APPLICATION ON b\(\mathring{g}\)-CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract:
In 1970, Levine defined generalized closed sets in Topological Spaces. In 2013, R.Subasree and M.Maria Singam introduced a new set namely b\(\mathring{g}\)-closed set which is defined as bcl(A) \(\subseteq\) U whenever A \(\subseteq\) U and U is \(\mathring{g}\)-open in X. Later, they have established function such as b\(\mathring{g}\)-continuous function that is the inverse image of every closed set is b\(\mathring{g}\)-closed in .b\(\mathring{g}\)-irresolute function if the inverse image every b\(\mathring{g}\)-closed set is b\(\mathring{g}\)-closed; b\(\mathring{g}\)-open map means the image of every open set is b\(\mathring{g}\)-open and b\(\mathring{g}\)-closed map means the image of every closed set is b\(\mathring{g}\)-closed. In 1973, Das defined semi-interior point and semi-limit point of a subset. Further, the semi-derived set of a topological space was defined and studied by him. Following this concept, we define b\(\mathring{g}\)-limit point, b\(\mathring{g}\)-derived set, b\(\mathring{g}\)-border, b\(\mathring{g}\)-frontier and b\(\mathring{g}\)-exterior of a subset of topological spaces using the concept of b\(\mathring{g}\)-closed sets which is defined as and we studied some of its properties.

Keywords: b\(\mathring{g}\)-limit point, b\(\mathring{g}\)-derived set, b\(\mathring{g}\)-border, b\(\mathring{g}\)-frontier, b\(\mathring{g}\)-exterior, b\(\mathring{g}\)-closed set.

INTRODUCTION:
In 2013, R. Subasree and M. Maria Singam introduced[3] a new set namely b\(\mathring{g}\)-closed set. In 1973, Das[2] defined semi-interior point and semi-limit point of a subset. Further, the semi-derived set of a topological space was defined and studied by him. Following this concept, we define b\(\mathring{g}\)-limit point, b\(\mathring{g}\)-derived set, b\(\mathring{g}\)-border, b\(\mathring{g}\)-frontier and b\(\mathring{g}\)-exterior of a subset of topological spaces using the concept of b\(\mathring{g}\)-closed sets which is defined as and we studied some of its properties.

PRELIMINARIES:
Throughout this paper \((X,\tau)\) (or simply X) represents topological space on which no separation axioms are assumed unless otherwise mentioned . For a subset A of \((X,\tau)\), Cl(A), Int(A), D(A), b(A) and Ext(A) denote the closure, interior, derived, border and exterior of A respectively. We are giving some basic definitions.

Definition 2.1: A subset A of a topological space \((X,\tau)\) is called
1) b\(\mathring{g}\)-closed set if bcl(A) \(\subseteq\) U whenever A \(\subseteq\) U and U is \(\mathring{g}\)-open in X. The collection of all b\(\mathring{g}\)-closed sets in \((X,\tau)\) is denoted by b\(\mathring{g}\)-C(X,\(\tau\)).
2) b\(\mathring{g}\)-open set if \(X \setminus A\) is b\(\mathring{g}\)-closed in A . The collection of all b\(\mathring{g}\)-open sets in \((X,\tau)\) is denoted by b\(\mathring{g}\)-O(X,\(\tau\)).

Definition 2.2: Let A be the subset of a space \((X,\tau)\). Then
1) The border of A is defined as b(A) = A \(\setminus\) Int(A).
2) The frontier of A is defined as Fr(A) = Cl(A) \(\setminus\) Int(A).
3) The exterior of A is defined as Ext(A) = Int(X\(\setminus\)A).

Theorem 2.3: [2]
1) Every closed set is b\(\mathring{g}\)-closed.
2) Every open set is b\(\mathring{g}\)-open.
3. PROPERTIES OF $b\tilde{g}$-INTERIOR AND $b\tilde{g}$-CLOSURE:

Definition 3.1: The $b\tilde{g}$-interior of $A$ is defined as the union of all $b\tilde{g}$-open sets of $X$ contained in $A$. It is denoted by $b\tilde{g}\text{Int}(A)$.

Definition 3.2: A point $x \in X$ is called a $b\tilde{g}$-interior point of $A$ if $A$ contains a $b\tilde{g}$-open sets containing $x$.

