Structure Theorem for B(1,2) $\bar{s}$ - Near Subtraction Semigroups

1. V. LOKANAYAKI, 2. V. MAHALAKSHMI and 3. S. USHA DEVI

1. P.G. Mathematics, A.P.C. Mahalaxmi college for women, Thoothukudi,
lokanyakimani@rediff.com

2. PG and Research Department of Mathematics, A.P.C. Mahalaxmi college for women, Thoothukudi,
maha.krishna86@gmail.com

3. Assistant Professor of Mathematics, Sri Parasakthi college for women, Courtallam,
ushadevinathan@gmail.com

Abstract: In this paper, we introduce the concept of B(1,2) $\bar{s}$-Near Subtraction Semigroups and give the structure theorem for the same. By x we mean a zero-symmetric near subtraction semigroups. Define x to be a $p_k(p'_k)$ near subtraction semigroups if $a^k X = aXA(Xa^k = aXA)$ for all $a \in X$ and a near subtraction semigroups x is said to be a $p_k(m,n)(p'_k(m,n))$ near subtraction semigroups if $a^k X = a^m Xa^n (Xa^k = a^m Xa^n)$ for all $a \in X$. Motivated by these concept we introduce B(1,2) near subtraction semigroups and their generalization and similarities. A near subtraction semigroups X is said to be a B(1,2) near subtraction semigroups is the right (left) X-subalgebra of near subtraction semigroups X generated by ‘a’.

Keywords: Left permutable, $\bar{s}$-near subtraction semigroups, $B_k(B'_k)$ near subtraction semigroups, B(1,2) near subtraction semigroups.

1. Introduction

B.M. Schein [10] considered system of the form $(X;0:\setminus)$, where X is set of functions closed under the composition “0” of functions (and hence $(X;0)$ is a function semigroups) and the set theoretic subtraction “\setminus” (and hence $(X;\setminus)$ is a subtraction algebra in the sense of [1]). He proved that every subtraction semigroups is isomorphic to a difference semigroups of invertible function B.Zelinka [11] discussed a problem proposed by B.M. Schein concerning the structure of multiplication in a subtraction semigroups. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. For basic definition one may refer to Pilz[8]. Motivated by the study of B(1,2) near subtraction semigroups in “A Study on Regularities in Near-ring” by S.Jayalakshmi. We introduced new concepts “B(1,2) near subtraction semigroups”.

2. Preliminary

Definition :2.1
A non empty set X together with two binary operation ‘−’ and ‘●’ is said to be $\bar{s}$ - near subtraction semigroups if it satisfies the following

i) $(X;\setminus)$ is a subtraction algebra.

ii) $(X;\bullet)$ is a semigroups

iii) $x(y - z) = xy - xz$ and $(x - y)z = xz - yz$ for every $x, y, z \in X$
Definition: 2.2
A near subtraction semigroups $X$ is said to have property($\alpha$) if $aX$ is a subalgebra of $(X, -)$ for every $a \in X$.

Definition: 2.3
A near subtraction semigroups $X$ is called a generalized near-field if for each $a \in X$ there exists unique $b \in X$ such that $a = aba$ and $b = bab$.

Theorem: 2.4
Let $X$ be a near subtraction semigroups. Then the following are equivalent.

i) $X$ is a GNF

ii) $X$ is a regular and each idempotent is central

iii) $X$ is regular and sub commutative

Lemma: 2.5
If $X$ is a $K(1,2)$ near subtraction semigroups, $E \subseteq C(X)$

Remark: 2.6
If $X$ is a s-near subtraction semigroups with property($\alpha$), then $< a >_r = aX$ and $< a >_l = Xa$, for all $a \in X$.

Lemma: 2.7
Let $X$ be a zero –symmetric near subtraction semigroups without non-zero nilpotent elements. Then $ab = 0$ implies $ba = 0$.

Remark: 2.8
Whenever a zero-symmetric near subtraction semigroups contains no non-zero nilpotent elements in view of above lemma2.7,$X$ has IFP.

Theorem: 2.9
Let $X$ be a $\bar{s}$- near subtraction semigroup with property ($\alpha$). If $X$ is a $B(1,2)$ near subtraction semigroups, then $M_1 \cap M_2 = M_1M_2$ for any two left $x$-subalgebra $M_1$ and $M_2$ of $X$.

Corollary: 2.10
Let $X$ be a $B(1,2)$ $\bar{s}$ - near subtraction semigroups with property($\alpha$). Then $X$ is strongly regular.

