REGULAR SEMI CONTINUOUS AND OPEN MAPPINGS ON INTUITIONISTIC FUZZY TOPOLOGICAL SPACES IN \( \hat{S} \) OSTAK’S SENSE

G. Saravanakumar\(^1\)
Research Scholar, Department of Mathematics, Annamalai University

S. Tamilselvan\(^2\)
Mathematics Section (FEAT), Annamalai University

A. Vadivel\(^3\)
Department of Mathematics, Government Arts College (Autonomous), Karur - 5

ABSTRACT: We introduce the concepts of fuzzy \((r, s)\)-regular semi (resp. \((r, s)\)-\(\alpha\), \((r, s)\)-\(\beta\)) continuous mappings and fuzzy \((r, s)\)-regular semi (resp. \((r, s)\)-\(\alpha\), \((r, s)\)-\(\beta\)) open mappings on intuitionistic fuzzy topological spaces in the sense of \(\hat{S}\) ostak's. Further we investigate some of their characteristic and respective properties.

KEYWORDS AND PHRASES: fuzzy \((r, s)\)-regular semi (resp. \((r, s)\)-\(\alpha\), \((r, s)\)-\(\beta\)) continuous mappings and fuzzy \((r, s)\)-regular semi (resp. \((r, s)\)-\(\alpha\), \((r, s)\)-\(\beta\)) open mappings.


1. Introduction
The concept of fuzzy sets was introduced by Zadeh [12]. Chang [2] defined fuzzy topological spaces. These spaces and their generalizations are later studied by several authors one of which, developed by \(\hat{S}\) ostak [12], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay, Hazra and Samanta [3], and by Ramadan [10]. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Coker and his collegues [4,6,7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Coker and Demirci [5] defined

Intuitionistic fuzzy topological spaces in \(\hat{S}\) ostak’s sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces. In this paper, we introduce the concepts of fuzzy \((r, s)\)-regular semi (resp. \((r, s)\)-\(\alpha\), \((r, s)\)-\(\beta\)) continuous mappings and fuzzy \((r, s)\)-regular semi (resp. \((r, s)\)-\(\alpha\), \((r, s)\)-\(\beta\)) open mappings on intuitionistic fuzzy topological spaces in the sense of \(\hat{S}\) ostak's. Further we investigate some of their characteristic and respective properties.

2. Preliminaries
Let \(I\) be the unit interval \([0,1]\) of the real line. A member \(\mu\) of \(I^X\) is called a fuzzy set of \(X\). By \(\tilde{0}\) and \(\tilde{1}\) we denote constant maps on \(X\) with value 0 and 1, respectively. For any \(\mu \in I^X\), \(\mu^c\) denotes the complement of \(\tilde{1} - \mu\). All other notations are standard notations of fuzzy set theory. Let \(X\) be a nonempty set. An intuitionistic fuzzy set \(A\) is an ordered pair \(A = (\mu_{A}, \gamma_{A})\) where the functions \(\mu_{A} : X \to I\) and \(\gamma_{A} : X \to I\) denote the degree of membership and degree of non-membership, respectively, and \(\mu_{A} + \gamma_{A} \leq \tilde{1}\). Obviously every fuzzy set \(\mu\) on \(X\) is an intuitionistic fuzzy set of the form \((\mu, \tilde{1} - \mu)\).

1. Let \(A = (\mu_{A}, \gamma_{A})\) and \(B = (\mu_{B}, \gamma_{B})\) be intuitionistic fuzzy sets on \(X\). Then when (\(i\)) \(A \subseteq B\) iff \(\mu_{A} \leq \mu_{B}\) and \(\gamma_{A} \geq \gamma_{B}\), (\(ii\)) \(A = B\) iff \(A \subseteq B\) and \(B \subseteq A\), (\(iii\)) \(A^{c} = (\gamma_{A}, \mu_{A})\), (\(iv\)) \(A \cap B = (\mu_{A} \wedge \mu_{B}, \gamma_{A} \vee \gamma_{B})\), (\(v\)) \(A \cup B = (\mu_{A} \vee \mu_{B}, \gamma_{A} \wedge \gamma_{B})\). \(\{0, 1\} = (\tilde{0}, \tilde{1})\) and \(\{1, 0\} = (\tilde{1}, \tilde{0})\). Let \(f\) be a map from a set \(X\) to a set \(Y\). Let \(A = (\mu_{A}, \gamma_{A})\) be a intuitionistic fuzzy set of \(X\) and \(B = (\mu_{B}, \gamma_{B})\) an intuitionistic fuzzy set of \(Y\). Then when \(f\) is an intuitionistic fuzzy set in \(Y\) defined by \(f(A) = (f(\mu_{A}), \tilde{1} - f(1 - \gamma_{A}))\). The inverse image of \(B\) under \(f\), denoted by \(f^{-1}(B)\) is an intuitionistic fuzzy set in \(X\) defined by \(f^{-1}(B) = (f^{-1}(\mu_{B}), f^{-1}(\gamma_{B}))\). \([10]\)
A smooth fuzzy topology on $X$ is a map $T: I^X \to I$ which satisfies the following properties:

\[ (i) \quad T(\emptyset) = T(\bar{1}) = 1, \]

\[ (iii) \quad T(\mu_1 \wedge \mu_2) \supseteq T(\mu_1) \wedge T(\mu_2), \]

\[ (iv) \quad T(\mu_1 \vee \mu_2) \supseteq \wedge(T(\mu_1), T(\mu_2)). \]

The pair $(X, T)$ is called a smooth fuzzy topological space. [5] An intuitionistic fuzzy topology on $X$ is a family $T$ of intuitionistic fuzzy sets in $X$ which satisfies the following properties:

\[ (i) \quad 0, 1 \in T, \]

\[ (ii) \quad \text{If } A_1, A_2 \in T, \text{ then } A_1 \cap A_2 \in T, \]

\[ (iii) \quad \text{If } A \in T \text{ for all } i, \text{ then } \bigcup A_i \in T. \]