Definition 3.3: The $b\tilde{g}$-closure of $A$ is defined as the intersection of all $b\tilde{g}$-closed sets of $X$ containing $A$. It is denoted by $b\tilde{g}\text{Cl}(A)$.

Theorem 3.4: If $A$ is a subset of $X$, then $b\tilde{g}\text{Int}(A)$ is the set of all $b\tilde{g}$-interior points of $A$.

Proof: If $x \in b\tilde{g}\text{Int}(A)$, then $x$ belongs to some $b\tilde{g}$-open subset $U$ of $A$. That is, $x$ is a $b\tilde{g}$-interior point of $A$.

Remark 3.5: If $A$ is any subset of $X$, $b\tilde{g}\text{Int}(A)$ is $b\tilde{g}$-open. In fact $b\tilde{g}\text{Int}(A)$ is the largest $b\tilde{g}$-open set contained in $A$.

Remark 3.6: A subset $A$ of $X$ is $b\tilde{g}$-open if and only if $b\tilde{g}\text{Int}(A) = (A)$.

Result 3.7: For the subset $A$ of a topological space $(X, \tau)$, $\text{Int}(A) \subseteq b\tilde{g}\text{Int}(A)$.

Proof: Since $\text{Int}(A)$ is the union of open sets and theorem 2.3, $\text{Int}(A)$ is $b\tilde{g}$-open. It is clear from the definition 3.1 that $\text{Int}(A) \subseteq b\tilde{g}\text{Int}(A)$.

Theorem 3.8: Let $A$ and $B$ be the subsets of a topological space $(X, \tau)$, then the following result holds:

1) $b\tilde{g}\text{Int}(\emptyset) = \emptyset$;
2) $b\tilde{g}\text{Int}(X) = X$;
3) $b\tilde{g}\text{Int}(A) \subseteq A$;
4) $A \subseteq B \Rightarrow b\tilde{g}\text{Int}(A) \subseteq b\tilde{g}\text{Int}(B)$;
5) $b\tilde{g}\text{Int}(A \cup B) \supseteq b\tilde{g}\text{Int}(A) \cup b\tilde{g}\text{Int}(B)$;
6) $b\tilde{g}\text{Int}(A \cap B) \subseteq b\tilde{g}\text{Int}(A) \cap b\tilde{g}\text{Int}(B)$;
7) $b\tilde{g}\text{Int}(\text{Int}(A)) = \text{Int}(A)$;
8) $\text{Int}(b\tilde{g}\text{Int}(A)) \subseteq \text{Int}(A)$;
9) $b\tilde{g}\text{Int}(b\tilde{g}\text{Int}(A)) = b\tilde{g}\text{Int}(A)$

Proof: (1), (2), and (3) follows from definition 3.1.

(4) From definition 3.1 we have, $b\tilde{g}\text{Int}(A) \subseteq A$. Since $A \subseteq B$, $b\tilde{g}\text{Int}(A) \subseteq B$. But $b\tilde{g}\text{Int}(B) \subseteq B$. By remark 3.5 $b\tilde{g}\text{Int}(A) \subseteq b\tilde{g}\text{Int}(B)$.

(5) Since $A \subseteq AU B; B \subseteq AU B$ and by (4) we have, $b\tilde{g}\text{Int}(A) \subseteq b\tilde{g}\text{Int}(A \cup B)$ and $b\tilde{g}\text{Int}(B) \subseteq b\tilde{g}\text{Int}(A \cup B)$. Therefore $b\tilde{g}\text{Int}(A) \cup b\tilde{g}\text{Int}(B) \subseteq b\tilde{g}\text{Int}(A \cup B)$.

(6) Since $A \cap B \subseteq A; A \cap B \subseteq B$ and by (4) we have, $b\tilde{g}\text{Int}(A \cap B) \subseteq b\tilde{g}\text{Int}(A)$ and $b\tilde{g}\text{Int}(A \cap B) \subseteq b\tilde{g}\text{Int}(B)$. Therefore $b\tilde{g}\text{Int}(A \cap B) \subseteq b\tilde{g}\text{Int}(A \cap B)$.

