complement of (iii), we have the result.(iv)$\Rightarrow$(i) Let (iv) holds and let $B\in I^Y$ be a (r,s)-fro set, since every (r,s)-fro set is (r,s)-fuzzy open. So $W_1(B)>r$, and $W_2(B)<s$. By (iv)

$f^{-1}(rint_{W_1,W_2}(B,r,s)) \subseteq rsint_{T_1,T_2}(f^{-1}(B),r,s)$ or $f^{-1}(B) \subseteq rsint_{T_1,T_2}(f^{-1}(B),r,s)$ (since $W_1(B) \geq r, W_2(B) \leq s$). But $rsint_{T_1,T_2}(f^{-1}(B),r,s) \subseteq f^{-1}(B)$. Then, $f^{-1}(B) = rsint_{T_1,T_2}(f^{-1}(B),r,s)$ and this means that $f^{-1}(B)$ is (r,s)-frso. Hence, f is (r,s) fuzzy regular semi continuous.

Theorem 3.3 Let (X,T_1,T_2) and (Y,W_1,W_2) be SoIFTSs. Let $f: X \to Y$ be a mapping. The following statements are equivalent: [(i)] f is fuzzy (r,s)-α continuous.[(ii)] $f(\alpha cl_{T_1,T_2}(A,r,s)) \subseteq cl_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X$.[(iii)] $\alpha cl_{T_1,T_2}(f^{-1}(B),r,s) \subseteq f^{-1}(cl_{W_1,W_2}(B,r,s))$, for each $B \in I^Y$.[(iv)] $f^{-1}(int_{W_1,W_2}(B,r,s)) \subseteq \alpha int_{T_1,T_2}(f^{-1}(B),r,s)$, for each $B \in I^Y$.

Theorem 3.4 Let (X,T_1,T_2) and (Y,W_1,W_2) be SoIFTSs. Let $f: X \to Y$ be a mapping. The following statements are equivalent: [(i)] f is fuzzy (r,s)-pre continuous.[(ii)] $f(pcl_{T_1,T_2}(A,r,s)) \subseteq cl_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X$.[(iii)] $pcl_{T_1,T_2}(f^{-1}(B),r,s) \subseteq f^{-1}(cl_{W_1,W_2}(B,r,s))$, for each $B \in I^Y$.[(iv)] $f^{-1}(int_{W_1,W_2}(B,r,s)) \subseteq pint_{T_1,T_2}(f^{-1}(B),r,s)$, for each $B \in I^Y$.

Theorem 3.5 Let (X,T_1,T_2) and (Y,W_1,W_2) be SoIFTSs. Let $f: X \to Y$ be a mapping. The following statements are equivalent: [(i)] f is fuzzy (r,s)-β continuous.

[(ii)] $f(\beta cl_{T_1,T_2}(A,r,s)) \subseteq cl_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X$.[(iii)] $\beta cl_{T_1,T_2}(f^{-1}(B),r,s) \subseteq f^{-1}(cl_{W_1,W_2}(B,r,s))$, for each $B \in I^Y$.[(iv)] $f^{-1}(int_{W_1,W_2}(B,r,s)) \subseteq \beta int_{T_1,T_2}(f^{-1}(B),r,s)$, for each $B \in I^Y$.

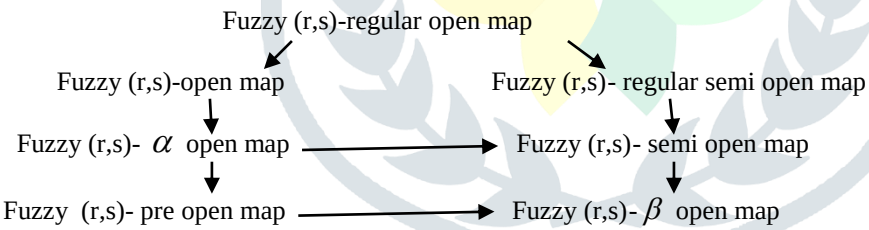
Proof. Proof of Theorem 3.3,3.4,3.5 it follows from Theorem 3.2