The pair $(X, T)$ is called an intuitionistic fuzzy topological space. Let $I(X)$ be a family of all intuitionistic fuzzy sets of $X$ and let $I \otimes I$ be the set of the pair $(r, s)$ such that $r, s \in I$ and $r + s \leq 1$. [6] Let $X$ be a nonempty set. An intuitionistic fuzzy topology in $\hat{S}$ ostak's sense (SoIFT for short) $T = (T_1, T_2)$ on $X$ is a map $T : I(X) \to I \otimes I$ which satisfies the following properties:

\[ (i) \quad T_1(0) = T_1(1) = 1 \quad \text{and} \quad T_2(0) = T_2(1) = 1, \]

\[ (ii) \quad T_1(A \cap B) \supseteq T_1(A) \wedge T_1(B) \quad \text{and} \quad T_2(A \cap B) \supseteq T_2(A) \vee T_2(B). \]

Then $T_1(\cup A) \supseteq \vee T_2(A)$ and $T_2(\cup A) \supseteq \wedge T_2(A)$. The $(X, T)$ is said to be an intuitionistic fuzzy topological space in $\hat{S}$ ostak's sense (SoIFTS for short). Also, we call $T_1(A)$ a gradation of openness of $A$ and $T_2(A)$ a gradation of nonopenness of $A$. [8] Let $A$ be an intuitionistic fuzzy set in a SoIFTS $(X, T_1, T_2)$ and $(r, s) \in I \otimes I$. Then $A$ is said to be

\[ (i) \quad \text{fuzzy } (r, s) - \text{open if } T_1(A) \supseteq r \quad \text{and} \quad T_2(A) \subseteq s, \]

\[ (ii) \quad \text{fuzzy } (r, s) - \text{closed if } T_1(A^c) \supseteq r \quad \text{and} \quad T_2(A^c) \subseteq s. \]

Let $(X, T_1, T_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the fuzzy $(r, s)$-interior is defined by

\[ \text{int}(r, s)(A) = \bigcup \{B \in I(X) \mid A \supseteq B, \text{is fuzzy } (r, s) - \text{open}\}. \]

and the fuzzy $(r, s)$-closure is defined by

\[ \text{cl}(r, s)(A) = \bigcap \{B \in I(X) \mid A \subseteq B, \text{is fuzzy } (r, s) - \text{closed}\}. \]

The operators $\text{int}(r, s) : I(X) \times I \otimes I \to I(X)$ and $\text{cl}(r, s) : I(X) \times I \otimes I \to I(X)$ are called the fuzzy interior operator and fuzzy closure operator in $(X, T_1, T_2)$, respectively. [8] For an intuitionistic fuzzy set $A$ in a SoIFTS $(X, T_1, T_2)$ and $(r, s) \in I \otimes I$, we have

\[ \text{int}(r, s)(A) = \bigcup \{B \in I(X) \mid A \supseteq B, \text{is fuzzy } (r, s) - \text{open}\}. \]

\[ \text{cl}(r, s)(A) = \bigcap \{B \in I(X) \mid A \subseteq B, \text{is fuzzy } (r, s) - \text{closed}\}. \]

Let $(X, T)$ be an intuitionistic fuzzy topological space in $\hat{S}$ ostak's sense. Then it is easy to see that for each $(r, s) \in I \otimes I$, the family $T_{(r, s)}$ defined by

\[ T_{(r, s)} = \{A \in I(X) \mid T_1(A) \supseteq r \text{ and } T_2(A) \subseteq s\} \]

is an intuitionistic fuzzy topology on $X$. [9] Let $A$ be an intuitionistic fuzzy set in a SoIFTS $(X, T_1, T_2)$ and $(r, s) \in I \otimes I$. Then $A$ is said to be

\[ (i) \quad \text{fuzzy } (r, s) - \text{semi open if there is a fuzzy } (r, s) - \text{open set } B \text{ in } X \text{ such that } B \subseteq A \subseteq \text{cl}_{(r, s)}(B, r, s), \]

\[ (ii) \quad \text{fuzzy } (r, s) - \text{semi closed if there is a fuzzy } (r, s) - \text{regular } B \text{ in } X \text{ such that } \text{int}_{(r, s)}(B, r, s) \subseteq A \subseteq B. \]

Let $A$ be an intuitionistic fuzzy set in a SoIFTS $(X, T_1, T_2)$ and $(r, s) \in I \otimes I$. Then $A$ is said to be

\[ (i) \quad \text{fuzzy } (r, s) - \text{regular open if } A = \text{int}_{(r, s)}(A, r, s), \]

\[ (ii) \quad \text{fuzzy } (r, s) - \text{regular closed if } A = \text{cl}_{(r, s)}(A, r, s), \]

\[ (iii) \quad \text{fuzzy } (r, s) - \alpha \text{ open if } A \subseteq \text{int}_{(r, s)}(A, r, s), \]

\[ (iv) \quad \text{fuzzy } (r, s) - \alpha \text{ closed if } A \supseteq \text{cl}_{(r, s)}(A, r, s), \]

\[ (v) \quad \text{fuzzy } (r, s) - \beta \text{ open if } A \supseteq \text{cl}_{(r, s)}(A, r, s), \]

\[ (vi) \quad \text{fuzzy } (r, s) - \beta \text{ closed if } A \supseteq \text{int}_{(r, s)}(A, r, s). \]

Let $(X, T_1, T_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the fuzzy $(r, s)$-regular semi (resp. $(r, s) - \alpha$, $(r, s) - \beta$) - interior is defined by

\[ \text{rsint}_{(r, s)}(A, r, s) = \bigcup \{B \in I(X) \mid A \supseteq B, \text{is fuzzy } (r, s) - \text{regular semi } (\text{resp. } (r, s) - \alpha, (r, s) - \beta) - \text{open}\}. \]
and the fuzzy \((r, s)\)-regular semi (resp. \((r, s)\)-pre and \((r, s)\)-\(\beta\) ) - closure is defined by
\[
\text{rscl}_{\alpha, \beta}(A, r, s) = \text{pre cl}_{\alpha, \beta}(A, r, s) \cup \text{cl}_{\alpha, \beta}(A, r, s) = \bigcap \{B \in I(X) \mid A \subseteq B, B \text{ is fuzzy } (r, s)\text{-regular semi}(\alpha, \text{pre and } \beta)\text{-closed}\}
\]

3. **Fuzzy \((r, s)\)-regular semi (resp. \(\alpha\), pre and \(\beta\) ) continuous mappings**

**Definition 3.1** Let \(f : (X, T_1, T_2) \rightarrow (Y, W_1, W_2)\) be a mapping from a SoIFTSet \(X\) to another SoIFTSet \(Y\) and \((r, s) \in I \otimes I\). Then \(f\) is said to be fuzzy \((r, s)\)-regular semi (resp. regular, \(\alpha\), pre and \(\beta\) ) continuous if \(f^{-1}(B)\) is a fuzzy \((r, s)\)-regular semi (resp. regular, \(\alpha\), pre and \(\beta\) ) open set of \(X\) for each fuzzy \((r, s)\)-open set \(B\) of \(Y\).