(7) Since $A \cap B \subseteq A; A \cap B \subseteq B$ and by (4) we have, $b\tilde{g}\text{Int}(A \cap B) \subseteq b\tilde{g}\text{Int}(A)$ and $b\tilde{g}\text{Int}(A \cap B) \subseteq b\tilde{g}\text{Int}(B)$. Therefore $b\tilde{g}\text{Int}(A \cap B) \subseteq b\tilde{g}\text{Int}(A \cap B)$.
(8) Since $\text{Cl}(A)$ is a closed set and by theorem 2.3 (1), $\text{Cl}(A)$ is $b\digamma$-closed. Therefore by remark 3.10, $b\digamma \text{Cl}(A) = \text{Cl}(A)$.

(9) Follows from remark 3.9 and 3.10.

4. APPLICATION OF $b\digamma$-OPEN SETS

Definition 4.1: Let $A$ be a subset of a topological space $X$. A point $x \in X$ is said $b\digamma$-limit point of $A$ if for every $b\digamma$-open set $U$ containing $x$, $\bigcap U \neq \emptyset$. The set of all $b\digamma$-limit points of $A$ is called an $b\digamma$-derived set of $A$ and is denoted by $b\digamma(A)$.

Example 4.2: Let $X = \{a,b,c\}$ with property $\tau = \{X, \emptyset, \{a\}, \{a,b\}\}$ and $b\digamma O (X) = \{X, \emptyset, \{a\}, \{b\}, \{a,c\}, \{a,b\}\}$. If $A = \{b\}$, then $b\digamma D(A) = \{b\}$.

Result 4.3: Let $A$ be a subset of a topological space $X$. Then,

1) $b\digamma \text{Cl}(X \setminus A) = X \setminus b\digamma \text{Int}(A)$

2) $b\digamma \text{Int}(X \setminus A) = X \setminus b\digamma \text{Cl}(A)$

Proof: 

1) Let $x \in X \setminus b\digamma \text{Int}(A)$. Then, $x \notin b\digamma \text{Int}(A)$. This implies that $x$ does not belong to any $b\digamma$-open subset of $A$. Let $F$ be a $b\digamma$-closed set containing $X \setminus A$. Then $X \setminus F$ is $b\digamma$-open set contained in $A$. Therefore, $x \in X \setminus F$ and so $x \in F$. Hence, $x \in b\digamma \text{Cl}(X \setminus A)$. This implies $X \setminus b\digamma \text{Int}(A) \subseteq b\digamma \text{Cl}(X \setminus A)$. On the other hand, let $x \notin b\digamma \text{Int}(A)$. Then $x$ belongs to every $b\digamma$-closed set containing $X \setminus A$. Hence, $x$ does not belong to any $b\digamma$-open subset of $A$. That is $x \notin b\digamma \text{Int}(A)$. This implies $x \in X \setminus b\digamma \text{Int}(A)$. Therefore, $b\digamma \text{Cl}(X \setminus A) \subseteq X \setminus b\digamma \text{Int}(A)$. Thus, $b\digamma \text{Cl}(X \setminus A) = X \setminus b\digamma \text{Int}(A)$.

2) can be proved by replacing $A$ by $X \setminus A$ in 1) and using set theoretic properties.

Theorem 4.4: For subsets $A,B$ of a space $X$, the following statement holds:

1) $D(A) \subseteq b\digamma D(A)$, where $D(A)$ is the derived set of $A$;

2) $b\digamma D(\emptyset) = \emptyset$;

3) If $A \subseteq B$, then $b\digamma D(A) \subseteq b\digamma D(B)$;

4) $b\digamma D(A \cup B) \supseteq b\digamma D(A) \cup b\digamma D(B)$;

5) $b\digamma D(A \cap B) \subseteq b\digamma D(A) \cap b\digamma D(B)$;

6) $b\digamma D(A) \subseteq b\digamma D(A \setminus \{x\})$.

Proof: 1) Let $x \in D(A)$. By the definition of $D(A)$, there exist an open set $U$ containing $x$ such that $U \cap (A \setminus \{x\}) \neq \emptyset$. By theorem 2.3 (2), $U$ is an $b\digamma$-open set containing $x$ such that $U \cap (A \setminus \{x\}) \neq \emptyset$. Therefore, $x \in b\digamma(A)$. Hence, $D(A) \subseteq b\digamma(A)$.

2) For all $b\digamma$-open set $U$ and for all $x \in U$, $U \cap (A \setminus \{x\}) \neq \emptyset$. Hence, $b\digamma D(\emptyset) = \emptyset$.