4. Fuzzy (r,s)-regular semi(resp. α, pre, β) open mappings

Definition 4.1 Let $f:(X,T_1,T_2) \to (Y,W_1,W_2)$ be a mapping from a SoIFTS X to another SoIFTS Y and $(r,s) \in I \otimes I$. Then f is said to be

[(i)] fuzzy (r,s)-regular semi (resp. regular, α, pre and β) open if $f(A)$ is a fuzzy (r,s)-regular semi (resp. regular, α, pre and β) open set of Y for each fuzzy (r,s)-open set A of X

[(ii)] fuzzy (r,s)-regular semi (resp. regular, α, pre and β) closed if $f(A)$ is a fuzzy (r,s)-regular semi (resp. regular, α, pre and β) closed set of Y for each fuzzy (r,s)-closed set A of X.



From the above definitions it is clear that the above implications are true for $(r,s) \in I \otimes I$

The converses of the above implications are not true as the following examples show:

Example 4.1 Let $X = \{a,b\} = Y$. Define $A_1 \in I(X)$ $B_1, B_2 \in I(Y)$ as follows

$$A_1(x) = B_1(x) = (0.6,0.3), A_1(y) = B_1(y) = (0.5,0.3), B_2(x) = (0.8,0.1), B_2(y) = (0.5,0.1)$$

Define $T: I(X) \to I \otimes I$ and $W: I(Y) \to I \otimes I$ by $T(A) = (T_1(A), T_2(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2},\frac{1}{2}) & \text{if } A = A_1 \\ (0,1) & \text{otherwise} \end{cases}$

$W(B) = (W_1(A), W_2(A)) = \begin{cases} (1,0) & \text{if } B = 0_\sim, 1_\sim, \\ (\frac{1}{2},\frac{1}{2}) & \text{if } B = B_1, B_2 \\ (0,1) & \text{otherwise} \end{cases}$ For $r = 1/2, s = 1/2$. Then the identity mapping $f:(X,T_1,T_2) \to (Y,W_1,W_2)$ is fuzzy $(1/2,1/2)$-open mapping which is not fuzzy $(1/2,1/2)$-regular open mapping. Since A_1 is fuzzy $(1/2,1/2)$- open in T but $f(A_1)$ is not fuzzy $(1/2,1/2)$-regular open in W.

Example 4.2 Let $X = \{a,b\} = Y$. Define $A_1 \in I(X)$, $B_1, B_2 \in I(X)$ as follows

$B_1(x)=(0.6,0.3), B_1(y)=(0.5,0.3), B_2(x)=A_1(x)=(0.8,0.1), B_2(y)=A_1(y)=(0.5,0.1)$

Define $T:I(X)\to I\otimes I$ and $W:I(Y)\to I\otimes I$ by $T(A)=(T_1(A),T_2(A))=\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{2}) & \text{if } A=A_1\\ (0,1) & \text{otherwise}\end{cases}$

$W(B)=(W_1(B),W_2(B))=\begin{cases}(1,0) & \text{if } B=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{2}) & \text{if } B=B_1,B_2\\ (0,1) & \text{otherwise}\end{cases}$ For $r=1/2, s=1/2$. Then the identity mapping $f:(X,T_1,T_2)\to(Y,W_1,W_2)$ is fuzzy $(1/2,1/2)$ - semi open mapping which is not fuzzy $(1/2,1/2)$ -regular semi open mapping. Since A_1 is fuzzy $(1/2,1/2)$ - open in T but $f(A_1)$ is not fuzzy $(1/2,1/2)$ -regular semi open in W.