**Example 3.1** Let \(X = \{a, b\} = Y\), and let \(A_1, A_2 \in I(X), B_1 \in I(Y)\) defined as
\[
A_1(x) = B_1(x) = (0.6, 0.3), A_1(y) = B_1(y) = (0.5, 0.3), A_2(x) = (0.8, 0.1), A_2(y) = (0.5, 0.1)
\]

Define \(T : I(X) \rightarrow I \otimes I\) and \(W : I(Y) \rightarrow I \otimes I\) by \(T(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0_1 \cdot 1_1, \\ (0, 1) & \text{otherwise} \end{cases}\)

\(W(B) = (W_1(B), W_2(B)) = \begin{cases} (1, 0) & \text{if } B = 0_1 \cdot 1_1, \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } B = B_1, \\ (0, 1) & \text{otherwise} \end{cases}\)

For \(r = 1/2, s = 1/2\). Then the identity mapping \(f : (X, T_1, T_2) \rightarrow (Y, W_1, W_2)\) is fuzzy \((1/2, 1/2)\)-continuous which is not fuzzy \((1/2, 1/2)\)-regular continuous. Since \(B_1\) is fuzzy \((1/2, 1/2)\)-open in \(W\) but \(f^{-1}(B_1)\) is not fuzzy \((1/2, 1/2)\)-regular open in \(T\).

**Example 3.2** Let \(X = \{a, b\} = Y\), and let \(A_1, A_2 \in I(X), B_1 \in I(Y)\) defined as
\[
A_1(x) = (0.6, 0.3), A_1(y) = (0.5, 0.3), A_2(x) = (0.8, 0.1), A_2(y) = (0.5, 0.1)
\]

Define \(T : I(X) \rightarrow I \otimes I\) and \(W : I(Y) \rightarrow I \otimes I\) by \(T(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0_1 \cdot 1_1, \\ (0, 1) & \text{otherwise} \end{cases}\)

\(W(B) = (W_1(B), W_2(B)) = \begin{cases} (1, 0) & \text{if } B = 0_1 \cdot 1_1, \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } B = B_1, \\ (0, 1) & \text{otherwise} \end{cases}\)

For \(r = 1/2, s = 1/2\). Then the identity mapping \(f : (X, T_1, T_2) \rightarrow (Y, W_1, W_2)\) is fuzzy \((1/2, 1/2)\)-semi continuous which is not fuzzy \((1/2, 1/2)\)-regular semi continuous. Since \(B_1\) is fuzzy \((1/2, 1/2)\)-open in \(W\) but \(f^{-1}(B_1)\) is not fuzzy \((1/2, 1/2)\)-regular semi open in \(T\).

**Example 3.3** Let \(X = \{a, b\} = Y\), and let \(A_1 \in I(X), B_1, B_2 \in I(Y)\) defined as
\[
A_1(x) = B_1(x) = (0.3, 0.6), A_1(y) = B_1(y) = (0.3, 0.5), B_2(x) = (0.4, 0.3), B_2(y) = (0.4, 0.3)
\]
Define $T : I(X) \to I \otimes I$ and $W : I(Y) \to I \otimes I$ by $T(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0, \quad 1, \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } A = A_1 \\ (0, 1) & \text{otherwise} \end{cases}$

$W(B) = (W_1(B), W_2(B)) = \begin{cases} (1, 0) & \text{if } B = 0, \quad 1, \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } B = B_1, B_2 \\ (0, 1) & \text{otherwise} \end{cases}$

For $r = 1/2$, $s = 1/2$. Then the identity mapping $f : (X, T_1, T_2) \to (Y, W_1, W_2)$ is fuzzy $(1/2, 1/2)$ - semi continuous, regular semi continuous which is not fuzzy $(1/2, 1/2)$-regular continuous and $\alpha$ -continuous. Since $B_2$ is fuzzy $(1/2, 1/2)$ - open in $W$ but $f^{-1}(B_2)$ is not fuzzy $(1/2, 1/2)$-open and $\alpha$ -open in $T$.

Example 3.4 Let $X = \{a, b\} = Y$, and let $A_t \in I(X)$, $B_1, B_2, B_1 \cup B_2, B_1 \cap B_2 \in I(Y)$ defined as

$A_t(x) = B_t(x) = (0.3, 0.1), A_t(y) = B_t(y) = (0.3, 0.1), B_1(x) = (0.4, 0.3), B_1(y) = (0.4, 0.3)$

$B_2(x) \cup B_2(x) = (0.4, 0.1), B_2(y) \cup B_2(y) = (0.4, 0.1), B_2(x) \cap B_2(x) = (0.3, 0.3), B_2(y) \cap B_2(y) = (0.3, 0.3)$

Define $T : I(X) \to I \otimes I$ and $W : I(Y) \to I \otimes I$ by

$T(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0, \quad 1, \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } A = A_1 \\ (0, 1) & \text{otherwise} \end{cases}$

$W(B) = (W_1(B), W_2(B)) = \begin{cases} (1, 0) & \text{if } B = 0, \quad 1, \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } B = B_1, B_2 \\ (0, 1) & \text{otherwise} \end{cases}$

For $r = 1/2$, $s = 1/2$. Then the identity mapping $f : (X, T_1, T_2) \to (Y, W_1, W_2)$ is fuzzy $(1/2, 1/2)$ - $\alpha$ -continuous which is not fuzzy $(1/2, 1/2)$ - continuous. Since $B_2$ is fuzzy $(1/2, 1/2)$ - open in $W$ but $f^{-1}(B_2)$ is not fuzzy $(1/2, 1/2)$-open in $T$.

