3 – GRAPHOIDAL PATH COVERS

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Abstract

The concept of graphoidal cover was introduced by B.D. Acharya and E. SampathKumar [1]. The concept of 2-graphoidal path cover was introduced by K.Nagarajan [7] et.al,. The concept of 3-graphoidal cover was introduced by T.Gayathri [4] et.al., A 3-graphoidal path cover of a graph G is a collection of paths in G such that every path in G has at least two vertices, every vertex of G is an internal vertex of at most three paths and every edge of G is in exactly one path. The minimum cardinality of it is a 3-graphoidal path covering number η_{3a}(G). In this paper we find η_{3a}(G) for some special graphs.

Keywords

Graphoidal cover, Acyclic graphoidal, 2-Graphoidal cover and 3-Graphoidal cover.

1. Introduction

A graph is a pair G=(V,E) , where V is the set of vertices and E is the set of edges. We consider only non-trivial, finite, connected, undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. For graph theoretic terminology we refer to Harary [5]. The concept of graphoidal cover was introduced by B.D Acharya and E.Sampathkumar[1] and the concept of acyclic graphoidal cover was introduced by Arumugam and Suresh Suseela[2]. The concept of 2-graphoidal path cover was introduced by P.K. Das and K.Ratan Singh[3].

Let P=(v_1 ,v_2 ,v_3 ,…,v_{n-1}, v_n) be a path in a graph G=(V,E). Then the vertices (v_2 ,v_3 ,…,v_{n-1}) are called internal vertices of P and v_1 and v_n are called external vertices of P. IfP=(v_1 ,v_2 ,…,v_{n-1}, v_n = v_1) is a closed path then v_1 may be taken as the external vertex. Let Ψ be the collection of internally edge disjoint paths in G. A vertex of G is said to be an internal vertex of Ψ if it is an internal vertex of some path(s) in Ψ otherwise it is called an external vertex of Ψ. The number of internal vertices of a path P in Ψ is denoted by i_Ψ(P); the number of internal vertices which appear exactly once in a path of Ψ by t_1(Ψ) and max t_1(Ψ)=t_1; the number of internal vertices which appear exactly twice in two paths of Ψ by t_2(Ψ) and max t_2(Ψ)= t_2 ; the number of internal vertices which appear exactly thrice in three paths of Ψ by t_3(Ψ) and max t_3(Ψ)= t_3 and the number of external vertices by t_Ψ and min t_Ψ = t. The reader may refer [5] and [1] for the terms not defined here.

1.1 Definition

Duplication of a vertex of a graph G produces a new graph G' by adding a new vertex v'such that N(v)=N(v'). In other words , a vertex v' is said to be duplication of v if all the vertices which are adjacent to v in G is also adjacent to v' in G'. The graph abtained by duplication of all the vertices of G is denoted by D(G).

1.2 Definition

An Udukkai graph U_n, n≥ 2 is a graph constructed by joining two fan gragh F_n, n≥ 2 with two paths P_n, n≥ 2 by sharing a common vertex at the centre.

1.3 Definition

An Octopus graph O_n, n≥ 2, can be constructed by joining a fan graph F_n, n≥ 2 with a star graph $K_{1,n}$, by sharing a common vertex which is the centre of the star.

1.4 Definition

A Butterfly graph C_n:C_m@W_K denote the two cycles of order m and n sharing a common vertex with k pendant edges attached at the common vertex.

1.5 Definition

The graph C_m@$K_{1,n}$ is obtained from the cycle C_m by attaching the root vertex of the star $K_{1,n}$ to any one vertex of the cycle C_m.

1.6 Definition

The graph W_{n+1}@P_m is obtained from the wheel $W_{n+1}$by attaching a terminal vertex of P_m..

1.7 Theorem:[3]

If a graph with $\delta \geq 3$, then there exixts a graphoidal cover Ψ of G such that every vertex of G is an internal vertex of some paths in Ψ.

1.8 Remark:[3]

If $\Delta \leq 3$, then t_2= 0 and hence

η_{2a} (G)= (G) , where is the minimum graphoidal covering number.

1.9 Theorem:[4]

For any graph with $\delta = 1$ and $\Delta \geq 6$, $\eta_{3a} \geq | q - p - (t_1 + t_2 + t_3) + t |$.

1.10 Definition

A 3-graphoidal path cover of a graph G is a collection Ψ of paths in G such that

i. Every path inΨ has at least two vertices.
ii. Every vertex of G is an internal vertex of at most three paths in Ψ.
iii. Every edge of G is in exactly one path in Ψ.
The minimum cardinality of a 3-graphoidal cover of G is called the 3-graphoidal covering number of G and it is denoted by η_{3a}(G) or η_{3a}.