Example 4.3 Let $X=\{a,b\}=Y$. Define $A_1,A_2\in I(X)$, $B_1\in I(X)$, as follows

$A_1(x)=B_1(x)=(0.3,0.6), A_1(y)=B_1(y)=(0.3,0.5), A_2(x)=(0.4,0.3), A_2(y)=(0.4,0.3)$

Define $T:I(X)\to I\otimes I$ and $W:I(Y)\to I\otimes I$ by $T(A)=(T_1(A),T_2(A))=\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{2}) & \text{if } A=A_1,A_2\\ (0,1) & \text{otherwise}\end{cases}$

$W(B)=(W_1(B),W_2(B))=\begin{cases}(1,0) & \text{if } B=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{2}) & \text{if } B=B_1\\ (0,1) & \text{otherwise}\end{cases}$

For $r=1/2, s=1/2$. Then the identity mapping $f:(X,T_1,T_2)\to(Y,W_1,W_2)$ is fuzzy $(1/2,1/2)$ - semi open mapping, regular semi open mapping which is not fuzzy $(1/2,1/2)$ -regular open mapping. Since A_2 is fuzzy $(1/2,1/2)$ - open in T but $f(A_2)$ is not fuzzy $(1/2,1/2)$ -regular open and α open in W.

Example 4.4 Let $X=\{a,b\}=Y$. Define $A_1,A_2,A_1\cup A_2,A_1\cap A_2\in I(X)$, $B_1\in I(Y)$ as follows

$A_1(x)=B_1(x)=(0.3,0.1), A_1(y)=B_1(y)=(0.3,0.1), A_1(x)\cap A_2(x)=(0.3,0.3), A_1(y)\cap A_2(y)=(0.3,0.3)$
$A_2(x)=(0.4,0.3), A_2(y)=(0.4,0.3)$ $A_1(x)\cup A_2(x)=(0.4,0.1), A_1(y)\cup A_2(y)=(0.4,0.1)$

Define $T:I(X)\to I\otimes I$ and $W:I(Y)\to I\otimes I$ by $T(A)=(T_1(A),T_2(A))=$

$\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{2}) & \text{if } A=A_1,A_2,A_1\cup A_2,A_1\cap A_2\\ (0,1) & \text{otherwise}\end{cases}$ $W(B)=(W_1(B),W_2(B))=\begin{cases}(1,0) & \text{if } B=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{2}) & \text{if } B=B_1\\ (0,1) & \text{otherwise}\end{cases}$

For $r=1/2, s=1/2$. Then the identity mapping $f:(X,T_1,T_2)\to(Y,W_1,W_2)$ is fuzzy $(1/2,1/2)$ - α open mapping which is not fuzzy $(1/2,1/2)$ -open mapping. Since A_2 is fuzzy $(1/2,1/2)$ - open in T but $f(A_2)$ is not fuzzy $(1/2,1/2)$ -open in W.

Example 4.5 Let $X=\{a,b\}=Y$. Define $A_1,A_2,A_1\cup A_2,A_1\cap A_2\in I(X)$, $B_1\in I(Y)$ as follows

$A_1(x)=B_1(x)=(0.3,0.6), A_1(y)=B_1(y)=(0.3,0.5), A_1(x)\cap A_2(x)=(0.1,0.6), A_1(y)\cap A_2(y)=(0.1,0.5)$
$A_2(x)=(0.1,0.2), A_2(y)=(0.1,0.2)$ $A_1(x)\cup A_2(x)=(0.3,0.2), A_1(y)\cup A_2(y)=(0.3,0.2)$

Define $T:I(X)\to I\otimes I$ and $W:I(Y)\to I\otimes I$ by $T(A)=(T_1(A),T_2(A))=$

$\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{2}) & \text{if } A=A_1,A_2,A_1\cup A_2,A_1\cap A_2\\ (0,1) & \text{otherwise}\end{cases}$ $W(B)=(W_1(B),W_2(B))=\begin{cases}(1,0) & \text{if } B=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{2}) & \text{if } B=B_1\\ (0,1) & \text{otherwise}\end{cases}$

For $r=1/2, s=1/2$. Then the identity mapping $f:(X,T_1,T_2)\to(Y,W_1,W_2)$ is fuzzy $(1/2,1/2)$ - β open mapping which is not fuzzy $(1/2,1/2)$ -semi open mapping. Since A_2 is fuzzy $(1/2,1/2)$ - open in T but $f(A_2)$ is not fuzzy $(1/2,1/2)$ -semi open in W.