3) Let $x \in b\digamma D(A)$. Then for each $b\digamma$-open set $U$ containing $x$, $U \cap (A \setminus \{x\}) \neq \emptyset$. Since $A \subseteq B$, $U \cap (B \setminus \{x\}) \neq \emptyset$. This implies that $x \in b\digamma(B)$. Hence $b\digamma D(A) \subseteq b\digamma D(B)$.

4) Let $x \in b\digamma D(A) \cup b\digamma D(B)$. Then $x \in b\digamma D(A)$ or $x \in b\digamma D(B)$. If $x \in b\digamma D(A)$, then for each $b\digamma$-open set $U$ containing $x$, $U \cap (A \setminus \{x\}) \neq \emptyset$. Since $A \subseteq B$, $U \cap (A \cup B \setminus \{x\}) \neq \emptyset$. This implies that $x \in b\digamma D(A \cup B)$. Hence $b\digamma D(A) \subseteq b\digamma D(A \cup B)$.

5) Similarly, if $x \in b\digamma D(A \cup B)$, then for each $b\digamma$-open set $U$ containing $x$, $U \cap (A \cup B \setminus \{x\}) \neq \emptyset$. This implies that $x \in b\digamma D(A \cup B)$. Hence, $b\digamma D(A \cup B) \subseteq b\digamma D(A \cup B)$.

6) Similarly, if $x \in b\digamma D(A \setminus \{x\})$, then for each $b\digamma$-open set $U$ containing $x$, $U \cap (A \setminus \{x\} \setminus \{x\}) \neq \emptyset$. This implies that $x \in b\digamma D(A \setminus \{x\})$. Hence, $b\digamma D(A \setminus \{x\}) \subseteq b\digamma D(A \setminus \{x\})$.

Definition 4.5: If $A$ is a subset of $X$, then the $b\digamma$-border of $A$ is defined by $b\digamma b(A) = A \setminus b\digamma \text{Int}(A)$.

Theorem 4.6: For a subset $A$ of a space $X$, the following statement holds:

1) $b\digamma b(\emptyset) = \emptyset$;

2) $b\digamma b(X) = X$;
3) \( b \cap b(A) \subseteq (A) \);
4) \( b \cap b(A) 
\subseteq b(A) \); where \( b(A) \) denotes the border of \( A \);
5) \( b \cap Int(A) \cup b \cap b(A) = A \);
6) \( b \cap Int(A) \cap b \cap b(A) = \emptyset \);
7) \( b \cap (b \cap Int(A)) = \emptyset \);
8) \( b \cap Int(b \cap b(A)) = \emptyset \);
9) \( b \cap b(b \cap b(A)) = b \cap b(A) \).

**Proof:** 1),2) and 3) follows from definition 4.5.

4) Let \( x \in b \cap b(A) \). Then by definition 4.5, \( x \in A \setminus b \cap Int(A) \). This implies that \( x \in A \) and \( x \notin b \cap Int(A) \). By result 3.7, \( x \in A \) and \( x \notin Int(A) \). This implies that \( x \in A \setminus Int(A) \). This implies that \( x \in b(A) \). Hence, \( b \cap b(A) \subseteq (A) \).

5) and 6) follows from definition 4.5.

7) \( b \cap (b \cap Int(A)) = b \cap Int(b \cap b(A)) = b \cap Int(b \cap Int(A)) = b \cap Int(b \cap Int(A)) \) by theorem 3.8(9)) which is \( \emptyset \). Hence, \( b \cap b(b \cap Int(A)) = \emptyset \).

8) Let \( x \in b \cap Int(b \cap b(A)) \). By theorem 3.8(3), \( x \in b \cap b(A) \). On the other hand, since \( b \cap b(A) \subseteq A \), \( x \in b \cap Int(A) \). Therefore, \( x \in b \cap b(A) \cap b \cap Int(A) \) which is a contradiction to (6). Hence, \( b \cap b(b \cap b(A)) = \emptyset \).

9) \( b \cap b(b \cap b(A)) = b \cap b(A) \setminus b \cap Int(b \cap b(A)) = b \cap b(A) \setminus \emptyset = b \cap b(A) \) (from 8). Hence, \( b \cap b(b \cap b(A)) = b \cap b(A) \).