Example 3.5 Let $X = \{a, b\} = Y$, and let $A_t \in I(X)$, $B_1, B_2, B_1 \cup B_2, B_1 \cap B_2 \in I(Y)$ defined as

$A_t(x) = B_t(x) = (0.3, 0.6), A_t(y) = B_t(y) = (0.3, 0.5), B_1(x) = (0.1, 0.2), B_1(y) = (0.1, 0.2)$

$B_2(x) \cup B_2(x) = (0.3, 0.2), B_2(y) \cup B_2(y) = (0.3, 0.2), B_2(x) \cap B_2(x) = (0.1, 0.6), B_2(y) \cap B_2(y) = (0.1, 0.5)$

Define $T : I(X) \to I \otimes I$ and $W : I(Y) \to I \otimes I$ by $T(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0, \quad 1, \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } A = A_1 \\ (0, 1) & \text{otherwise} \end{cases}$

$W(B) = (W_1(B), W_2(B)) = \begin{cases} (1, 0) & \text{if } B = 0, \quad 1, \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } B = B_1, B_2 \\ (0, 1) & \text{otherwise} \end{cases}$

For $r = 1/2$, $s = 1/2$. Then the identity mapping $f : (X, T_1, T_2) \to (Y, W_1, W_2)$ is fuzzy $(1/2, 1/2)$ - $\beta$ continuous which is not fuzzy $(1/2, 1/2)$ - semi continuous. Since $B_2$ is fuzzy $(1/2, 1/2)$ - open in $W$ but $f^{-1}(B_2)$ is not fuzzy $(1/2, 1/2)$-semi open in $T$.

Example 3.6 Let $X = \{a, b\} = Y$, and let $A_t \in I(X)$, $B_1, B_2, B_1 \cup B_2, B_1 \cap B_2 \in I(Y)$ defined as

$A_t(x) = B_t(x) = (0.3, 0.6), A_t(y) = B_t(y) = (0.3, 0.5), B_1(x) = (0.1, 0.3), B_1(y) = (0.1, 0.3)$

$B_2(x) \cup B_2(x) = (0.3, 0.3), B_2(y) \cup B_2(y) = (0.3, 0.3), B_2(x) \cap B_2(x) = (0.1, 0.6), B_2(y) \cap B_2(y) = (0.1, 0.5)$

Define $T : I(X) \to I \otimes I$ and $W : I(Y) \to I \otimes I$ by $T(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0, \quad 1, \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } A = A_1 \\ (0, 1) & \text{otherwise} \end{cases}$

$W(B) = (W_1(B), W_2(B)) = \begin{cases} (1, 0) & \text{if } B = 0, \quad 1, \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } B = B_1, B_2 \\ (0, 1) & \text{otherwise} \end{cases}$
mapping $f : (X, T_1, T_2) \to (Y, W_1, W_2)$ is fuzzy $(1/2, 1/2) \cdot \beta$ continuous which is not fuzzy $(1/2, 1/2)$-pre continuous. Since $B_2$ is fuzzy $(1/2, 1/2)$-open in $W$ but $f^{-1}(B_2)$ is not fuzzy $(1/2, 1/2)$-pre open in $T$.

Example 3.7 Let $X = \{a, b\} = Y$, and let $A_1 \in I(X), B_1, B_2, B_3 \cup B_4, B_1 \cap B_2 \in I(Y)$ defined as

$$A_1(x) = B_1(x) = (0.1, 0.3), A_1(y) = B_1(y) = (0.1, 0.6), B_2(x) \cap B_2(y) = (0.1, 0.5)$

$B_2(x) = (0.3, 0.6), B_2(y) = (0.3, 0.5)$

$(1, 0) = A_1 \cup A_2, B_1 \cup B_2, B_1 \cap B_2$ For $r = 1/2, s = 1/2$. Then the identity mapping $f : (X, T_1, T_2) \to (Y, W_1, W_2)$ is fuzzy $(1/2, 1/2)$-pre continuous which is not fuzzy $(1/2, 1/2) \cdot \alpha$ continuous. Since $B_2$ is fuzzy $(1/2, 1/2)$-open in $W$ but $f^{-1}(B_2)$ is not fuzzy $(1/2, 1/2)$-pre open in $T$.

It need not be true that $f$ and $g$ are fuzzy $(r, s)$-regular semi (resp. regular, $\alpha$, pre and $\beta$) continuous mappings then so is $g \circ f$. But we have the following theorem.

Theorem 3.1 Let $(X, T_1, T_2), (Y, W_1, W_2)$ and $(Z, V_1, V_2)$ be SoFTSs and let $f : X \to Y$ and $g : Y \to Z$ be mappings and $(r, s) \in I \otimes I$. If $f$ is a fuzzy $(r, s)$-regular semi (resp. regular, $\alpha$, pre and $\beta$) continuous mapping and $g$ is a fuzzy $(r, s)$-continuous mapping, then $g \circ f$ is a fuzzy $(r, s)$-regular semi (resp. regular, $\alpha$, pre and $\beta$) continuous mapping.

Proof. Straightforward.