Example 4.6 Let $X=\{a,b\}=Y$. Define $A_1, A_2, A_1\cup A_2, A_1\cap A_2 \in I(X)$, $B_1 \in I(Y)$ as follows

$A_1(x)=B_1(x)=(0.3,0.6), A_1(y)=B_1(y)=(0.3,0.5), A_1(x)\cap A_2(x)=(0.1,0.6), A_1(y)\cap A_2(y)=(0.1,0.5)$
$A_2(x)=(0.1,0.3), A_2(y)=(0.1,0.3)$ $A_1(x)\cup A_2(x)=(0.3,0.3), A_1(y)\cup A_2(y)=(0.3,0.3)$

Define $T: I(X)\to I\otimes I$ and $W: I(Y)\to I\otimes I$

by $T(A)=(T_1(A),T_2(A))=\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{2}) & \text{if } A=A_1,A_2,A_1\cup A_2,A_1\cap A_2\\ (0,1) & \text{otherwise}\end{cases}$ $W(B)=(W_1(B),W_2(B))=$

$\begin{cases}(1,0) & \text{if } B=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{2}) & \text{if } B=B_1\\ (0,1) & \text{otherwise}\end{cases}$

For $r=1/2, s=1/2$. Then the identity mapping $f:(X,T_1,T_2)\to(Y,W_1,W_2)$ is fuzzy $(1/2,1/2)$- β open mapping which is not fuzzy $(1/2,1/2)$-pre open mapping. Since A_2 is fuzzy $(1/2,1/2)$- open in T but $f(A_2)$ is not fuzzy $(1/2,1/2)$-pre open in W.

Example 4.7 Let $X=\{a,b\}=Y$. Define $A_1, A_2, A_1\cup A_2, A_1\cap A_2 \in I(X)$, $B_1 \in I(Y)$ as follows

$A_1(x)=B_1(x)=(0.1,0.3), A_1(y)=B_1(y)=(0.1,0.3), A_1(x)\cap A_2(x)=(0.1,0.6), A_1(y)\cap A_2(y)=(0.1,0.5)$
$A_2(x)=(0.3,0.6), A_2(y)=(0.3,0.5)$ $A_1(x)\cup A_2(x)=(0.3,0.3), A_1(y)\cup A_2(y)=(0.3,0.3)$

Define $T: I(X)\to I\otimes I$ and $W: I(Y)\to I\otimes I$

by $T(A)=(T_1(A),T_2(A))=\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{2}) & \text{if } A=A_1,A_2,A_1\cup A_2,A_1\cap A_2\\ (0,1) & \text{otherwise}\end{cases}$ $W(B)=(W_1(B),W_2(B))=$

$\begin{cases}(1,0) & \text{if } B=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{2}) & \text{if } B=B_1\\ (0,1) & \text{otherwise}\end{cases}$

For $r=1/2, s=1/2$. Then the identity mapping $f:(X,T_1,T_2)\to(Y,W_1,W_2)$ is fuzzy $(1/2,1/2)$- pre open mapping which is not fuzzy $(1/2,1/2)$-α open mapping. Since A_2 is fuzzy $(1/2,1/2)$- open in T but $f(A_2)$ is not fuzzy $(1/2,1/2)$-α open in W.

It need not be true that f and g are fuzzy (r,s)-regular semi (resp. regular, α, pre and β) open (resp. (r,s)-regular semi (resp. regular, α, pre and β) closed) mappings then so is $g\circ f$. But we have the following theorem.

Theorem 4.1 Let $(X,T_1,T_2),(Y,W_1,W_2)$and (Z,V_1,V_2) be SoIFTSs and let $f:X\to Y$ and $g:Y\to Z$ be mappings and $(r,s)\in I\otimes I$. Then the following statements are true:

[(i)] If f is a fuzzy (r,s)-open mapping and g is a fuzzy (r,s)-regular semi (resp. regular, α, pre and β) open mapping, then $g\circ f$ is a fuzzy (r,s)-regular semi (resp. regular, α, pre and β) open mapping.[(ii)] If f is a fuzzy (r,s)-closed mapping and g is a fuzzy (r,s)-regular semi (resp. regular, α, pre and β) closed mapping, then $g\circ f$ is a fuzzy (r,s)-regular semi (resp. regular, α, pre and β) closed mapping.