2. MAIN RESULTS

2.1 Proposition

Let G be a C_m@$K_{1,n}$ graph, then

$$\eta_{3a}(G)= \begin{cases} n & if\ n = 4,5 \\ n-1 & if\ n \geq 6 \end{cases}$$

Proof

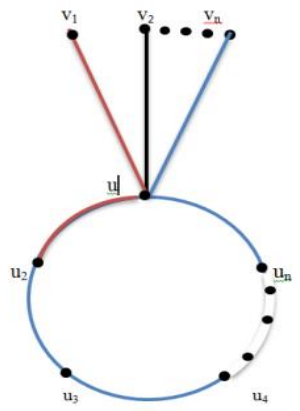

Figure 1

Consider, C_m@$K_{1,n}$ as in figure 1.

For n=4,5 $\eta_{2a} = \eta_{3a}$=5.

For n≥6, q=m+n, p= m+n , $\Delta \geq 6$, $t_1 = t_2 = 0$, $t_3 \leq 1$, $t \geq n$.

Then, $\eta_{3a} \geq | q - p - (t_1 + t_2 + t_3) + t |$.

$\geq |m+n - (m+n) - (0+0+1) + n |$

$\eta_{3a} \geq n-1$.

Let $P_1 = \langle v_1, u_1, \ldots, u_m \rangle$

$P_2 = \langle v_2, u_1, u_m \rangle$, $P_3 = \langle v_3, u_1, v_4 \rangle$

The paths P_1, P_2, P_3 together with the remaining edges form a 3-graphoidal path cover Ψ of C_m@$K_{1,n}$

$|\Psi| = 3 + |E(C_m@K_{1,n}) - \{E(P_1) + E(P_2) + E(P_3)\}|$

$= 3 + |(m+n) - \{(m-2-2)\}|$

$= 3 + |n-4| = n-1$

$\eta_{3a} \leq n-1$.

Hence, η_{3a}=n-1.

2.2 Proposition

Let G be a W_{n+1}@P_m graph, then

η_{3a}(G)= n-2 if $n \geq 4$

Proof

Consider $W_{n+1}@P_m$,

Here, $\Delta \geq 6$, q=m+2n-1, p = m+n +1, $t_1=0$ $t_2 \leq 1$, $t_3 \leq 1$, $t \geq 1$.

Then, $\eta_{3a} \geq | q - p - (t_1 + t_2 + t_3) + t |$.

$\geq |m+2n-1 - (m+n) - (0+1+1)+1 |$

$\eta_{3a} \geq$n-2.

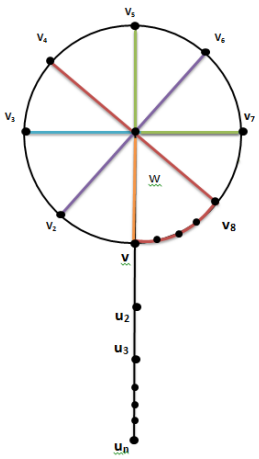

Figure 2

Let P_1= <$v_1,v_2,\ldots v_9,w,v_{10}$>

P_2=<$v_9,..,v_n,v_1,w,v_2$ >, P_3=<v_3,w,v_4>

The paths P_1, P_2, P_3 together with the remaining edges form a 3-graphoidal path cover Ψ of G

$|\Psi|=3+|E(G)-\{E(P_1)+E(P_2)+E(P_3)\}|$

$=3+|(m+2n-1)-\{(m+n+4)\}|$

$=3+|n-5| =n-2 \Rightarrow \eta_{3a} \leq$ n-2.

Hence, η_{3a}=n-2.

2.3 Proposition

Let G denote the Franklin graph with 12 vertices and 18 edges as given in figure. Then η_{3a}(G)= 6.

Proof

Here, $\Delta \geq 6$; q=18, p= 12 , $t_1=0$, $t_2=0$, $t_3 \leq 1$ since the graph forms a cycle,at least one vertex must appear as an external vertex $t \geq 1$.

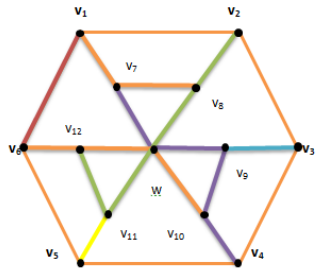

Figure 3

Then, $\eta_{3a} \geq | q - p - (t_1 + t_2 + t_3) + t |$.