Theorem 3.2 Let $(X, T_1, T_2)$ and $(Y, W_1, W_2)$ be SoFTSs. Let $f : X \to Y$ be a mapping. The following statements are equivalent:

$$[(i) f \text{ is } \ (r, s) \text{-fuzzy regular semi continuous}, [(ii) f(rcl_{t_1, t_2}((A, r, s))) = rcl_{w_1, w_2}(f(A), r, s)), \text{ for each } A \in I^X.] [(iii) rscl_{t_1, t_2}(f^{-1}(B), r, s) \subseteq f^{-1}(rcl_{w_1, w_2}(B, r, s)) \text{, for each } B \in I^Y.] [(iv) f^{-1}(rscl_{w_1, w_2}(B, r, s)) \subseteq rscl_{t_1, t_2}(f^{-1}(B), r, s), \text{ for each } B \in I^V.]$$

Proof. (i) $\Rightarrow$ (ii) Let (i) holds and let $A \in I^X$. Suppose that there exists $A \in I^X$ and $(r, s) \in I \otimes I$ such that $f(rcl_{t_1, t_2}((A, r, s))) \subseteq rcl_{w_1, w_2}(f(A), r, s))$. So there exists $y \in Y, t \in (0, 1]$ such that $f(rcl_{t_1, t_2}((A, r, s)))(y) > t > rcl_{w_1, w_2}(f(A), r, s)(y)$. If $f^{-1}(y) = \emptyset$, then, $f(rcl_{t_1, t_2}((A, r, s)))(y) = 0$. So, there exists $x \in f^{-1}(y)$ such that: $f(rcl_{t_1, t_2}((A, r, s)))(y) = rcl_{t_1, t_2}((A, r, s))(x) > t > RC_{w_1, w_2}(f(A), r, s)(y)$

Since $rcl_{w_1, w_2}(f(A), r, s)(y) < t$, there exists $(r, s)$-fr set $B$ such that $f(A) = B$. Then, $rcl_{w_1, w_2}(f(A), r, s)(y) \subseteq B$ and this implies $rcl_{w_1, w_2}(f(A), r, s)(f(x)) \subseteq B(f(x)) < t$. Moreover, $f(A) \subseteq B$ implies $A \subseteq f^{-1}(B)$. Since $f^{-1}(\{1, 0\} \cap B) = (r, s)$-frso, then from (i), $f^{-1}(\{1, 0\} \cap B) \subseteq rscl_{t_1, t_2}((1, 0, r, s))$ or $1_r-rc_{t_1, t_2}((1, 0, r, s)) \subseteq 1_r-f^{-1}(\{1, 0\} \cap B)$.

Then, $rscl_{t_1, t_2}((A, r, s)) \subseteq f^{-1}(B)$. This gives $rscl_{t_1, t_2}((A, r, s))(x) \subseteq f^{-1}(B)(x) = B(f(x)) < t$.

But the last relation contradicts the relation (3.1) above. Hence, the result in (ii) is true.

(ii) $\Rightarrow$ (iii) Take $A = f^{-1}(B)(B \in I^V)$ and apply (ii) we have the required result.

(iii) $\Rightarrow$ (iv) Taking the complement of (iii), we have the result.(iv) $\Rightarrow$ (i) Let (iv) holds and let $B \in I^V$ be a $(r, s)$-fr set, since every $(r, s)$-fr set is $(r, s)$-fuzzy open. So $W_1(B) > r, and W_2(B) < s$. By (iv)
\( f^{-1}(\text{rint}_{W,B}(B,r,s)) \subseteq \text{rsint}_{T,T}(f^{-1}(B),r,s) \) or \( f^{-1}(B) \subseteq \text{rsint}_{T,T}(f^{-1}(B),r,s) \) (since \( W(B) \geq r, W(B) \leq s \)). But \( \text{rsint}_{T,T}(f^{-1}(B),r,s) \subseteq f^{-1}(B) \). Then, \( f^{-1}(B) = \text{rsint}_{T,T}(f^{-1}(B),r,s) \) and this means that \( f^{-1}(B) \) is \( (r,s) \)-fsso. Hence, \( f \) is \( (r,s) \) fuzzy regular semi continuous.

**Theorem 3.3** Let \( (X,T_1,T_2) \) and \( (Y,W_1,W_2) \) be SoIFTSs. Let \( f : X \rightarrow Y \) be a mapping. The following statements are equivalent: \( [\text{(i)}] \) \( f \) is fuzzy \((r,s)-\alpha \) continuous, \( [\text{(ii)}] \) \( f(\text{accl}_{T,T}(A,r,s)) \subseteq \text{cl}_{W,W}(f(A),r,s) \), for each \( A \in I^X \). \( [\text{(iii)}] \) \( \alpha \text{cl}_{T,T}(f^{-1}(B),r,s) \subseteq f^{-1}(\text{cl}_{W,W}(B,r,s)) \), for each \( B \in I^Y \). \( [\text{(iv)}] \) \( f^{-1}(\text{int}_{W,W}(B,r,s)) \subseteq \text{aint}_{T,T}(f^{-1}(B),r,s) \), for each \( B \in I^Y \).

**Theorem 3.4** Let \( (X,T_1,T_2) \) and \( (Y,W_1,W_2) \) be SoIFTSs. Let \( f : X \rightarrow Y \) be a mapping. The following statements are equivalent: \( [\text{(i)}] \) \( f \) is fuzzy \((r,s)-\alpha \) pre continuous, \( [\text{(ii)}] \) \( f(\text{pcl}_{T,T}(A,r,s)) \subseteq \text{cl}_{W,W}(f(A),r,s) \), for each \( A \in I^X \). \( [\text{(iii)}] \) \( \alpha \text{cl}_{T,T}(f^{-1}(B),r,s) \subseteq f^{-1}(\text{cl}_{W,W}(B,r,s)) \), for each \( B \in I^Y \). \( [\text{(iv)}] \) \( f^{-1}(\text{int}_{W,W}(B,r,s)) \subseteq \text{aint}_{T,T}(f^{-1}(B),r,s) \), for each \( B \in I^Y \).

**Theorem 3.5** Let \( (X,T_1,T_2) \) and \( (Y,W_1,W_2) \) be SoIFTSs. Let \( f : X \rightarrow Y \) be a mapping. The following statements are equivalent: \( [\text{(i)}] \) \( f \) is fuzzy \((r,s)-\beta \) continuous. \( [\text{(ii)}] \) \( f(\text{cl}_{T,T}(A,r,s)) \subseteq \text{cl}_{W,W}(f(A),r,s) \), for each \( A \in I^X \). \( [\text{(iii)}] \) \( \beta \text{cl}_{T,T}(f^{-1}(B),r,s) \subseteq f^{-1}(\text{cl}_{W,W}(B,r,s)) \), for each \( B \in I^Y \). \( [\text{(iv)}] \) \( f^{-1}(\text{int}_{W,W}(B,r,s)) \subseteq \text{aint}_{T,T}(f^{-1}(B),r,s) \), for each \( B \in I^Y \).