Corollary: 2.11
Let $X$ be a $B(1,2)$ $\bar{s}$ - near subtraction semigroups with property($\alpha$). Then $X$ is regular.

3. Structure Theorem for $B(1,2)$ $\bar{s}$-Near Subtraction Semigroups
In this section first we study certain properties involving structure theorem for $B(1,2)$ in the class of near subtraction semigroups.

Definition: 3.1
We say that a near subtraction semigroups $X$ has the property $B(m,n)$, if there exist positive integers $m,n$ such that $< a >_r^mX = X < a >_l^n$, for all $a$ in $X$.

Example: 3.2
Let $X = \{0, a, b, 1\}$ in which “$-$” and “$\bullet$” are defined by,

\[
\begin{array}{ccc}
- & 0 & a & b & 1 \\
0 & 0 & 0 & 0 & 0 \\
a & a & 0 & a & 0 \\
b & b & b & 0 & 0 \\
1 & 1 & b & a & 0 \\
\end{array}
\]

One can check the $< 0 >_r = < 0 >_l = \{0\}$, $< a >_r = < a >_l = \{0, a\}$, $< b >_l = \{0, b\}$, $< 1 >_r = < 1 >_l = \{0, a, b, 1\}$ and so this $X$ is $B(m,n)$ near subtraction semigroups, for all positive integer $m$ and $n$.

Proposition: 3.3
Let X be a $\tilde{s}$ -near subtraction semigroups. Then X is a B (1,2) near subtraction semigroups with property($\alpha$) if and only if X is a K(1,2) near subtraction semigroups.

**Proof:**

Assume that X is a B(1,2) near subtraction semigroups with property($\alpha$). By corollary2.11, X is regular and so, for $a \in X$, $a=axa$, for some $x \in X$. Now $aX=axaX \subseteq aXXX \subseteq aXX = < a >_r X = X < a >^2 \subseteq XXaXa \subseteq Xa \text{ ie, } aX \subseteq Xa$.

Similarly $Xa \subseteq aX$ and so $aX = Xa$ ie, X is a sub commutative. Since X is regular and sub commutative by Theorem 2.4, X is regular and $E \subseteq C(X)$. Let $X_1 \in aX$. Then for $y \in X$, $X_1 = ay = axay = ax(xay) = ax(xa) = axa \subseteq Xa^2$. Trivially $Xa^2 \subseteq Xa = aX$. Thus $aX = Xa^2$, for all a in X ie., X is a K(1,2) near subtraction semigroups. Conversely, Let X be a K(1,2) near subtraction semigroups.

Since X is a $\tilde{s}$ -near subtraction semigroups $a \in aX = Xa^2$. ie., X is strongly regular and so X is regular. Since X is a K(1,2) near subtraction semigroups, by Lemma2.5, $E \subseteq C(X)$. Then, by Theorem2.4, X is regular and sub commutative. In the review of Remark2.6 $< a >_r X = aXX = XaX = XXaXa \subseteq XXaXa = X < a >^2 \subseteq XXaXa \subseteq XaXa = XaX = aXX = aXX = < a >_r X$. ie, $X < a >^2 \subseteq aX$. Also $X < a >^2 \subseteq XXaXa \subseteq XXaXa = XaX = aXX = < a >_r X$ . These two imply that $< a >_r X = X < a >^2$. ie., X is a B(1,2) near subtraction semigroups.

**Proposition:3.4**

Let X be a $\tilde{s}$ -near subtraction semigroups with property ($\alpha$). Then X is a B(1,2) near subtraction semigroups if and only if X is a GNF.

**Proof:**

Assume that X is a GNF. Now for $a \in X$, $< a >_r X = aXX = axaXX \subseteq aXX = XXaXa = X < a >^2$ . (ie) $< a >_r X \subseteq X < a >^2$. Similarly $X < a >^2 = XXaXa \subseteq XXa = XaX = aXX = aXX = < a >_r X < a >^2 \subseteq XaXaXa \subseteq XXa = XaX = aXX = < a >_r X$. From these, we get that $< a >_r X = X < a >^2$. (ie) X is a B(1,2) near subtraction semigroups. Conversely, assume that X is a B(1,2) near subtraction semigroups. Since X is a $\tilde{s}$ -near subtraction semigroups with property ($\alpha$) and by Corollary2.11, X is regular. By Theorem2.4, X is a K(1,2) near subtraction semigroups. Again by Lemma2.5, $E \subseteq C(X)$. X is GNF.