Proof:. Straighforward

Theorem 4.2 Let (X,T_1,T_2) and (Y,W_1,W_2) be smooth topological space's. Let $f:X\to Y$ be a mapping. The following statements are equivalent: [(i)] f is fuzzy (r,s)-regular semi open. [(ii)] $f(int_{T_1,T_2}(A,r,s))\subseteq rsint_{W_1,W_2}(f(A),r,s)$, for each $A\in I^X$.[(iii)] $int_{T_1,T_2}(f^{-1}(B),r,s)\subseteq f^{-1}(rsint_{W_1,W_2}(B,r,s))$, for each $B\in I^Y$.[(iv)] For any $A\in I^Y$ and any $B\in I^X$ such that $T_1(1_\sim-B)\geq r$, $T_2(1_\sim-B)\geq s, B\supseteq f^{-1}(A)$, there exists (r,s)-frsc set $C\in I^Y$ with $A\subseteq C$ and $B\supseteq f^{-1}(C)$.

Proof. (i)$\Rightarrow$(ii) Since $int_{T_1,T_2}(A,r,s)\subseteq A,\ (A\in I^X)$, then $f(int_{T_1,T_2}(A,r,s))\subseteq f(A)$. But $f(int_{T_1,T_2}(A,r,s))$ is (r,s)-frso (by (i)), then $f(I_{T_1,T_2}(A,r,s))\subseteq rsintI_{W_1,W_2}(f(A),r,s)$.

(ii)$\Rightarrow$(iii) Let (ii) holds. Put $A=f^{-1}(B), B\in I^Y$ and apply (ii), we have $f(int_{T_1,T_2}(f^{-1}(B),r,s))\subseteq rsint_{W_1,W_2}(ff^{-1}(B),r,s)\subseteq rsint_{W_1,W_2}(B,r,s)$. Hence, $int_{T_1,T_2}(f^{-1}(B),r,s)\subseteq f^{-1}(rsint_{W_1,W_2}(B,r,s))$ (iii)$\Rightarrow$(iv) Let (iii) holds and let $A\in I^Y$ and $B\in I^X$ such that $T_1(\bar{1}-B)\geq r$ and $T_2(\bar{1}-B)\geq s$ and $B\supseteq f^{-1}(A)$. Since $1_\sim-B\subseteq 1_\sim-f^{-1}(A)=f^{-1}(1_\sim-A)$, it implies $I_{T_1,T_2}(1_\sim-B)=1_\sim-B\subseteq I_{T_1,T_2}(f^{-1}(1_\sim-A),r,s)$ or $1_\sim-B\subseteq f^{-1}(rsint_{W_1,W_2}(1_\sim-A,r,s))$. Then, $B\supseteq 1_\sim-f^{-1}(rsint_{W_1,W_2}(1_\sim-A,r,s))=f^{-1}(rscl_{W_1,W_2}(A,r,s))$. Take $C=rscl_{W_1,W_2}(A,r,s)$. Hence C is (r,s)-frsc and $C\supseteq A$ such that $B\supseteq f^{-1}(C)$.(iv)$\Rightarrow$(i) Let D be a fuzzy set such that $D\in I^X$ and $T_1(D)\geq r$ and $T_2(D)\geq s$. Put $B=1_\sim-D$ and $A=1_\sim-f(D)$. Then $B\supseteq f^{-1}(A)$. So there exists (r,s)-frsc set C such that $C\supseteq A$ and $B\supseteq f^{-1}(C)$. Then $1_\sim-D\supseteq f^{-1}(C)$ implies $D\subseteq f^{-1}(1_\sim-C)$. Thus $f(w)\subseteq 1_\sim-C$. Also, $A\subseteq C$, $f(D)=1_\sim-A\supseteq 1_\sim-C$. Hence $f(D)=1_\sim-C$ and therefore $f(D)$ is (r,s)-frso. So f is fuzzy (r,s)-regular semi open.

