IFP-IDEALS IN NEAR SUBTRACTION SEMIGROUPS

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Abstract:
In this paper we introduced the concept of IFP-ideals in near subtraction semigroup, i.e) X is said to fulfill the insertion-of-factors property (IFP) provided that for all a, b, x \in X: ab = 0, implies axb = 0. Such a near subtraction semigroups are called IFP-near subtraction semigroups. An ideal P of a near subtraction semigroup X is called an IFP-ideal if it is provided that ab \in P \forall a, b \in X implies axb \in P, \forall x \in X.

Introduction

B.M. Schein [5] considered systems of the form (X; \circ, \cdot), where X is a set of functions closed under the composition “\circ” of functions (and hence (X;\circ) is a function semigroup) and the set theoretic subtraction “\cdot” (and hence (X;\cdot) is a subtraction algebra in the sense of [1]). He proved that every subtraction semigroup is isomorphic to a difference semigroup of invertible functions. B.Zelinka [6] discussed a problem proposed by B.M. Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. For basic definition one may refer to Pilz [4]. Motivated by the study of IFP-ideal in near rings by Akin Osman Atagun. We introduced the new concept IFP-ideals in near subtraction semigroups.

2. Preliminary definitions and results

Throughout X will denote a right near subtraction semigroup. It is assumed that the reader is familiar with the basic definitions of right near subtraction semigroup, zero-symmetric near subtraction semigroup, and ideal.

Definition 2.1:

A non-empty set X together with binary operation “\cdot” and is said to be subtraction algebra if satisfies the following:

1) \( x - (y - x) = x \).
2) \( x - (x - y) = y - (y - x) \).
3) \( (x - y) - z = (x - z) - y \ \forall x, y, z \in X \).
Definition 2.2:

A non-empty set $X$ together with two binary operations “-” and “•” is said to be a subtraction semigroup if it satisfies the following:

1) $(X, -)$ is a subtraction algebra.
2) $(X, •)$ is a semigroup.
3) $x(y - z) = xy - xz$ and $(x - y)z = xz - yz, \forall x, y, z \in X$

Definition 2.3:

A non-empty set $X$ together with two binary operations “-” and “•” is said to be a near subtraction semigroup (right) if it satisfies the following:

1) $(X, -)$ is a subtraction algebra.
2) $(X, •)$ is a semigroup.
3) $(x - y)z = xz - yz, \forall x, y, z \in X$.

Definition 2.4:

If $X$ is said to fulfill the insertion-of-property (IFP) provided that for all $a, b, n \in X$: $ab = 0$ implies $anb = 0$. Such near subtraction semigroup are called IFP-near subtraction semigroups.

Definition 2.5:

If an ideal $P$ of a near subtraction semigroup $X$ is called an IFP-ideal if it is provided that $ab \in P, \forall a, b \in X$ implies $anb \in P, \forall n \in X$.

Definition 2.6:

The ideal $P$ of $X$ is called a 0-prime ideal if for every $A, B \lhd X, AB \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$.

Definition 2.7:

If $P \lhd X$ is called a 3-prime (3-semiprime) ideal if for $a,b \in X$, $aXb \subseteq P$ $(aXa \subseteq P)$ implies $a \in P$ or $b \in P$ $(a \in P)$.

Definition 2.8:

If for $a,b \in X$, $ab \in P$ $(a \in P)$ implies $a \in P$ or $b \in P$ $(a \in P)$, then $P \lhd X$ is called a completely prime (completely semi prime) ideal.

Definition 2.9:

If the zero ideal of $X$ is $v$-prime $(v = 0, 3, \text{completely})$, then $X$ is called a $v$-prime near subtraction semigroups.

Lemma 2.10:

We have shown that a reduced near subtraction semigroup then $X$ is weakly regular iff every ideal is completely semiprime and $X \setminus P$ is left weakly regular for all prime ideal $P$ of $X$.

Lemma 2.11:

Let $X$ be a finite near subtraction semigroup

1) If $X$ is a zero symmetric right permut able near subtraction semigroup, then $X$ is a near-field iff $X$ is an equiprime near subtraction semigroup.
2) If $X$ is a left permut able near subtraction semigroup, then $X$ is a near-field iff $X$ is an equiprime near subtraction semigroup.

It easily seen that any completely prime near subtraction semigroup has no non-zero nilpotent elements. If for all $a, b, c, d \in X$, (resp. $abc = bac$, $abcd = acbd$), then $X$ is called a right permut able (resp. left permut able, medial) near subtraction semigroup. If $abc = abac$ (resp. $abc = acbc$), then...
X is called a left self distributive (resp. right self distributive) near subtraction semigroup.

For some definitions of strongly regular near subtraction semigroup, reduced subtraction semigroup, Boolean near subtraction semigroup and p-near subtraction semigroup, left weakly regular near subtraction semigroup, left (right) strongly regular near subtraction semigroups.

3. Prime ideals and IFP ideals

Proposition 3.1:

If P is an IFP-ideal and a 3-(semi) prime ideal of X, then P is a completely (semi) prime ideal.

Proof:

Let ab ∈ P for a, b ∈ X. Since P is an IFP-ideal, then aXb ⊆ P. Hence a ∈ P or b ∈ P, since P is a 3-prime ideal. Therefore P is a completely prime ideal. To prove the semiprime case, it is enough to take a = b.

From now on, all near subtraction semigroup will be zero-symmetric in this section.

Proposition 3.2:

Let P be a completely semiprime ideal of X. Then P is an IFP-ideal.

Proof:

Assume P is a completely semiprime ideal of X and ab ∈ P for a, b ∈ X. It is easily seen that XP ⊆ P, since X is zero-symmetric. Then (ba)^2 = baba ∈ XPX ⊆ P and then ba ∈ P since P is completely semiprime. Hence (anb)^2 = anbanb ∈ XPX ⊆ P for all n ∈ X, whence anb ∈ P, since P is completely semiprime. Therefore P is an IFP-ideal.