$\geq | 18 - 12 - (0+0+1)+1 |$

$\eta_{3a} \geq 6$.

P_1= <v_8,v_7,v_1,v_2 ,v_3,v_4v_5,v_6, v_{12},w,v_{10}>

P_2=<v_{12},v_{11},w,v_8,v_2 >,P_3=<v_7,w,v_9, v_{10} ,v_4>

P_1= <v_1,v_6> ,P_2=<v_5,v_{11}> ,P_3=<v_3,v_9>

we have, $\eta_{3a} \leq 6$

Hence, η_{3a} =6.

2.4 Proposition

Let G be a D($K_{1,n}$) graph. Then η_{3a} (G)= 2n-3.

Proof

Let $u_1,u_2,\ldots,u_n$ be the vertices of the star graph and u be its centre. Then $u'_1,u'_2,\ldots,u'_n$ be its duplication and u' be the duplication of u.

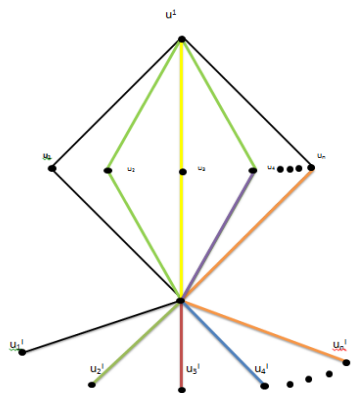

Figure 4

Since, d(u)=2n , we have $t_3 \leq 1$, p=2n+2, q=3n , $t \geq n$

$\eta_{3a} \geq | q - p - (t_1 + t_2 + t_3) + t |$.

$\geq | 3n - 2n-2 - (0+0+1)+n|$

≥2n-3.

η_{3a}≥2n-3

Let P_1= <$u_{1'}$,u,u_1,u',u_n>

P_2=<$u_{2'}$,u,u_2,u',u_{n-1}>

P_3=<$u_{3'}$,u,u_3,u',u_{n-2}> , P_4=<u,u_4,u' >

P_5=<u,u_5,u' >…..P_{n-3}=<u,u_{n-3},u' >

The paths P_1, P_2,…, P_{n-3} together with the remaining edges form a 3-graphoidal path cover Ψ of $D(K_{1,n})$

|Ψ|=n-3+|E(D($K_{1,n}$)-{E(P_1)+E(P_2)+E(P_3)}|

=n-3+|(3n)-{(12+2(n-6)}|

=n-3+|n| =2n-3.

η_{3a} ≤ 2n-3.

Hence, η_{3a} = 2n-3.

2.5 Proposition

For a graph U_n, n≥ 2, η_{3a}(U_n)= 2n-1

Proof

Here Δ≥ 6, q=6n-4, p = 4n-1, t_1=0 t_2=0, t_3≤ 1 , since the graph contains cycle, at least one vertex must appear as an external vertex, t ≥ 3.

Then, η_{3a}≥ | q – p – (t_1+ t_2+ t_3)+t |.

≥|6n-4-4n+1- (0+0+1)+3 |

η_{3a}≥2n-1.

Now, P_1= <v_1,v_2,…v_n,w,$t_1t_1t_2$,…,t_n>

P_2=<s_n,s_{n-1},…s_1,w,u_1,u_2,…,u_n>

P_3=<u_1,w,v_n>

The paths P_1, P_2, P_3 together with the remaining edges form a 3-graphoidal path cover Ψ of U_n.

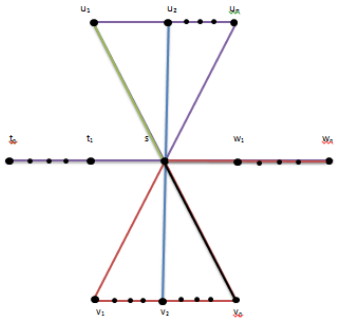

Figure 5

| Ψ|=3+|E(U_n)-{E(P_1)+E(P_2)+E(P_3)}|

=3+|(6n-4)-{(4n)}|

=3+|2n-4| =2n-1.

η_{3a} ≤ 2n-1.

Hence, η_{3a}(U_n)= 2n-1

2.6 Proposition

For a graph O_n, n≥ 2 , η_{3a}(O_n)= 2n-3

Proof

Here Δ≥ 6, q=3n-1, p = 2n+1, t_1=0 t_2=0, t_3≤ 1 , t ≥ n.

Then, η_{3a}≥ | q – p – (t_1+ t_2+ t_3)+t |.

≥|3n-1-(2n+1)- (0+0+1)+n |

η_{3a}≥2n-3.