**Proposition:3.5**

Let X be a B(1,2) $\tilde{s}$ - near subtraction semigroups with property($\alpha$) and let A and B be any two left X -subalgebra of X. Then we have the following:

i. $\sqrt{A} = A$

ii. $A \cap B = AB$

iii. $A^2 = A$

iv. If $A \subseteq B$ then $AB = A$

v. $A \cap XB = AB$

vi. If A is proper, then each element of A is a zero divisor
vii. $A$ is a completely semiprime ideal of $X$.

**Proof:**

i) For $x \in \sqrt{A}$, there exists some positive integer $k$ such that $x^k \in A$. Since $x$ is a $B(1,2)$ -near subtraction semigroups with property($\alpha$) . By Corollary 2.10, $X$ is strongly regular. If $x \in X$, then $x=ax^2$, for some $a \in X$. This implies $x=ax^2=(ax)x=a(ax^2)x=a^2x^3=\ldots=a^{k-1}x^k \in XA \subseteq A$. (ie ) $x \in A$. Thus $\sqrt{A} \subseteq A$ obviously $A \subset \sqrt{A}$ and so $A = \sqrt{A}$

ii) Since $X$ is a $\bar{s}$ -near subtraction semigroups with property($\alpha$), by the Theorem 2.9, $AB=\cap B$

iii) Taking $B=A$ in (ii) we get $A=A^2$

iv) Suppose that $A \subset B$. Then $A \cap B = A$ and (ii) gives $A=AB$

v) $A \cap XB \subset A \cap B$ and so $A \cap XB \subset AB$ (by(ii)). Also $AB = A \cap B \subset A$ and $AB \subset XB$. Therefore $AB \subset A \cap XB$. Hence $AB = A \cap XB$

vi) By the Remark 2.8, $X$ has the IFP. Then the concept of left zero –divisors, right zero-divisors and zero–divisors are equivalent in $X$. Thus we need only to prove that $A^*$ consists of only zero-divisors. Let $a \in A^*$by (iii) for the principal left $x$-subalgebra $Xa$, $Xa= (Xa)^2=XaXa$

Consequently, for any $x \in X$, there exists $y,z \in X$ such that $xa=yaza$. (ie)($x-yaz)a=0$.

Similarly ($yaz-x)a=0$ .If $a$ is not a zero-divisor , then $x-yaz=0$ and $yaz-x=0$. This implies $x=yaz \in XAX \subset A$ (ie) $X \subset A$ . Hence $X=A$ which is a contradiction to the hypothesis that $A$ is proper.Thus $a \in A^*$.

Hence ‘a’ is a zero-divisor.

vii) Let $a^2 \in A.X$ has strong IFP. So $axa \in A$ By Corollary 2.11 $X$ is regular. Then $a \in A$. Hence $A$ is completely semi prime.

**Proposition:3.6**

Let $X$ be a $\bar{s}$ - near subtraction semigroups with property($\alpha$) .Then $X$ is a $B(m,n)$ near subtraction semigroups, for all positive integer $m,n$ if and only if $X$ is a $B(1,2)$ near subtraction semigroups.

**Proof:**

Assume that $X$ is a $B(1,2)$ near subtraction semigroups. By Proposition 3.3, $X$ is a GNF. Therefore by Theorem 2.4. $X$ is regular and $E \subseteq C(X)$. Let $a \in X$, here $< a >_r^m X =< a >_r < a >_{r-1}^m \subseteq < a >_r X = X < a >_r < a >_{r-1}^m Xa =$

$X < a >_r Xaxa \subseteq Xxa \subseteq X(xa)^n \in X(Xa)^n = X < a >_r^n$. Similary $X < a >_r^n = X < a >_r < a >_{r-2}^n < a >_{r-1}^n \subseteq X < a >_{r-1}^n = < a >_r X \subseteq aXX = axaXX \in axX =$

$(ax)^mX \in (aX)^mX = \,< a >_r^m X \text{ i.e.,} X < a >_r^n \subseteq X < a >_{r-1}^n$. So $< a >_{r-1}^n X = X < a >_{r-1}^n$ and hence $X$ is $B(m,n)$ near subtraction semigroups, for all positive integer $m,n$. Converse part is trivial.
References


[9] S. Seyadali Fathima, *k(r,m) near subtraction semigroups*, International Journal of Algebra,

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