Potential of Crank Nicolson Finite Difference Scheme and Variational terationmethod to Solve Time Fractional Mobile-Immobile Equation

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Abstract: In the present paper we develop the Crank-Nicolson finite difference scheme for time fractional Mobile Immobile equation (MIM). Also, we prove solution of the scheme is unconditionally stable and convergent. Furthermore, the same equation is solved by the variational iteration method. The results obtained from both the schemes are compared for better understanding. A numerical example proves the accuracy of the developed scheme.

IndexTerms: Time fractional Mobile Immobile equation, Crank Nicolson Finite differencescheme,Variational Iteration Method, Mathematica

I. INTRODUCTION

Fractional calculus is useful mathematical tool for applied sciences but it is,somehowhard to tackle and become the topic of active research among the researchers. Nowdays, number of authors have study time and space fractional diffusion equations and contributed to the field [**1-17**]. In the present scenario fractional calculus plays animportant role in the various fields of scientific and engineering problems. Fractionalcalculus is a field of mathematical study that grows out of the traditional definitionsof the calculus integral and derivative operators[**2**]. The increase of the subject iswitnessed by series of conferences, research papers. Fractional diffusion equationshave been used in modeling turbulent flow, chaotic dynamics of classical conservativesystem, groundwater contaminant transport, and applications in biology, physics,chemistry, finance etc. Advection diffusion equation has a wide range of practicaland industrial applications[**8**]. Solute transport in rivers, streams and ground wateris controlled by the physical features or heterogeneity in different reaches[**7**]. Theadvection diffusion equation and its extensions Mobile-Immobile or transient storagemodels have taken attention of hydrologist on study the transport in differentmedia/zones [6, 7,8]. Solute transport in fractured rocks is of major interest in manyapplications from the petroleum industry to groundwater management[**3**]. HosseinPourbashash[**8**] discussed the spectral method for solving time fractional MIM. TheMIM approach is based on a simple hypothesis: not all the pore spaces in a geological medium contribute to global flow. But, most of the fractional differential equationsdo not have exact analytical solutions; hence approximation and numerical techniquesmust be used. Therefore, fractional diffusion equations have been topic of interest,studied by many researchers for developing fractional order finite difference schemes.With the importance of the advection diffusion equation, the present paper solvesand analyzes MIM equation using Finite difference scheme and variational iterationmethod.In the year 1967, Caputo, introduced new and useful definition of fractional derivativepopularly known as Caputo fractional derivative. We define it as [**1**, **8**] follows

Definition 1.1 *The Caputo time fractional derivative of order α,* $(0 < \alpha \le 1)$ *isdefined as follows.*

$$\frac{\partial^\alpha U(x,t)}{\partial t^\alpha} = \begin{cases} \dfrac{1}{\Gamma(1-\alpha)} \displaystyle\int_0^t \frac{\partial U(x,t)}{\partial \xi} \frac{d\xi}{(t-\xi)^\alpha}, & 0 < \alpha < 1 \\ \dfrac{\partial U(x,t)}{\partial t}, & \alpha = 1 \end{cases}$$

Where, Γ(.) is a Gamma function.

We organize this paper as follows: In section 2, The Crank-Nicolson fractional orderfinite difference scheme for time fractional Mobile Immobile equation is developed.The stability of the solution is proved in section 3, the concept of convergence isdiscussed in section 4. In section 5, the numerical solution of time fractional MobileImmobile equation is obtained and it is represented graphically by Mathematica softwareand in the last section the solution of mobile immobile equation is obtained by
the variational iteration method.The comparison of the solution by both the schemesis also studied.

II. FINITE DIFFERENCE SCHEME

We consider the following time fractional Mobile Immobile equation (TFMIME) withinitial and boundary conditions

$$A\frac{\partial u(x,t)}{\partial t} + B\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = -V\frac{\partial u(x,t)}{\partial x} + D\frac{\partial^2 u(x,t)}{\partial x^2} + f(x,t), (x,t) \in [0,L] \times [0,T], 0 < \alpha \le 1, t > 0 \quad (2.1)$$

Initial Condition: u(x,0) = g(x), $0 \le x \le L$ (2.2)

Boundary Condition: u(0,t) = $\emptyset_1(t)$, u(L, t) = $\emptyset_2(t)$, t≥ 0 (2.3)

For the time fractional order Crank-Nicolson implicit numerical approximation scheme we define $h = \frac{x_R - x_L}{N} = \frac{L}{N}$ and$= \frac{T}{N}$, the space and time steps respectively, suchthat $t_k = k\tau$; k = 0,1,...,N,& $0 \le t_k \le T$ and $x_i = x_L + ih$ for i = 0,1, ..., N.

Define $u_i^k = u(x_i, t_k)$ and let u_i^kdenote the numerical approximation tothe exact solution $u(x_i, t_k)$.

In the differential equation (**2.1**), the time fractional derivative term is approximatedby the following scheme,

$$\frac{\partial^\alpha u(x_i, t_{k+1})}{\partial t^\alpha} \approx \frac{1}{\Gamma(1-\alpha)} \int_0^{t_{k+1}} \frac{1}{(t_{k+1}-\xi)^\alpha} \frac{\partial u(x_i,\xi)}{\partial \xi} \partial\xi$$

$$= \frac{1}{\Gamma(1-\alpha)} \sum_{j=0}^{k} \frac{u(x_i,t_{j+1})-u(x_i,t_j)}{\Delta t} \int_{j\tau}^{(j+1)\tau} \frac{1}{(t_{k+1}-\xi)^{\alpha}}\, \partial\xi$$

$$= \frac{1}{\Gamma(1-\alpha)} \sum_{j=0}^{k} \frac{u(x_i,t_{j+1})-u(x_i,t_j)}{\Delta t} \int_{j\tau}^{(j+1)\tau} \frac{1}{(t_{k+1}-\xi)^{\alpha}}\, \partial\xi$$

$$= \frac{1}{\Gamma(1-\alpha)} \sum_{j=0}^{k} \frac{u(x_i,t_{j+1})-u(x_i,t_j)}{\tau} \int_{(k-j)\tau}^{(k+1-j)\tau} \frac{d\eta}{\eta^{\alpha}}$$

$$= \frac{\tau^{1-\alpha}}{\Gamma(2-\alpha)} \sum_{j=0}^{k} \frac{u(x_i,t_{k+1-j})-u(x_i,t_{k-j})}{\tau} \times [(j+1)^{1-\alpha}-j^{1-\alpha}]$$

$$\frac{\partial^{\alpha} u(x_i,t_{k+1})}{\partial t^{\alpha}} = \frac{\tau^{-\alpha}}{\Gamma(2-\alpha)}[u_i^{k+1}-u_i^{k}] + \frac{\tau^{-\alpha}}{\Gamma(2-\alpha)} \sum_{j=1}^{k} b_j [u_i^{k+1-j}-u_i^{k-j}]$$

Where $b_j = [(j+1)^{1-\alpha}-j^{1-\alpha}]$, j= 1, 2, …..,k

For $\frac{\partial^2 u(x,t)}{\partial x^2}$, we adopt the second order central difference scheme in space for eachinterior grid points *xi*, 0 <*i*< *M*. Therefore, we approximate$\frac{\partial^2 u(x,t)}{\partial x^2}$, as follows [**?**]

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{2}[\delta_x^2 u_i^{k+1} + \delta_x^2 u_i^{k}]$$

$$= \frac{1}{2}\left[\frac{u_{i-1}^{k+1}-2u_i^{k+1}+u_{i+1}^{k+1}}{h^2} + \frac{u_{i-1}^{k}-2u_i^{k}+u_{i+1}^{k}}{h^2}\right] + O(h^2)$$

where $\delta x$is central difference operator. For partial derivative with respect to time and space, forward difference and backward difference are used respectively.

$$\frac{\partial u(x,t)}{\partial x} = \frac{u(x_i,t_{k+1})-u(x_{i-1},t_{k+1})}{h} + O(h)$$

$$= \frac{u_i^{k+1}-u_{i-1}^{k+1}}{h} + O(h)$$

$$\frac{\partial u(x,t)}{\partial t} = \frac{u(x_i,t_{k+1})-u(x_i,t_k)}{h} + O(\tau)$$

.

$$= \frac{u_i^{k+1}-u_i^{k}}{\tau} + O(\tau)$$

Now by using time fractional approximation, the Crank-Nicolson implicit type numerical approximation to equation (**2.1**) − (**2.3**) is given as follows

$$A\frac{u_i^{k+1}-u_i^{k}}{\tau} + B\frac{\tau^{-\alpha}}{\Gamma(2-\alpha)}[u_i^{k+1}-u_i^{k}] + B\frac{\tau^{-\alpha}}{\Gamma(2-\alpha)}\sum_{j=1}^{k} b_j[u_i^{k+1-j}-u_i^{k-j}]$$

$$= -V\frac{u_i^{k+1}-u_{i-1}^{k+1}}{h} + D\frac{1}{2}\left[\frac{u_{i-1}^{k+1}-2u_i^{k+1}+u_{i+1}^{k+1}}{h^2} + \frac{u_{i-1}^{k}-2u_i^{k}+u_{i+1}^{k}}{h^2}\right] + f_i^k$$

The initial condition is approximated as$u_i^0 = g(x_i)$, *i*= 1, 2, ...,*M*.

For the two boundary points x_0 and x_M the corresponding discretization schemes are$u_0^k = \Phi_1(t)$ and $u_M^k = \Phi_2(t)$, k=0,1,2….N

After solving this we get,

$$u_i^{k+1}-u_i^{k} + \frac{B\tau^{1-\alpha}}{A\Gamma(2-\alpha)}[u_i^{k+1}-u_i^{k}] + \frac{B\tau^{1-\alpha}}{A\Gamma(2-\alpha)}\sum_{j=1}^{k} b_j[u_i^{k+1-j}-u_i^{k-j}]$$

$$= -\frac{V\tau}{Ah}[u_i^{k+1}-u_{i-1}^{k+1}] + \frac{D\tau}{2Ah^2}[u_{i-1}^{k+1}-2u_i^{k+1}+u_{i+1}^{k+1}+u_{i-1}^{k}-2u_i^{k}+u_{i+1}^{k}] + \frac{\tau}{A}f_i^k$$

$$u_i^{k+1}-u_i^{k} + r_1[u_i^{k+1}-u_i^{k}] + r_1\sum_{j=1}^{k} b_j[u_i^{k+1-j}-u_i^{k-j}] = -r_2[u_i^{k+1}-u_{i-1}^{k+1}] + r[u_{i-1}^{k+1}-2u_i^{k+1}+u_{i+1}^{k+1}+u_{i-1}^{k}-2u_i^{k}+u_{i+1}^{k}] + r_3 f_i^k$$

where $r = \frac{D\tau}{2Ah^2}$, $r_1 = \frac{B\tau^{1-\alpha}}{A\Gamma(2-\alpha)}$, $r_2 = \frac{V\tau}{Ah}$, $r_3 = \frac{\tau}{A}$, $b_j = [(j+1)^{1-\alpha}-j^{1-\alpha}]$,for i=1, ...,$M-1$, j = 1, 2, ..., $k$and$k$ = 0, 1, 2, ... ,N

On further simplification we get,

$$-(r+r_2)u_{i-1}^{k+1} + (1+r_1+r_2+2r)u_i^{k+1} - ru_{i+1}^{k+1} - r_1\sum_{j=1}^{k-1}(b_j-b_{j+1})u_i^{k-j}$$

$$= ru_{i-1}^{k} + (1+(1-b_1)r_1-2r)u_i^{k} + ru_{i+1}^{k} + r_1 b_k u_i^0 + r_3 f_i^k$$

Therefore, the complete discretization of the time fractional Mobile Immobile equation is,

$$-(r+r_2)u_{i-1}^{k+1} + (1+r_1+r_2+2r)u_i^{k+1} - ru_{i+1}^{k+1} - r_1\sum_{j=1}^{k-1}(b_j-b_{j+1})u_i^{k-j} = ru_{i-1}^{k} + (1+(1-b_1)r_1-2r)u_i^{k} + ru_{i+1}^{k} + r_1 b_k u_i^0 + r_3 f_i^k \quad (2.4)$$

initial condition: $u_i^0 = f(ih)$, *i*= 1, 2, ...,M −1 (2.5) boundary conditions: $u_k^0 = \Phi_1(k\tau)$ and $u_k^M = \Phi_2(k\tau)$, k = 0, 1, 2, ...N (2.6)

where $r = \frac{D\tau}{2Ah^2}$, $r_1 = \frac{B\tau^{1-\alpha}}{A\Gamma(2-\alpha)}$, $r_2 = \frac{V\tau}{Ah}$, $r_3 = \frac{\tau}{A}$, $b_j = [(j+1)^{1-\alpha} - j^{1-\alpha}]$, for $i = 1, \ldots, M-1$, $j = 1, 2, \ldots, k$ and $k = 0, 1, 2, \ldots, N$

Therefore, the approximated time fractional Mobile Immobile equation (**2.4**) − (**2.6**)can be written in the following matrix equation form….

$$AU^1 = BU^0 + S, \qquad for\ k = 0$$

$$AU^{k+1} = XU^k + r\sum_{j=1}^{k-1}(b_j - b_{j+1})U^{k-j} + r_1 b_k U^0 + S, \qquad for\ k \geq 1$$

Where, $U^k = [u_1^k, u_2^k, \ldots\ldots, u_N^k]$, k = 0, 1, 2, ...N and

$$A = \begin{pmatrix} 1+r_1+r_2+2r & -r & \cdots & & \cdots & 0 \\ -(r+r_2) & 1+r_1+r_2+2r & -r & \cdots & & \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ 0 & & -(r+r_2) & 1+r_1+r_2+2r & -r\cdots & \\ \vdots & \ddots & \ddots & \vdots & & \\ 0 & \cdots & & & -(r+r_2) & 1+r_1+r_2+2r \end{pmatrix}, \quad S = \begin{pmatrix} (r+r_2)u_0^{k+1} + ru_0^k + r_3 f_1^1 \\ r_3 f_2^1 \\ r_3 f_3^1 \\ r_3 f_4^1 \\ \vdots \\ \vdots \\ ru_N^{k+1} + ru_N^k + r_3 f_2^1 \end{pmatrix}$$

$$B = \begin{pmatrix} (1+r_1-2r) & r & \cdots & & \cdots & 0 \\ r & (1+r_1-2r) & r & \cdots & & \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ 0 & & r & (1+r_1-2r) & r\cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ 0 & \cdots & & & r & (1+r_1-2r) \end{pmatrix} X \begin{pmatrix} c & r & \cdots & & \cdots & \cdots & 0 \\ r & c & r & \cdots & & & \\ \vdots & \ddots & \ddots & \ddots & & & \vdots \\ 0 & & & & & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & & & \vdots \\ 0 & \cdots & & & & r & c \end{pmatrix}$$

III. STABILITY

Theorem 3.1

The solution of the discretised scheme (**2.4**) − (**2.6**) for the time fractional Mobile Immobile equation (**2.1**) − (**2.3**) is unconditionally stable.

Proof: We assume that $\| E^k \|_\infty = |\epsilon_l^k| = \max_{1\leq i\leq M-1} |\epsilon_i^k|$, α, r, $r1$, $r2$, $r3$ >0, 1 = $b1 > b2 >, \ldots > 0$ for $i = 1, 2, \ldots, M-1$, $k = 1, 2, \ldots, N$.

Therefore, from equation (**2.4**) for $k = 0$, we get

$$\begin{aligned} |\epsilon_l^1| &= |-(r+r_2)\epsilon_{i-1}^1 + (1+r_1+r_2+2r)\epsilon_i^1 - r\epsilon_{i+1}^1| \\ &= |-(r+r_2)\epsilon_{i-1}^0 + (1+r_1+r_2+2r)\epsilon_i^0 - r\epsilon_{i+1}^0| \\ &\leq |-(r+r_2) + (1+r_1+r_2+2r) - r||\epsilon_i^0| \\ &\leq (1+r_1)|\epsilon_l^0| \\ &\therefore ||E^1||_\infty \leq (1+r_1)|\epsilon_l^0| \\ &\therefore ||E^1||_\infty \leq K||E^0||_\infty \end{aligned}$$

Suppose that,

$$\| E^k \|_\infty \leq \| E^0 \|_\infty.$$

From equation (**2.4**), we get

$$\begin{aligned} |\epsilon_l^{k+1}| &= |-(r+r_2)\epsilon_{i-1}^{k+1} + (1+r_1+r_2+2r)\epsilon_i^{k+1} - r\epsilon_{i+1}^{k+1}| \\ &= |r\epsilon_{i-1}^k + (1+(1-b_1)r_1 - 2r)\epsilon_i^k - r\epsilon_{i+1}^k| \\ &\leq r|\epsilon_i^k| + (1+(1-b_1)r_1 - 2r)|\epsilon_i^k| + r|\epsilon_i^k| \\ &\leq r|\epsilon_i^k| + (1+(1-b_1)r_1 - 2r)|\epsilon_i^k| + r|\epsilon_i^k| \\ &\leq (r+1+(1-b_1)r_1 - 2r + r)|\epsilon_i^k| \\ &\leq (1+r_1)|\epsilon_l^k| \leq K|\epsilon_l^k| \\ &\| E^{k+1} \|_\infty \leq K \| E^0 \|_\infty. \end{aligned}$$

Hence, by induction we prove that the scheme is unconditionally stable.

IV. CONVERGENCE

Theorem: The fractional Crank-Nicolson finite difference scheme (2.4)−(2.6) is convergent and $U(x_i, t_k)$ be the exact solution of the time fractional diffusion equation (**2.1**) − (**2.3**)and U_i^kbe the solution of the discrete equation (**2.4**) − (**2.6**) at mesh points(x_i,t_k), here i= 1, 2, ...,M − 1 and k = 1, 2, ...,N, satisfy

$$\| U(x_i,t_k) - U_i^k \| \le \| E^k \|_\infty + O(\tau^{1-\alpha} + h^2), i = 1,\dots, M-1, k = 1,2,\dots,N.$$

Proof:

Suppose,

$\left|e_l^k\right| = \max_{1\le i\le M-1}\left|\epsilon_i^k\right| = \| E^k \|_\infty, for\ i = 1,2,...,M-1$, and $T_j^n = h^2[O(\tau^{1-\alpha}) + O(h^2)]$, $T_l^k = \max_{1\le i\le M-1}|T_i^k|$

Then we have,

$$|\epsilon_i^1| = \left|-(r+r_2)\epsilon_{i-1}^1 + (1+r_1+r_2+2r)\epsilon_i^1 - r\epsilon_{i+1}^1\right| = \left|-r\epsilon_{i-1}^0 + (1+r_1-2r)\epsilon_i^0 - r\epsilon_{i+1}^0\right| \le (1+r_1)\,|\epsilon_l^0| + |T_l^1|$$

$$\| E^1 \|_\infty \le K \| E^0 \|_\infty + h^2[O(\tau^{1-\alpha}) + O(h^2)].$$

Suppose that, $\| E^k \|_\infty \le K \| E^0 \|_\infty + h^2[O(\tau^{1-\alpha}) + O(h^2)]$.

Therefore we have, $\left|\epsilon_l^{k+1}\right| = \left|-(r+r_2)\epsilon_{i-1}^{k+1} + (1+r_1+r_2+2r)\epsilon_i^{k+1} - r\epsilon_{i+1}^{k+1}\right| = \left|r\epsilon_{i-1}^k + (1+(1-b_1)r_1 - 2r)\epsilon_i^k - r\epsilon_{i+1}^k\right|$

$$\le r\left|\epsilon_i^k\right| + (1+(1-b_1)r_1-2r)\left|\epsilon_i^k\right| + r\left|\epsilon_i^k\right| \le (1+r_1)\left|\epsilon_l^k\right| + |T_l^k| \le K\left|\epsilon_l^k\right| + h^2[O(\tau^{1-\alpha}) + O(h^2)]$$

$$\| E^{k+1} \|_\infty \le K \| E^k \|_\infty + h^2[O(\tau^{1-\alpha}) + O(h^2)]$$

Hence by induction hypothesis, we prove that $\| E^{k+1} \|_\infty \le K \| E^k \|_\infty + h^2[O(\tau^{1-\alpha}) + O(h^2)]$

Therefore, $\| U(x_i,t_k) - U_i^k \| \le \| E \|_\infty + O(\tau^{1-\alpha} + h^2), i = 1,\dots,M-1, k = 1,2,\dots,N.$

V. NUMERICAL SOLUTIONS

In this section, we obtain the approximated solution of time fractional anomalous diffusion equation with initial and boundary conditions. To obtain the numerical solution of the time fractional anomalous diffusion equation (TFADE) by the finite difference scheme, it is important to use some analytical model. Therefore, we present an example to demonstrate that TFADE can be applied to simulate behaviour of a fractional diffusion equation by using Mathematica Software. We consider the following, dimensionless one-dimensional time fractional anomalous diffusion equationwith suitable initial and boundary conditions

$$\frac{\partial u(x,t)}{\partial t} + \frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial u(x,t)}{\partial x}, (x,t)\epsilon[0,2\pi]X[0,T], 0 < \alpha \le 1, t > 0 \quad (5.1)$$

initial condition : $U(x, 0) = cos(x), 0 \le x \le 2\pi$ (5.2)

Boundary conditions :$U(0, t) = 1, U(2\pi, t) = 1, t \ge 0$ (5.3)

The numerical solutions are obtained at $t = 0.05$ by considering the parameters$\tau = 0.005$, $h = 0.628319$, in the following figure.

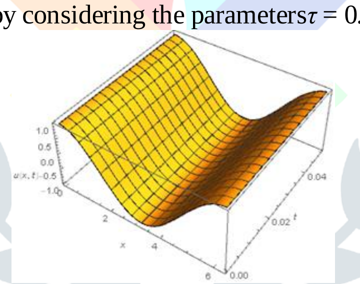

Fig.5.1: The Exact solution of fractional mobile immobile equation

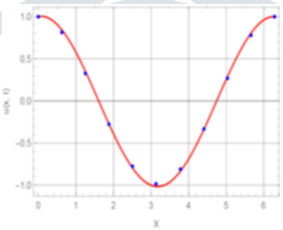

Fig.5.2: Solution of CNTFDE for α = 0.9 and α = 0.86 variational Iteration Method for MIM

We consider the following time fractional Mobile Immobile equation (TFMIME) withinitial and boundary conditions

$$A\frac{\partial u(x,t)}{\partial t} + B\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = -V\frac{\partial u(x,t)}{\partial x} + D\frac{\partial^2 u(x,t)}{\partial x^2} + f(x,t), (x,t)\epsilon[0,L]X[0,T], 0 < \alpha \le 1, t > 0 \quad (6.1)$$

$$initialcondition: u(x,0) = g(x), 0 \le x \le L \quad (6.2)$$

$$Boundaryconditions: u(0,t) = \Phi_1(t), u(L,t) = \Phi_2(t), t \ge 0. \quad (6.3)$$

The basic structure of He's VIM is the construction of correction functional for (**6.1**)is given by

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\xi)\left[A\frac{\partial \tilde{u}_n(x,t)}{\partial t} + B\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} + V\frac{\partial \tilde{u}_n(x,t)}{\partial x} - D\frac{\partial^2 \tilde{u}_n(x,t)}{\partial x^2} - f(x,t)\right]d\xi.$$

where λ is general Lagrange multiplier which can be identified optimally via, Variational theory and ˜un is restricted variation which means $\delta \tilde{u}_n = 0$.Substituting the value of the Lagrange multiplier into the correction functional,weget the following iteration formula

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t [\,A\frac{\partial \tilde{u}_n(x,t)}{\partial t} + B\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} + V\frac{\partial \tilde{u}_n(x,t)}{\partial x} - D\frac{\partial^2 \tilde{u}_n(x,t)}{\partial x^2} - f(x,t)]d\xi \quad (6.4)$$

The successive approximations $u_n(x, t)$, $n > 0$ of solution $u(x, t)$ will be rapidly obtainedusing any selective function $u_0(x, t)$, consequently, the solution is

$$u(x,t) = \lim_{n\to\infty} u_n(x,t) \quad (6..5)$$

The above MIM test problem is solved by VIM and the solution is represented graphically asfollows:

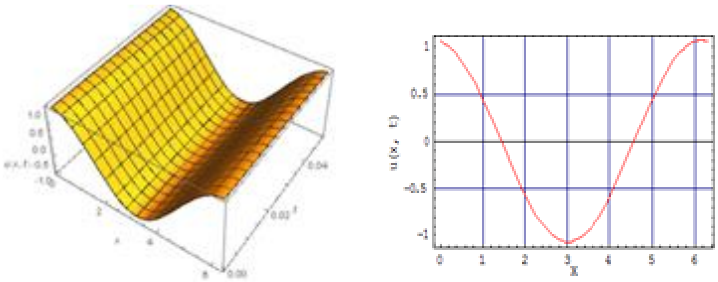

Fig.5.3: Solution of TFDE by Variational Iteration Method

VI.CONCLUSIONS

- We successfully develop fractional order Crank-Nicolson finite difference scheme and Variational iteration method for solving time fractional Mobile Immobile equation in a bounded domain.
- Efficiency and applicability of both the methods can be observed from the figures5.1-5.3.
- Stability and convergence of the Crank Nicolson Scheme is also demonstrated.
- The variational iteration method required less computation..

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