**Definition 4.7:** If \( A \) is a subset of \( X \), then the \( b \cap -frontier \) of \( A \) is defined by \( b \cap Fr(A) = b \cap Cl(A) \setminus b \cap Int(A) \).

**Theorem 4.8:** Let \( A \) be a subset of a space \( X \). Then the following statements hold:

1) \( b \cap Fr(\emptyset) = \emptyset \);
2) \( b \cap Fr(X) = \emptyset \);
3) \( b \cap Fr(A) \subseteq b \cap Cl(A) \);
4) \( b \cap Cl(A) = b \cap Int(A) \cup b \cap Fr(A) \);
5) \( b \cap Int(A) \cap b \cap Fr(A) = \emptyset \);
6) \( b \cap b(A) \subseteq b \cap Fr(A) \);
7) \( b \cap Fr(b \cap Int(A)) \subseteq b \cap Fr(A) \);
8) \( b \cap Cl(b \cap Fr(A)) \subseteq b \cap Cl(A) \);
9) \( b \cap Int(A) \subseteq b \cap Cl(A) \);
10) \( b \cap Int(b \cap Fr(A)) \subseteq b \cap Cl(A) \);
11) \( X = b \cap Int(A) \cup b \cap Int(X \setminus A) \cup b \cap Fr(A) \);
12) \( b \cap Fr(A) = b \cap Cl(A) \cap b \cap Cl(X \setminus A) \);
13) \( b \cap Fr(A) = b \cap Fr(X \setminus A) \).

**Proof:** (1), (2), (3) and (4) follows from definition 4.7.

5) \( b \cap Int(A) \cap b \cap Fr(A) = b \cap Int(A) \cap (b \cap Cl(A) \setminus b \cap Int(A) \subseteq A \cap (b \cap Cl(A) \setminus A) \) (by theorem 3.8(3)). \( b \cap Int(A) \cap b \cap Fr(A) \subseteq b \cap Cl(A) \cap b \cap Cl(A) \) (by theorem 3.11(3)). \( b \cap Fr(A) \subseteq b \cap Cl(A) \cap b \cap Cl(A) \) (by theorem 3.8(3),(9)) which is \( b \cap Fr(A) \).

6) Let \( x \in b \cap b(A) \). Then \( x \in A \setminus b \cap Int(A) \).

**Theorem 4.10:** Let \( A \) be a subset of a space \( X \). Then the \( b \cap -Exterior \) of \( A \) is defined by \( b \cap Ext(A) = b \cap Int(X \setminus A) \).

**Definition 4.9:** If \( A \) be a subset of a space \( X \). Then the \( b \cap -Exterior \) of \( A \) is defined by \( b \cap Ext(A) = b \cap Int(X \setminus A) \).

**Theorem 4.10:** Let \( A \) be a subset of a space \( X \). Then the following statements hold:

1) \( b \cap Ext(\emptyset) = X \);
2) \( b \cap Ext(X) = \emptyset \);
3) \( Ext(A) = b \cap Ext(A) \);
4) \( b \cap Ext(A) = X \setminus b \cap Cl(A) \);
5) \( A \) is closed iff \( b \cap Ext(A) = X \setminus A \);
6) \( A \subseteq B \), then \( b \cap Ext(A) \supseteq b \cap Ext(B) \);
7) $b \tilde{g} \text{ Ext}(A \cup B) \subseteq b \tilde{g} \text{ Ext}(A) \cap b \tilde{g} \text{ Ext}(B);
8) b \tilde{g} \text{ Ext}(A \cap B) \supseteq b \tilde{g} \text{ Ext}(A) \cup b \tilde{g} \text{ Ext}(B)$
9) $b \tilde{g} \text{ Ext}(A)$ is $b \tilde{g}$-open;
10) $b \tilde{g} \text{ Ext}(X \setminus b \tilde{g} \text{ Ext}(A)) = b \tilde{g} \text{ Ext}(A)$;
11) $b \tilde{g} \text{ Ext}(b \tilde{g} \text{ Ext}(A)) = b \tilde{g} \text{ Int}(b \tilde{g} \text{ Cl}(A));$
12) $b \tilde{g} \text{ Int}(A) \subseteq b \tilde{g} \text{ Ext}(b \tilde{g} \text{ Ext}(A));$
13) $X = b \tilde{g} \text{ Int}(A) \cup b \tilde{g} \text{ Ext}(A) \cup b \tilde{g} \text{ Fr}(A)$.