Theorem 4.3 Let (X,T_1,T_2) and (Y,W_1,W_2) be smooth topological space's. Let $f:X\to Y$ be a mapping. The following statements are equivalent:[(i)] f is fuzzy (r,s)-α open.

[(ii)] $f(int_{T_1,T_2}(A,r,s))\subseteq \alpha int_{W_1,W_2}(f(A),r,s)$, for each $A\in I^X$.[(iii)] $int_{T_1,T_2}(f^{-1}(B),r,s)\subseteq f^{-1}(\alpha int_{W_1,W_2}(B,r,s))$, for each $B\in I^Y$.[(iv)] For any $A\in I^Y$ and any $B\in I^X$ such that $T_1(1_\sim-B)\geq r$, $T_2(1_\sim-B)\geq s, B\supseteq f^{-1}(A)$, there exists fuzzy (r,s)-α open set $C\in I^Y$ with $A\subseteq C$ and $B\supseteq f^{-1}(C)$.

Theorem 4.4 Let (X,T_1,T_2) and (Y,W_1,W_2) be smooth topological space's. Let $f:X\to Y$ be a mapping. The following statements are equivalent: [(i)] f is fuzzy (r,s)-pre open.

[(ii)] $f(int_{T_1,T_2}(A,r,s))\subseteq pint_{W_1,W_2}(f(A),r,s)$, for each $A\in I^X$.[(iii)] $int_{T_1,T_2}(f^{-1}(B),r,s)\subseteq f^{-1}(pint_{W_1,W_2}(B,r,s))$, for each $B\in I^Y$.[(iv)] For any $A\in I^Y$ and any $B\in I^X$ such that $T_1(1_\sim-B)\geq r$, $T_2(1_\sim-B)\geq s, B\supseteq f^{-1}(A)$, there exists fuzzy (r,s)-pre open set $C\in I^Y$ with $A\subseteq C$ and $B\supseteq f^{-1}(C)$.

Theorem 4.5 Let (X,T_1,T_2) and (Y,W_1,W_2) be smooth topological space's. Let $f:X\to Y$ be a mapping. The following statements are equivalent:

[(i)] f is fuzzy (r,s)-β open. [(ii)] $f(int_{T_1,T_2}(A,r,s))\subseteq \beta int_{W_1,W_2}(f(A),r,s)$, for each $A\in I^X$.

[(iii)] $int_{T_1,T_2}(f^{-1}(B),r,s)\subseteq f^{-1}(\beta int_{W_1,W_2}(B,r,s))$, for each $B\in I^Y$.[(iv)] For any $A\in I^Y$ and any $B\in I^X$ such that $T_1(1_\sim-B)\geq r$, $T_2(1_\sim-B)\geq s, B\supseteq f^{-1}(A)$, there exists fuzzy (r,s)-β open set $C\in I^Y$ with $A\subseteq C$ and $B\supseteq f^{-1}(C)$.

Proof. Proof of theorem 4.3,4.4,4.5 it follows from Theorem 4.2

Theorem 4.6 Let (X,T_1,T_2) and (Y,W_1,W_2) be SoIFTS`s. Let $f:X\to Y$ be a mapping. The following statements are equivalent: [(i)] f is fuzzy (r,s)-regular semi closed.

[(ii)] $rscl_{W_1,W_2}(f(A),r,s))\subseteq f(rcl_{T_1,T_2}(A,r,s))$, for each $A\in I^X$.

Proof. (i)$\Rightarrow$(ii) Let (i) holds and let $A\in I^X$. Since $A\subseteq rcl_{T_1,T_2}(A,r,s)$, then $f(A)\subseteq f(rcl_{T_1,T_2}(A,r,s))$ and therefore, $rscl_{W_1,W_2}(f(A),r,s)\subseteq f(RC_{T_1,T_2}(A,r,s))$.