Let P_1= <u_n,u_{n-1},…u_1,w,v_1>

P_2= <u_1,w,v_n> , P_3= <u_n,w,v_1>

The paths P_1, P_2, P_3 together with the remaining edges form a 3-graphoidal path cover Ψ of O_n.

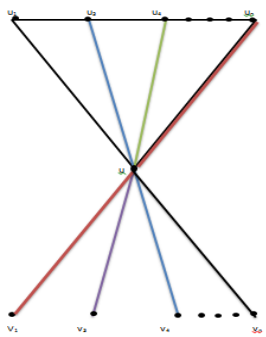

Figure 6

|Ψ|=3+|E(O_n)-{E(P_1)+E(P_2)+E(P_3)}|

$=3+|(3n-1)-\{(n+5)\}|$

$=3+|2n-6| =2n-3.$

$\eta_{3a} \leq 2n-3.$

Hence, $\eta_{3a}(O_n)= 2n-3.$

2.7 Proposition

For a graph $F_{2,n}$, $\eta_{3a}(F_{2,n})= 2n-3$

Proof

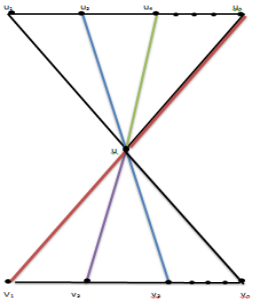

Figure 7

Here $\Delta \geq 6$, $q=4n-2$, $p = 2n+1$, $t_1=0$ $t_2=0$, $t_3 \leq 1$, since the graph contains cycles at least one vertex must appear as an external vertex, $t \geq 1$.

Then, $\eta_{3a} \geq | q - p - (t_1+ t_2+ t_3)+t |.$

$\geq |4n-2-(2n+1)- (0+0+1)+1 |$

$\eta_{3a} \geq 2n-3.$

Now, $P_1= \langle u_n,u_{n-1},\ldots u_1,u,v_n,v_{n-1},\ldots,v_1\rangle$

$P_2= \langle u_2,u,v_3\rangle$, $P_3= \langle u_n,u,v_1\rangle$

The paths P_1, P_2, P_3 together with the remaining edges form a 3-graphoidal path cover Ψ of $F_{2,n}$.

$|\Psi|=3+|E(F_{2,n})-\{E(P_1)+E(P_2)+E(P_3)\}|$

$=3+|(4n-2)-\{(2n+2+2)\}|$

$=3+|2n-6| =2n-3$

$\eta_{3a}(F_{2,n}) \leq 2n-3.$

Hence, $\eta_{3a}(F_{2,n})= 2n-3.$

2.8 Proposition

Let G be a butterfly graph $C_m{:}C_n@W_k$, then $\eta_{3a}(C_m{:}C_n@W_k)= k+1.$

Proof

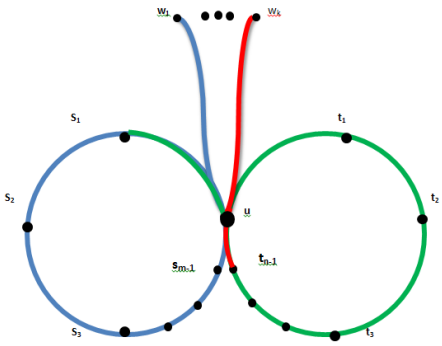

Figure 8

Here $\Delta \geq 6$, $q= m+n+k$, $p = m+n+k-1$, $t_1=0$ $t_2=0$, $t_3 \geq 1$, since the graph contains cycle, at least one vertex must appear as Nexternal vertex in the path, $t \leq k + 1$.

$\eta_{3a} \geq | q - p - (t_1+ t_2+ t_3)+t |.$

$\geq |m+n+k-(m+n+k-1)-(0+0+1)+k+1|$

$\eta_{3a} \geq k+1.$

$P_1= (w_1,u,s_{n-1},\ldots,s_1)$ $P_2= (s_1,u,t_1,\ldots,t_{n-1})$

$P_3= (w_k,u,t_{n-1})$

The paths P_1, P_2, P_3 together with the remaining edges form a 3-graphoidal path cover Ψ of $C_m{:}C_n@W_k$.

$|\Psi|=3+|E(G)-\{E(P_1)+E(P_2)+E(P_3)\}|$

$=3+|(m+n+k)-\{(m+n+2)\}|$

$=3+|k-2| =k+1.$

$\eta_{3a}(C_m{:}C_n@W_k) \leq k+1.$

Hence, $\eta_{3a}(C_m{:}C_n@W_k)=k+1.$

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