**Proof.** Proof of Theorem 3.3, 3.4, 3.5 it follows from Theorem 3.2

4. **Fuzzy \((r,s)\)-regular semi (resp. \(\alpha \), pre, \(\beta \) ) open mappings**

**Definition 4.1** Let \( f : (X,T_1,T_2) \rightarrow (Y,W_1,W_2) \) be a mapping from a SoIFTS \( X \) to another SoIFTS \( Y \) and \((r,s) \in I \otimes I \). Then \( f \) is said to be \( \alpha \)-open if \( f \) is fuzzy \((r,s)-\alpha \) open (resp. \(\alpha \), pre and \(\beta \) ) open if \( f \) is fuzzy \((r,s)-\beta \) open. \( f \) is fuzzy \((r,s)-\alpha \) open if \( f(A) \) is a fuzzy \((r,s)-\alpha \) regular semi (resp. \(\alpha \), pre and \(\beta \) ) open set of \( Y \) for each fuzzy \((r,s)-\alpha \) open set \( A \) of \( X \).

From the above definitions it is clear that the above implications are true for \((r,s) \in I \otimes I \)

The converses of the above implications are not true as the following examples show:

**Example 4.1** Let \( X = [a,b] = Y \). Define \( A_i \in I(X) \), \( B_i \in I(Y) \) as follows
\[
A_1(x) = B_1(x) = (0.6,0.3), A_1(y) = B_1(y) = (0.5,0.3), B_2(x) = (0.8,0.1), B_2(y) = (0.5,0.1)
\]

Define \( T : I(X) \rightarrow I \otimes I \) and \( W : I(Y) \rightarrow I \otimes I \) by \( T(A) = (T_1(A),T_2(A)) = \begin{cases} (1,0) & \text{if } A = A_1, \\ \left( \frac{1}{2}, \frac{1}{2} \right) & \text{if } A = A_1 \\ (0,1) & \text{otherwise} \end{cases} \)

\[
W(B) = (W_1(A),W_2(A)) = \begin{cases} (1,0) & \text{if } B = B_1, B_2 \\ \left( \frac{1}{2}, \frac{1}{2} \right) & \text{if } B = B_1, B_2 \text{ for } r = 1/2, s = 1/2. \text{ Then the identity mapping} \\ (0,1) & \text{otherwise} 
\end{cases}
\]

\( f : (X,T_1,T_2) \rightarrow (Y,W_1,W_2) \) is fuzzy \((1/2,1/2)\)-open mapping which is not fuzzy \((1/2,1/2)\)-regular open mapping. Since \( A_i \) is fuzzy \((1/2,1/2)\) - open in \( T \) but \( f(A_i) \) is not fuzzy \((1/2,1/2)\)-regular open in \( W \).

**Example 4.2** Let \( X = [a,b] = Y \). Define \( A_i \in I(X) \), \( B_i \in I(X) \) as follows

\[
A_1(x) = B_1(x) = (0.6,0.3), A_1(y) = B_1(y) = (0.5,0.3), B_2(x) = (0.8,0.1), B_2(y) = (0.5,0.1)
\]

Define \( T : I(X) \rightarrow I \otimes I \) and \( W : I(Y) \rightarrow I \otimes I \) by \( T(A) = (T_1(A),T_2(A)) = \begin{cases} (1,0) & \text{if } A = A_1, \\ \left( \frac{1}{2}, \frac{1}{2} \right) & \text{if } A = A_1 \\ (0,1) & \text{otherwise} \end{cases} \)

\[
W(B) = (W_1(A),W_2(A)) = \begin{cases} (1,0) & \text{if } B = B_1, B_2 \\ \left( \frac{1}{2}, \frac{1}{2} \right) & \text{if } B = B_1, B_2 \text{ for } r = 1/2, s = 1/2. \text{ Then the identity mapping} \\ (0,1) & \text{otherwise} 
\end{cases}
\]

\( f : (X,T_1,T_2) \rightarrow (Y,W_1,W_2) \) is fuzzy \((1/2,1/2)\)-open mapping which is not fuzzy \((1/2,1/2)\)-regular open mapping.
Let $X = \{a, b\} = Y$. Define $A_1, A_2 \in I(X), B_1 \in I(Y)$ as follows

\[ A_1(x) = B_1(x) = (0.3, 0.1), A_2(y) = B_2(y) = (0.3, 0.5), A_2(x) = (0.4, 0.3), A_2(y) = (0.4, 0.3) \]

Define $T : I(X) \rightarrow I \otimes I$ and $W : I(Y) \rightarrow I \otimes I$ by $T(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0_{12}, \\ \left(\frac{1}{2}, 1\right) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise} \end{cases}$

\[ W(B) = (W_1(B), W_2(B)) = \begin{cases} (1, 0) & \text{if } B = 0_{12}, \\ \left(\frac{1}{2}, 1\right) & \text{if } B = B_1, \\ (0, 1) & \text{otherwise} \end{cases} \]

For $r = 1/2, s = 1/2$. Then the identity mapping $f : (X, T_1, T_2) \rightarrow (Y, W_1, W_2)$ is fuzzy $(1/2, 1/2)$-open semi open mapping which is not fuzzy $(1/2, 1/2)$-regular semi open mapping. Since $A_1$ is fuzzy $(1/2, 1/2)$-open in $T$ but $f(A_1)$ is not fuzzy $(1/2, 1/2)$-regular semi open in $W$.

**Example 4.4** Let $X = \{a, b\} = Y$. Define $A_1, A_2, A_3, A_4 \in I(X), B_1 \in I(Y)$ as follows

\[ A_1(x) = B_1(x) = (0.3, 0.1), A_2(y) = B_2(y) = (0.3, 0.5), A_3(x) = (0.2, 0.1), A_4(y) = (0.2, 0.1) \]

Define $T : I(X) \rightarrow I \otimes I$ and $W : I(Y) \rightarrow I \otimes I$ by $T(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0_{12}, \\ \left(\frac{1}{2}, 1\right) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise} \end{cases}$

\[ W(B) = (W_1(B), W_2(B)) = \begin{cases} (1, 0) & \text{if } B = 0_{12}, \\ \left(\frac{1}{2}, 1\right) & \text{if } B = B_1, \\ (0, 1) & \text{otherwise} \end{cases} \]

For $r = 1/2, s = 1/2$. Then the identity mapping $f : (X, T_1, T_2) \rightarrow (Y, W_1, W_2)$ is fuzzy $(1/2, 1/2)$-open semi open mapping which is not fuzzy $(1/2, 1/2)$-open mapping. Since $A_2$ is fuzzy $(1/2, 1/2)$-open in $T$ but $f(A_2)$ is not fuzzy $(1/2, 1/2)$-open in $W$.