(ii)$\Rightarrow$(i) Let (ii) holds and let $A\in I^X$ be a (r,s)-frc set. Then $rscl_{W_1,W_2}(f(A),r,s)\subseteq f(rcl_{T_1,T_2}(A,r,s))$. It implies $rscl_{W_1,W_2}(f(A),r,s)\subseteq f(A)$. But $rscl_{W_1,W_2}(f(A),r,s)\supseteq f(A)$. Then, $f(A)=rscl_{W_1,W_2}(f(A),r,s)$ and it follows that $f(A)$ is (r,s)-frsc. Hence, f is fuzzy (r,s)-regular semi closed.

Theorem 4.7 Let (X,T_1,T_2) and (Y,W_1,W_2) be SoIFTS`s. Let $f:X\to Y$ be a mapping. The following statements are equivalent: [(i)] f is fuzzy (r,s)-α closed. [(ii)] $\alpha cl_{W_1,W_2}(f(A),r,s))\subseteq f(cl_{T_1,T_2}(A,r,s))$, for each $A\in I^X$.

Theorem 4.8 Let (X,T_1,T_2) and (Y,W_1,W_2) be SoIFTS`s. Let $f:X\to Y$ be a mapping. The following statements are equivalent: [(i)] f is fuzzy (r,s)-pre closed. [(ii)] $pcl_{W_1,W_2}(f(A),r,s))\subseteq f(cl_{T_1,T_2}(A,r,s))$, for each $A\in I^X$.

Theorem 4.9 Let (X,T_1,T_2) and (Y,W_1,W_2) be SoIFTS`s. Let $f:X\to Y$ be a mapping. The following statements are equivalent: [(i)] f is fuzzy (r,s)-β closed. [(ii)] $\beta cl_{W_1,W_2}(f(A),r,s))\subseteq f(cl_{T_1,T_2}(A,r,s))$, for each $A\in I^X$.

Proof. Proof of theorem 4.7,4.8,4.9 it follows from Theorem 4.6

References

[1] K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **20** (1986), 87-96.

[2] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl., **24** (1968), 182-190.

[3] K. C. Chattopadhyay, R. N. Harza and S. K. Samanta, *Gradation of openness: fuzzy topology*, Fuzzy Sets and Systems **49** (1992), 237-242.

[4] D. Coker and A. Haydar Es, *On fuzzy compactness in intuitionistic fuzzy topological spaces*, J. Fuzzy Math **3** (1995), 899-909.

[5] D. Coker, M. Demirci *An introduction to intuitionistic fuzzy topologica spaces in Sostak's sense*,BUSEFAL, **67** (1996), 67-76.

[6] D. Coker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems **88** (1997), 81-89.

[7] H. Gurcay, D. Coker and A. Haydar Es, *On fuzzy continuity in intuitionistic fuzzy topological spaces*, J. Fuzzy Math **5** (1997), 365-378.

[8] E.P. Lee and Y.B. Im, *Mated fuzzy topological spaces*, J. of Fuzzy Logic and Intelligent Systems **11** (2001), 161-165.

[9] E.P. Lee, *Semiopen sets on intuitionistic fuzzy topological spaces in* $\hat{S}$ *ostak's sense*, Journal of Korean institute of Intelligent Systems **11** (2) (2004), 234-238.

[10] A. A. Ramadan, *Smooth topological spaces*, Fuzzy Sets and Systems, **48** (1992), 371-375.

[11]G. Saravanakumar, S. Tamilselvan and A. Vadivel, *Regular Semi open sets on intuitionistic fuzzy topological spaces in* $\hat{S}$ *ostak`s sense*. (submitted)

[12]A. $\hat{S}$ ostak, *On a fuzzy topological structure*, Supp. Rend. Circ. Math. palermo (Ser.II), **11** (1985), 89-103.

[13]L. A. Zadeh, *Fuzzy sets*, Information and Control, **8** (1965), 338-353.