Proof:
1) $b \tilde{g} \text{ Ext}(\emptyset) = b \tilde{g} \text{ Int}(X \setminus A) = b \tilde{g} \text{ Int}(X) = X$ (by theorem 3.8 (2)).
2) $b \tilde{g} \text{ Ext}(X) = b \tilde{g} \text{ Int}(X \setminus X) = b \tilde{g} \text{ Int}(\emptyset) = \emptyset$ (by theorem 3.8(1)).
3) Let $x \in \text{ Ext}(A)$. Then by definition 2.2 (3), $x \in \text{ Int}(X \setminus A)$. By theorem 3.7, $x \in \text{ Int}(X \setminus A) = b \tilde{g} \text{ Ext}(A)$. Hence, $\text{ Ext}(A) \subseteq b \tilde{g} \text{ Ext}(A)$.
4) Let $x \in b \tilde{g} \text{ Ext}(A) \iff x \in b \tilde{g} \text{ Int}(X \setminus A) \iff x \in X \setminus b \tilde{g} \text{ Cl}(A)$ (by result 4.3 (2)). Hence, $b \tilde{g} \text{ Ext}(A) = X \setminus b \tilde{g} \text{ Cl}(A)$.
5) Let $A$ be $b \tilde{g}$-closed. Then $X \setminus A$ is $b \tilde{g}$-open. By remark 3.6, $b \tilde{g} \text{ Int}(X \setminus A) = X \setminus A$. Conversely, let $b \tilde{g} \text{ Ext}(A) = X \setminus A$. Then $b \tilde{g} \text{ Int}(X \setminus A) = X \setminus A$. Again by remark 3.6, $X \setminus A$ is $b \tilde{g}$-open. Hence, $A$ is $b \tilde{g}$-closed.
6) $b \tilde{g} \text{ Ext}(A) = b \tilde{g} \text{ Int}(X \setminus A) = X \setminus b \tilde{g} \text{ Cl}(A)$ (by result 4.3). $X \setminus b \tilde{g} \text{ Cl}(B)$ (since $A \subseteq B$ and by theorem 3.11(4)), $b \tilde{g} \text{ Int}(X \setminus B) = b \tilde{g} \text{ Ext}(B)$ (by definition 4.9). Hence, $b \tilde{g} \text{ Ext}(A) \subseteq b \tilde{g} \text{ Ext}(B)$.
7) Since $A \subseteq A \cup B$ and by (6), $b \tilde{g} \text{ Ext}(A \cup B) \subseteq b \tilde{g} \text{ Ext}(A)$. Similarly since $B \subseteq A \cup B$ and by (6), $b \tilde{g} \text{ Ext}(A \cup B) \subseteq b \tilde{g} \text{ Ext}(B)$. Hence, $b \tilde{g} \text{ Ext}(A \cup B) \subseteq b \tilde{g} \text{ Ext}(A \cup B)$.
8) Since $A \cap B \subseteq A$ and by (6), $b \tilde{g} \text{ Ext}(A \cap B) \subseteq b \tilde{g} \text{ Ext}(A \cap B)$. Similarly since $A \cap B \subseteq B$ and by (6), $b \tilde{g} \text{ Ext}(A \cap B) \subseteq b \tilde{g} \text{ Ext}(A \cap B)$. Hence, $b \tilde{g} \text{ Ext}(A) \cup b \tilde{g} \text{ Ext}(B) = b \tilde{g} \text{ Ext}(A \cap B)$.
9) Follows from definition 4.9 and theorem 3.8(2).
10) $b \tilde{g} \text{ Ext}(X \setminus b \tilde{g} \text{ Ext}(A)) = b \tilde{g} \text{ Ext}(X \setminus b \tilde{g} \text{ Int}(X \setminus A)) = b \tilde{g} \text{ Int}(b \tilde{g} \text{ Int}(X \setminus A)) = b \tilde{g} \text{ Int}(X \setminus A)$ (by theorem 3.8(9)) which is $b \tilde{g} \text{ Ext}(A)$. Hence, $b \tilde{g} \text{ Ext}(X \setminus b \tilde{g} \text{ Ext}(A)) = b \tilde{g} \text{ Ext}(A)$.

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