**Example 4.5** Let $X = \{a, b\} = Y$. Define $A_1, A_2, A_3, A_4 \in I(X), B_1 \in I(Y)$ as follows

\[ A_1(x) = B_1(x) = (0.3, 0.6), A_2(y) = B_2(y) = (0.3, 0.5), A_3(x) = (0.1, 0.6), A_4(y) = (0.1, 0.5) \]

Define $T : I(X) \rightarrow I \otimes I$ and $W : I(Y) \rightarrow I \otimes I$ by $T(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0_{12}, \\ \left(\frac{1}{2}, 1\right) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise} \end{cases}$

\[ W(B) = (W_1(B), W_2(B)) = \begin{cases} (1, 0) & \text{if } B = 0_{12}, \\ \left(\frac{1}{2}, 1\right) & \text{if } B = B_1, \\ (0, 1) & \text{otherwise} \end{cases} \]

For $r = 1/2, s = 1/2$. Then the identity mapping $f : (X, T_1, T_2) \rightarrow (Y, W_1, W_2)$ is fuzzy $(1/2, 1/2)$-open semi open mapping which is not fuzzy $(1/2, 1/2)$-open mapping. Since $A_3$ is fuzzy $(1/2, 1/2)$-open in $T$ but $f(A_3)$ is not fuzzy $(1/2, 1/2)$-open in $W$.
Example 4.6 Let $X = \{a, b\} = Y$. Define $A_1, A_2, A_1 \cup A_2, A_1 \cap A_2 \in I(X), B_i \in I(Y)$ as follows:

$A_1(x) = B_1(x) = (0.3, 0.6), A_1(y) = B_1(y) = (0.3, 0.5), A_2(x) \cap A_2(x) = (0.1, 0.6), A_1(y) \cap A_2(y) = (0.1, 0.5)$

$A_2(x) = (0.1, 0.3), A_2(y) = (0.1, 0.3) \quad A_1(x) \cup A_2(x) = (0.3, 0.3), A_1(y) \cup A_2(y) = (0.3, 0.3)$

Define $T : I(X) \rightarrow I \otimes I$ and $W : I(Y) \rightarrow I \otimes I$.

by $T(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0, 1, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_1, A_2, A_1 \cup A_2, A_1 \cap A_2 \\ (0, 1) & \text{otherwise} \end{cases}$

$W(B) = (W_1(B), W_2(B)) = \begin{cases} (1, 0) & \text{if } B = 0, 1, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } B = B_1 \\ (0, 1) & \text{otherwise} \end{cases}$

For $r = 1/2, s = 1/2$. Then the identity mapping $f : (X, T_1, T_2) \rightarrow (Y, W_1, W_2)$ is fuzzy $(1/2, 1/2)$-$\alpha$ open mapping and which is not fuzzy $(1/2, 1/2)$-$\alpha$ open mapping. Since $A_2$ is fuzzy $(1/2, 1/2)$-open in $T$ but $f(A_2)$ is not fuzzy $(1/2, 1/2)$-pre open in $W$.

Example 4.7 Let $X = \{a, b\} = Y$. Define $A_1, A_2, A_1 \cup A_2, A_1 \cap A_2 \in I(X), B_i \in I(Y)$ as follows:

$A_1(x) = B_1(x) = (0.1, 0.3), A_1(y) = B_1(y) = (0.1, 0.3), A_2(x) \cap A_2(x) = (0.1, 0.6), A_1(y) \cap A_2(y) = (0.1, 0.5)$

$A_2(x) = (0.3, 0.6), A_2(y) = (0.3, 0.5) \quad A_1(x) \cup A_2(x) = (0.3, 0.3), A_1(y) \cup A_2(y) = (0.3, 0.3)$

Define $T : I(X) \rightarrow I \otimes I$ and $W : I(Y) \rightarrow I \otimes I$.

by $T(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0, 1, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_1, A_2, A_1 \cup A_2, A_1 \cap A_2 \\ (0, 1) & \text{otherwise} \end{cases}$

$W(B) = (W_1(B), W_2(B)) = \begin{cases} (1, 0) & \text{if } B = 0, 1, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } B = B_1 \\ (0, 1) & \text{otherwise} \end{cases}$

For $r = 1/2, s = 1/2$. Then the identity mapping $f : (X, T_1, T_2) \rightarrow (Y, W_1, W_2)$ is fuzzy $(1/2, 1/2)$-pre open mapping which is not fuzzy $(1/2, 1/2)$-$\alpha$ open mapping. Since $A_2$ is fuzzy $(1/2, 1/2)$-open in $T$ but $f(A_2)$ is not fuzzy $(1/2, 1/2)$-$\alpha$ open in $W$.

It need not be true that $f$ and $g$ are fuzzy $(r, s)$-regular semi (resp. regular, $\alpha$, pre and $\beta$) open (resp. $(r, s)$-regular semi (resp. regular, $\alpha$, pre and $\beta$) closed) mappings then so is $g \circ f$. But we have the following theorem:

Theorem 4.1 Let $(X, T_1, T_2), (Y, W_1, W_2)$ and $(Z, V_1, V_2)$ be SolFTSS and let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be mappings and $(r, s) \in I \otimes I$. Then the following statements are true:

[i] If $f$ is a fuzzy $(r, s)$-open mapping and $g$ is a fuzzy $(r, s)$-regular semi (resp. regular, $\alpha$, pre and $\beta$) open mapping, then $g \circ f$ is a fuzzy $(r, s)$-regular semi (resp. regular, $\alpha$, pre and $\beta$) open mapping.\[ii] If $f$ is a fuzzy $(r, s)$-closed mapping and $g$ is a fuzzy $(r, s)$-regular semi (resp. regular, $\alpha$, pre and $\beta$) closed mapping, then $g \circ f$ is a fuzzy $(r, s)$-regular semi (resp. regular, $\alpha$, pre and $\beta$) closed mapping.