Corollary 3.3:

Let P be a completely prime ideal of X. Then P is an IFP-ideal.

Proof:

If P is a completely prime ideal of X, then it is completely semiprime. Hence the result follows proposition 3.5.

4. IFP ideals occurring naturally in some near subtraction semigroups

The class consisting of zero-symmetric near subtraction semigroups (resp., consisting of the near subtraction semigroups with identity) will be denoted by R_0 (resp., R_1).

Proposition 4.1:

Let X ∈ R_0 be a Boolean near subtraction semigroup and P ⊲ X. Then P is an IFP-ideal.

Proof:

Assume ab ∈ P for a, b ∈ X. Since ba = (ba)^2 = baba ∈ XPX ⊆ P, then anb = (anb)^2 = anbanb ∈ XPX ⊆ P for all n ∈ X. Therefore P is an IFP-ideal of X.

Proposition 4.2:

Let X ∈ R_0 be a p-near subtraction semigroup and P ⊲ X. Then P is an IFP-ideal.

Proof:

If p=2, the result follows Proposition 4.1. Assume p > 2 and that ab ∈ P for a, b ∈ X. Since ba = (ba)^p ∈ XPX ⊆ P, then anb = (anb)^p ∈ XPX ∈ P for all n ∈ X. Therefore P is an IFP-ideal of X.
We have the following:

**Proposition 4.3:**

Let \( X \) be an IFP near subtraction semigroup. Then for all \( x \in X \), \((0:x)\) is an IFP-ideal of \( X \).

**Proof:**

Clearly \((0:x) \trianglelefteq X\) for all \( x \in X \) when \( X \) is an IFP near subtraction semigroup. Let \( ab \in (0:x) \) for all \( a, b \in X \). Then \( abx = 0 \). Since \( X \) is an IFP near subtraction semigroup, then \( an(bx) = 0 \) for all \( n \in X \). Hence \( anb \in (0:x) \) for all \( n \in X \), i.e \((0:x)\) is an IFP-ideal of \( X \).

**Proposition 4.5:**

If \( P \) is an IFP-ideal of a near subtraction semigroup \( X \), then \((P:P)\) is an ideal of \( X \). Furthermore \((P:P)\) is also an IFP-ideal.

**Proof:**

To prove \((P:P) \trianglelefteq X\), it is enough to show that \((P:P)X \subseteq (P:P)\). Let \( y \in (P:P)X\). Then there exist \( a \in (P:P) \) and \( n \in X \) such that \( y = an \). Since \( a \in (P:P) \), then \( ap \in P \) for all \( p \in P \). Since \( P \) is an IFP-ideal, then \( anp \in P \) for all \( n \in X \). Then \( yp \in P \) for all \( p \in P \). Hence \( y \in (P:P) \). Now, we show that \((P:P)\) is an IFP-ideal. Assume \( xy \in (P:P) \) for \( x, y \in X \). Then \( xyp \in P \) for all \( p \in P \). Since \( P \) is an IFP-ideal, \( xyp \in P \) for all \( n \in X \) and for all \( p \in P \). Therefore, \( xyp \in (P:P) \), which completes the proof.

**Proposition 4.6:**

Let \( X \in R_0 \cap R_1 \) be a reduced left weakly regular near subtraction semigroup and let \( P \trianglelefteq X \). Then \( P \) is an IFP-ideal of \( X \).

**Proof:**

The result follows from Lemma 2.10 and Proposition 3.5.

**Proposition 4.7:**

Let \( P \trianglelefteq X \). Then,

a) If \( X \) is right permutable, then \( P \) is an IFP-ideal.

b) If \( X \) is left permutable, then \( P \) is an IFP-ideal.

c) If \( X \) is right self distributive, then \( P \) is an IFP-ideal.

d) If \( X \in R_0 \) is left self distributive, then \( P \) is an IFP-ideal.

**Proof:**

For \( a, b \in X \), assume \( ab \in P \). Then for all \( n \in X \),

a) \( anb = abn \in PX \subseteq P \).

b) \( anb = nab \in XP \subseteq P \), since \( X \in R_0 \) [Lemma 2.11]

c) \( anb = abn \in PX \subseteq P \).

d) \( anb = anab \in XP \subseteq P \), since \( X \in R_0 \) [Lemma 2.11]

**Proposition 4.8:**

Let \( X \) be a medial near subtraction semigroup and \( P \trianglelefteq X \). Then,

a) If \( X \) is regular, then \( P \) is an IFP-ideal.

b) If \( X \) is right strongly regular, then \( P \) is an IFP-ideal.

c) If \( X \in R_0 \) is left strongly regular, then \( P \) is an IFP-ideal.

**Proof:**

For \( x, y \in X \), assume \( xy \in P \).
a) Since X is regular, then there exist a, b ∈ X such that x = xax and y = yby. Then for all n ∈ X, xny = xaxnyby = x(axn)y(by). Since X is medial, then xny = xy(axn)by ∈ PN ⊆ P.

b) Since X is right strongly regular, then there exist a, b ∈ X such that x = x^2a and y = y^2b. Then for all n ∈ X, xny = xaxnyby = x(xan)y(by). Since X is medial, then xny = xy(xan)(by) ∈ PX ⊆ P.

c) Since X ∈ R_0 is left strongly regular, then there exist a, b ∈ X such that x = ax^2 and y = by^2. Then for all n ∈ X, xny = axxnbyy = (ax)(xnb)y(y). Since X is medial, then xny = (ax)y(xnb)y = a(xy)xnbxy ∈ XPX ⊆ P.

Reference:


