SOME TRANSLATIONS IN (T, S)-INTUITIONISTIC FUZZY SUBFIELD OF A FIELD

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ABSTRACT: In this paper, some translations of (T, S)-intuitionistic fuzzy subfield of a field are given. These translations are giving a new algebraic structure and this type of translations is very useful for convert to the one intuitionistic fuzzy algebraic structure to another intuitionistic fuzzy algebraic structure.

KEY WORDS: (T, S)- norm, fuzzy subset, intuitionistic fuzzy subset, (T, S)-intuitionistic fuzzy subfield, some translations.

INTRODUCTION: After the introduction of fuzzy sets by L.A.Zadeh[7], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K.T.Atanassov[1], as a generalization of the notion of fuzzy set. Hur.K, et.al.[3] have given the idea about the (T, S)-intuitionistic fuzzy ideals of a ring. Jianming Zhan[4] introduced the properties of fuzzy left h-ideals in hemiring with t-norms. Jun.Y.B, et.al.[5] gave the idea in intuitionistic nil radicals of (T, S)-intuitionistic fuzzy ideals and euclidean (T, S)-intuitionistic fuzzy ideals in rings. Vasu.M, et.al[6] have introduced the intuitionistic L-fuzzy subfields of a field. The above papers was useful for developing this paper. In this paper, some translations theorem of (T, S)-intuitionistic fuzzy subfield of a field are given.

1. PRELIMINARIES:

Definition 1.1[3]. A (T, S)-norm is a binary operations T: [0, 1]×[0, 1] → [0, 1] and S: [0, 1]×[0, 1] → [0, 1] satisfying the following requirements;
(i) T(0, x) = 0, T(1, x) = x (boundary condition)
(ii) T(x, y) = T(y, x) (commutativity)
(iii) T(x, T(y, z)) = T(T(x, y), z) (associativity)
(iv) if x ≤ y and w ≤ z, then T(x, w) ≤ T(y, z) (monotonicity).
(v) S(0, x) = x, S(1, x) = 1 (boundary condition)
(vi) S(x, y) = S(y, x) (commutativity)
(vii) S(x, S(y, z)) = S(S(x, y), z) (associativity)
(viii) if x ≤ y and w ≤ z, then S(x, w) ≤ S(y, z) (monotonicity).

Definition 1.2[7]. Let X be a non-empty set. A fuzzy subset A of X is a function A: X→[0, 1].

Definition 1.3[1]. An intuitionistic fuzzy subset (IFS) A of a set X is defined as an object of the form A = { (x, μA(x), νA(x)) / x ∈ X }, where μA: X → [0, 1] and νA: X → [0, 1] define the degree of membership and the degree of non-membership of the element x in X respectively and for every x in X satisfying 0 ≤ μA(x) + νA(x) ≤ 1.

Definition 1.4. Let (F, +, ·) be a field. An intuitionistic fuzzy subset A of F is said to be a (T, S)-intuitionistic fuzzy subfield of F if the following conditions are satisfied:
(i) μA(x−y) ≥ T(μA(x), μA(y)), for all x and y in F,
(ii) μA(xy⁻¹) ≥ T(μA(x), μA(y)), for all x and y≠ e in F,
(iii) νA(x−y) ≤ S(νA(x), νA(y)), for all x and y in F,
(iv) νA(xy⁻¹) ≤ S(νA(x), νA(y)), for all x and y≠ e in F.
Example 1.5. Consider the field $\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$ with addition modulo 5 and multiplication modulo 5 operations. Then $A = \{\langle 0, 0.7, 0.1 \rangle, \langle 1, 0.5, 0.4 \rangle,$ $\langle 2, 0.5, 0.4 \rangle, \langle 3, 0.5, 0.4 \rangle, \langle 4, 0.5, 0.4 \rangle\}$ is a $(T, S)$-intuitionistic fuzzy subfield of $\mathbb{Z}_5$.

Definition 1.6[2]. Let A and B be intuitionistic fuzzy subsets of X. Then the following translations and operations are defined as

(i) $\Lambda(A) = \{\langle x, \min\{\frac{1}{2}, \mu_{A}(x)\}\rangle, \max\{\frac{1}{2}, \nu_{A}(x)\}\}$ for all $x \in X$.

(ii) $\Theta(A) = \{\langle x, \max\{\frac{1}{2}, \mu_{A}(x)\}\rangle, \min\{\frac{1}{2}, \nu_{A}(x)\}\}$ for all $x \in X$.

(iii) $Q_{\alpha, \beta}(A) = \{\langle x, \min\{\alpha, \mu_{A}(x)\\}, \max\{\beta, \nu_{A}(x)\}\}$ for all $x \in X$, $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$.

(iv) $P_{\alpha, \beta}(A) = \{\langle x, \max\{\alpha, \mu_{A}(x)\}, \min\{\beta, \nu_{A}(x)\}\}$ for all $x \in X$, $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$.

(v) $G_{\alpha, \beta}(A) = \{\langle x, q_{\alpha, \beta}(A), \beta \nu_{A}(x)\rangle, \gamma \nu_{A}(x)\}$ for all $x \in X$ and $\alpha, \beta \in [0, 1]$.

(vi) $A \cap B = \{\langle x, \min\{\mu_{A}(x), \mu_{B}(x)\}, \max\{\nu_{A}(x), \nu_{B}(x)\}\}$ for all $x \in X$.

2- PROPERTIES:

Theorem 2.1 If A is a $(T, S)$-intuitionistic fuzzy subfield of a field $F$, then $\Lambda A$ is a $(T, S)$-intuitionistic fuzzy subfield of $F$.

Proof. For every $x, y \in F$, then $\mu_{\Lambda A}(x-y) = \min\{\frac{1}{2}, \mu_{A}(x-y)\} \geq \min\{\frac{1}{2}, T(\mu_{A}(x), \mu_{A}(y))\}$

$\geq T(\mu_{\Lambda A}(x), \mu_{\Lambda A}(y)) = T(\mu_{A}(x), \mu_{A}(y))$ for all $x, y \in F$. For every $x, y \in F$, then $\nu_{\Lambda A}(x-y) = \min\{\nu_{A}(x-y)\} \leq \min\{\nu_{A}(x), \nu_{A}(y)\}\leq S(\min\{\nu_{A}(x), \nu_{A}(y)\})\leq S(\min\{\nu_{A}(x), \nu_{A}(y)\})$ for all $x, y \in F$. For every $x, y \in F$, then $\mu_{\Lambda A}(xy^{-1}) = \min\{\mu_{A}(xy^{-1})\} \geq \min\{\mu_{A}(x), \mu_{A}(y)\}\geq T(\mu_{\Lambda A}(x), \mu_{\Lambda A}(y)) = T(\mu_{A}(x), \mu_{A}(y))$ for all $x, y \in F$. For every $x, y \in F$, then $\nu_{\Lambda A}(xy^{-1}) = \min\{\nu_{A}(xy^{-1})\} \leq \min\{\nu_{A}(x), \nu_{A}(y)\}\leq S(\min\{\nu_{A}(x), \nu_{A}(y)\})\leq S(\min\{\nu_{A}(x), \nu_{A}(y)\})$ for all $x, y \in F$. Hence $\Lambda A$ is a $(T, S)$-intuitionistic fuzzy subfield of $F$.

Theorem 2.2 If A is a $(T, S)$-intuitionistic fuzzy subfield of a field $F$, then $\Theta A$ is a $(T, S)$-intuitionistic fuzzy subfield of $F$.

Proof. For every $x, y \in F$, then $\mu_{\Theta A}(x-y) = \min\{\frac{1}{2}, \mu_{A}(x-y)\} \geq \min\{\frac{1}{2}, T(\mu_{A}(x), \mu_{A}(y))\}$

$\geq T(\min\{\mu_{A}(x), \mu_{A}(y)\}) = T(\mu_{\Theta A}(x), \mu_{\Theta A}(y))$ for all $x, y \in F$. For every $x, y \in F$, then $\nu_{\Theta A}(x-y) = \min\{\nu_{A}(x-y)\} \leq \min\{\nu_{A}(x), \nu_{A}(y)\}\leq S(\min\{\nu_{A}(x), \nu_{A}(y)\})\leq S(\min\{\nu_{A}(x), \nu_{A}(y)\})$ for all $x, y \in F$. For every $x, y \in F$, then $\mu_{\Theta A}(xy^{-1}) = \min\{\mu_{A}(xy^{-1})\} \geq \min\{\mu_{A}(x), \mu_{A}(y)\}\geq T(\mu_{\Theta A}(x), \mu_{\Theta A}(y)) = T(\mu_{A}(x), \mu_{A}(y))$ for all $x, y \in F$. For every $x, y \in F$, then $\nu_{\Theta A}(xy^{-1}) = \min\{\nu_{A}(xy^{-1})\} \leq \min\{\nu_{A}(x), \nu_{A}(y)\}\leq S(\min\{\nu_{A}(x), \nu_{A}(y)\})\leq S(\min\{\nu_{A}(x), \nu_{A}(y)\})$ for all $x, y \in F$. Hence $\Theta A$ is a $(T, S)$-intuitionistic fuzzy subfield of $F$.

Theorem 2.3. If A is a $(T, S)$-intuitionistic fuzzy subfield of a field F, then $Q_{\alpha, \beta}(A)$ is a $(T, S)$-intuitionistic fuzzy subfield of F.

Proof. For every $x, y \in F$ and $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$, then $\mu_{Q_{\alpha, \beta}(A)}(x-y) = \min\{\alpha, \mu_{A}(x-y)\} \geq \min\{\alpha, T(\mu_{A}(x), \mu_{A}(y))\}$

$\geq T(\min\{\alpha, \mu_{A}(x)\}, \min\{\alpha, \mu_{A}(y)\}) = T(\mu_{Q_{\alpha, \beta}(A)}(x), \mu_{Q_{\alpha, \beta}(A)}(y))$ for all $x, y \in F$. And $\nu_{Q_{\alpha, \beta}(A)}(x-y) = \min\{\beta, \nu_{A}(x-y)\} \leq \min\{\beta, S(\nu_{A}(x), \nu_{A}(y))\} \leq S(\min\{\beta, \nu_{A}(x)\}, \min\{\beta, \nu_{A}(y)\}) = S(\nu_{Q_{\alpha, \beta}(A)}(x), \nu_{Q_{\alpha, \beta}(A)}(y))$ for all $x, y \in F$. For every $x, y \in F$ and $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$, then $\mu_{Q_{\alpha, \beta}(A)}(xy^{-1}) = \min\{\alpha, \mu_{A}(xy^{-1})\}$

$\geq \min\{\alpha, T(\mu_{A}(x), \mu_{A}(y))\} \geq T(\min\{\alpha, \mu_{A}(x)\}, \min\{\alpha, \mu_{A}(y)\}) = T(\mu_{Q_{\alpha, \beta}(A)}(x), \mu_{Q_{\alpha, \beta}(A)}(y))$ for all $x, y \in F$. And $\nu_{Q_{\alpha, \beta}(A)}(xy^{-1}) = \min\{\beta, \nu_{A}(xy^{-1})\} \leq \min\{\beta, S(\nu_{A}(x), \nu_{A}(y))\}$

$\leq S(\min\{\beta, \nu_{A}(x)\}, \min\{\beta, \nu_{A}(y)\}) = S(\nu_{Q_{\alpha, \beta}(A)}(x), \nu_{Q_{\alpha, \beta}(A)}(y))$ for all $x, y \in F$. Hence $Q_{\alpha, \beta}(A)$ is a $(T, S)$-intuitionistic fuzzy subfield of F.

Theorem 2.4 If A is a $(T, S)$-intuitionistic fuzzy subfield of a field F, then $P_{\alpha, \beta}(A)$ is a $(T, S)$-intuitionistic fuzzy subfield of F.
Proof. For every $x, y \in F$ and $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$, then $\mu_{R_{\alpha, \beta}(A)}(x-y) = \max\{\alpha, \mu_A(x-y)\} \geq \min\{\alpha, \mu_A(x), \mu_A(y)\} \geq \mu_A(x) \mu_A(y)$ for all $x, y \in F$. And $\nu_{R_{\alpha, \beta}(A)}(x-y) = \min\{\beta, \nu_A(x-y)\}$ for all $x, y \in F$. For every $x, y \neq e \in F$, then $\mu_{R_{\alpha, \beta}(A)}(xy^{-1}) = \max\{\alpha, \mu_A(xy^{-1})\} \geq \max\{\alpha, \mu_A(x), \mu_A(y)\}$ for all $x, y \neq e \in F$. For every $x, y \neq e \in F$, then $\nu_{R_{\alpha, \beta}(A)}(xy^{-1}) = \min\{\beta, \nu_A(xy^{-1})\} \leq \min\{\beta, \nu_A(x), \nu_A(y)\}$ for all $x, y \neq e \in F$. Hence $P_{\alpha, \beta}(A)$ is a $(T, S)$-intuitionistic fuzzy subfield of $F$.

Theorem 2.5. If $A$ is a $(T, S)$-intuitionistic fuzzy subfield of a field $F$, then $G_{\alpha, \beta}(A)$ is a $(T, S)$-intuitionistic fuzzy subfield of $F$.

Proof. For every $x, y \in F$ and $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$, then $\mu_{G_{\alpha, \beta}(A)}(x-y) = \alpha \mu_A(x-y) \geq \alpha \mu_A(x) \mu_A(y)$ for all $x, y \in F$. And $\nu_{G_{\alpha, \beta}(A)}(x-y) = \beta \nu_A(x-y) \leq \beta \nu_A(x) \nu_A(y)$ for all $x, y \in F$. For every $x, y \neq e \in F$, then $\mu_{G_{\alpha, \beta}(A)}(xy^{-1}) = \alpha \mu_A(xy^{-1}) \geq \alpha \mu_A(x) \mu_A(y)$ for all $x, y \neq e \in F$. And $\nu_{G_{\alpha, \beta}(A)}(xy^{-1}) = \beta \nu_A(xy^{-1}) \leq \beta \nu_A(x) \nu_A(y)$ for all $x, y \neq e \in F$. Hence $G_{\alpha, \beta}(A)$ is a $(T, S)$-intuitionistic fuzzy subfield of $F$.

Theorem 2.6. If $A$ and $B$ are $(T, S)$-intuitionistic fuzzy subfields of a field $F$, then $A \cap B$ is also a $(T, S)$-intuitionistic fuzzy subfield of $F$.

Corollary 2.7. If $A$ and $B$ are $(T, S)$-intuitionistic fuzzy subfields of a field $F$, then $\Theta(A \cap B) = \Theta(A) \cap \Theta(B)$ is also a $(T, S)$-intuitionistic fuzzy subfield of $F$.

Corollary 2.8. If $A$ and $B$ are $(T, S)$-intuitionistic fuzzy subfields of a field $F$, then $\Lambda(A \cap B) = \Lambda(A) \cap \Lambda(B)$ is also a $(T, S)$-intuitionistic fuzzy subfield of $F$.

Corollary 2.9. If $A$ is a $(T, S)$-intuitionistic fuzzy subfield of a field $F$, then $\Theta(A) = \Lambda(A)$ is also a $(T, S)$-intuitionistic fuzzy subfield of $F$.

Corollary 2.10. If $A$ and $B$ are $(T, S)$-intuitionistic fuzzy subfields of a field $G$, then $P_{\alpha, \beta}(A \cap B) = P_{\alpha, \beta}(A) \cap P_{\alpha, \beta}(B)$ is also a $(T, S)$-intuitionistic fuzzy subfield of $F$.

Corollary 2.11. If $A$ and $B$ are $(T, S)$-intuitionistic fuzzy subfields of a field $F$, then $Q_{\alpha, \beta}(A \cap B) = Q_{\alpha, \beta}(A) \cap Q_{\alpha, \beta}(B)$ is also a $(T, S)$-intuitionistic fuzzy subfield of $F$.

Corollary 2.12. If $A$ is a $(T, S)$-intuitionistic fuzzy subfield of a field $F$, then $P_{\alpha, \beta}(A \cap B) = Q_{\alpha, \beta}(P_{\alpha, \beta}(A))$ is also a $(T, S)$-intuitionistic fuzzy subfield of $F$.

Corollary 2.13. If $A$ and $B$ are $(T, S)$-intuitionistic fuzzy subfields of a field $F$, then $G_{\alpha, \beta}(A \cap B) = G_{\alpha, \beta}(A) \cap G_{\alpha, \beta}(B)$ is also a $(T, S)$-intuitionistic fuzzy subfield of $F$.

Corollary 2.14. If $A$ is a $(T, S)$-intuitionistic fuzzy subfield of a field $F$, then $Q_{\alpha, \beta}(A \cap B) = Q_{\alpha, \beta}(A) \cap Q_{\alpha, \beta}(B)$ is also a $(T, S)$-intuitionistic fuzzy subfield of $F$. 
REFERENCES