Proof: Straightforward
Proof. (i) $\Rightarrow$ (ii) Since \( \text{int}_{T_1,T_2}(A,r,s) \subseteq A, (A \in I^X) \), then \( f(\text{int}_{T_1,T_2}(A,r,s)) \subseteq f(A) \). But \( f(\text{int}_{T_1,T_2}(A,r,s)) \) is \((r,s)\)-frsc. Hence, \( f(\text{int}_{T_1,T_2}(f^{-1}(B),r,s)) \subseteq \text{rscl}_{W_1,W_2}(f(A),r,s) \).

(iii) $\Rightarrow$ (iii) Let (ii) holds. Then, \( f(I_{T_1,T_2}(A,r,s)) \subseteq \text{rscl}_{W_1,W_2}(f(A),r,s) \).

(iii) $\Rightarrow$ (iv) Let (iii) holds and let \( A \in I^Y \) and apply (ii), we have \( f(\text{int}_{T_1,T_2}(f^{-1}(B),r,s)) \subseteq \text{rscl}_{W_1,W_2}(f(A),r,s) \).

Theorem 4.3 Let \((X,T_1,T_2)\) and \((Y,W_1,W_2)\) be smooth topological space's. Let \( f: X \to Y \) be a mapping. The following statements are equivalent: 

(i) \( f \) is fuzzy \((r,s)\)-open.
(ii) \( f(\text{int}_{T_1,T_2}(A,r,s)) \subseteq \text{aint}_{W_1,W_2}(f(A),r,s) \), for each \( A \subseteq I^X \).

Theorem 4.4 Let \((X,T_1,T_2)\) and \((Y,W_1,W_2)\) be smooth topological space's. Let \( f: X \to Y \) be a mapping. The following statements are equivalent: 

(i) \( f \) is fuzzy \((r,s)\)-pre-open.
(ii) \( f(\text{int}_{T_1,T_2}(A,r,s)) \subseteq \text{pint}_{W_1,W_2}(f(A),r,s) \), for each \( A \subseteq I^X \).

Theorem 4.5 Let \((X,T_1,T_2)\) and \((Y,W_1,W_2)\) be smooth topological space's. Let \( f: X \to Y \) be a mapping. The following statements are equivalent: 

(i) \( f \) is fuzzy \((r,s)\)-\(\beta\) open.
(ii) \( f(\text{int}_{T_1,T_2}(A,r,s)) \subseteq \text{bint}_{W_1,W_2}(f(A),r,s) \), for each \( A \subseteq I^X \).

Proof. Proof of theorem 4.3,4.4,4.5 it follows from Theorem 4.2

Theorem 4.6 Let \((X,T_1,T_2)\) and \((Y,W_1,W_2)\) be SoIFT's. Let \( f: X \to Y \) be a mapping. The following statements are equivalent: 

(i) \( f \) is fuzzy \((r,s)\)-regular semi closed.
(ii) \( \text{rscl}_{W_1,W_2}(f(A),r,s) \subseteq \text{rccl}_{T_1,T_2}(A,r,s) \), for each \( A \subseteq I^X \).

Proof. (i) $\Rightarrow$ (ii) Let (i) holds and let \( A \subseteq I^X \). Since \( \text{cl}_{T_1,T_2}(A,r,s) \subseteq \text{rccl}_{T_1,T_2}(A,r,s) \), then \( f(\text{cl}_{T_1,T_2}(A,r,s)) \subseteq f(\text{rccl}_{T_1,T_2}(A,r,s)) \) and therefore, \( \text{rscl}_{W_1,W_2}(f(A),r,s) \subseteq f(\text{rccl}_{T_1,T_2}(A,r,s)) \).

(ii) $\Rightarrow$ (i) Let (ii) holds and let \( A \subseteq I^X \) be a \((r,s)\)-fr set. Then \( \text{rscl}_{W_1,W_2}(f(A),r,s) \subseteq f(\text{rccl}_{T_1,T_2}(A,r,s)) \). It implies \( \text{rscl}_{W_1,W_2}(f(A),r,s) \subseteq f(A) \). But \( \text{rscl}_{W_1,W_2}(f(A),r,s) \subseteq f(A) \). Then \( f(A) = \text{rscl}_{W_1,W_2}(f(A),r,s) \) and it follows that \( f(A) \) is \((r,s)\)-frsc. Hence, \( f \) is fuzzy \((r,s)\)-regular semi closed.
Theorem 4.7 Let \((X, T_1, T_2)\) and \((Y, W_1, W_2)\) be SoIFTS’s. Let \(f : X \to Y\) be a mapping. The following statements are equivalent: [i] \(f\) is fuzzy \((r, s) - \alpha\) closed. [ii] \(\alpha cl_{W_1, W_2}(f(A), r, s)) \subseteq f(cl_{T_1, T_2}(A, r, s)), \) for each \(A \in I^X\).

Theorem 4.8 Let \((X, T_1, T_2)\) and \((Y, W_1, W_2)\) be SoIFTS’s. Let \(f : X \to Y\) be a mapping. The following statements are equivalent: [i] \(f\) is fuzzy \((r, s) - \text{pre closed.}\) [ii] \(\text{precl}_{W_1, W_2}(f(A), r, s)) \subseteq f(cl_{T_1, T_2}(A, r, s)),\) for each \(A \in I^X\).

Theorem 4.9 Let \((X, T_1, T_2)\) and \((Y, W_1, W_2)\) be SoIFTS’s. Let \(f : X \to Y\) be a mapping. The following statements are equivalent: [i] \(f\) is fuzzy \((r, s) - \beta\) closed. [ii] \(\beta cl_{W_1, W_2}(f(A), r, s)) \subseteq f(cl_{T_1, T_2}(A, r, s)), \) for each \(A \in I^X\).

Proof. Proof of theorem 4.7, 4.8, 4.9 it follows from Theorem 4.6.

References