Fuzzy *e*-closed and Generalized Fuzzy *e*-closed Sets in Double Fuzzy Topological Spaces

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Abstract

The purpose of this paper is to introduce and study a new class of fuzzy sets called (r,s)-generalized fuzzy e-closed sets in double fuzzy topological spaces. Furthermore, the relationship between the new concepts are introduced and established with some interesting examples.

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1.Introduction

A progressive development of fuzzy sets [9] has been made to discover the fuzzy analogues of the crisp sets theory. On the other hand, the ideal of intuitionistic fuzzy sets was first introduced by Atanassov [2]. Later on, Coker [3] presented the notion of intuitionistic fuzzy topology. Samanta and Mondal [7], introduced and characterized the intuitonistic gradation of openness of fuzzy sets which is a generalization of smooth topology and the topology of intuitionistic fuzzy sets. The name ``intuitionistic" is disconnected in mathematics and applications. Gracia and Rodabaugh [5] conclued that they word under the name ``double". In 2008, Erdal Ekici [4] introduced e-open sets in general topology. In 2014, Seenivasan et. al [8] introduce fuzzy e-open sets in fuzzy topological spaces. As a generalization of the results in References [4, 8], we introduce and study (r,s)-fuzzy e-closed sets and (r,s)-generalized fuzzy e-closed sets in double fuzzy topological spaces. Also the relationship between (r,s)-fuzzy e-closed (resp. (r,s)-generalized fuzzy e-closed) sets and and some other sets are introduced and established with some interesting couter examples.

2. Preliminaries

Throughout this paper, X will be a non-empty set, $I=[0,1], I_0=(0,1]$ and $I_1=[0,1)$. A fuzzy set A is quasi-coincident with a fuzzy set B (denoted by, AqB) iff there exists $x\in X$ such that $A(x)+B(x)>1$ and they are not quasi-coincident otherwise (denoted by, $A\bar{q}B$). The family of all fuzzy sets on X is denoted by I^X. By $\underline{0}$ and $\underline{1}$, we denote the smallest and the greatest fuzzy sets on X. For a fuzzy set $A\in I^X, \underline{1}-A$ denotes its complement. All other notations are standard notations of fuzzy set theory.
Now, we recall the following definitions which are useful in the sequel.

Definition 2.1 *[7] A double fuzzy topology $(T,T^{å})$ on X is a pair of maps $T,T^{å}:I^X\to I$, which satisfies the following properties:*

(i) $T(A)\le \underline{1}-T^{å}(A)$ for each $A\in I^X$.

(ii) $T(A_1\wedge A_2)\ge T(A_1)\wedge T(A_2)$ and $T^{å}(A_1\wedge A_2)\le T^{å}(A_1)\vee T^{å}(A_2)$, for each $A_1,A_2\in I^X$.

(iii) $T(\vee_{i\in\Gamma}A_i)\geq\wedge_{i\in\Gamma}T(A_i)$ and $T^{å}(\vee_{i\in\Gamma}A_i)\leq\vee_{i\in\Gamma}T^{å}(A_i)$ for each $A_i\in I^X, i\in\Gamma$.

The triplet $(X,T,T^{å})$ is called a double fuzzy topological space (briefly, dfts). A fuzzy set A is called an (r,s)-fuzzy open (briefly, (r,s)-fo) if $T(A)\geq r$ and $T^{å}(A)\leq s$. A fuzzy set A is called an (r,s)-fuzzy closed (briefly, (r,s)-fc) set iff $\underline{1}-A$ is an (r,s)-fo set.

Theorem 2.1 *[6] Let $(X,T,T^{å})$ be a dfts. Then double fuzzy closure operator and double fuzzy interior operator of $A\in I^X$ are defined by*

$C_{T,T^{å}}(A,r,s)=\wedge\{B\in I^X \mid A\leq B, T(\underline{1}-B)\geq r, T^{å}(\underline{1}-B)\leq s\}$.

$I_{T,T^{å}}(A,r,s)=\vee\{B\in I^X \mid A\geq B, T(B)\geq r, T^{å}(B)\leq s\}$.

Where $r\in I_0$ and $s\in I_1$ such that $r+s\leq 1$.

Definition 2.2 *[1] Let $(X,T,T^{å})$ be a dfts. For each $A\in I^X, r\in I_0$ and $s\in I_1$. A fuzzy set A is called an (r,s)-generalized fuzzy closed (briefly, (r,s)-gfc) if $C_{T,T^{å}}(A,r,s)\leq B, A\leq B, T(B)\geq r$ and $T^{å}(B)\leq s$. A is called an (r,s)-generalized fuzzy open (briefly (r,s)-gfo) iff $\underline{1}-A$ is (r,s)-gfc set.*

3. (r,s) fuzzy e-closed and (r,s)-generalized fuzzy e-closed sets

Definition 3.1 *Let $(X,T,T^{å})$ be a dfts. Then for each $A\in I^X, r\in I_0$ and $s\in I_1$. A fuzzy set A is called an (r,s)-fuzzy regular open (briefly, (r,s)- fro) if $A=I_{T,T^{å}}(C_{T,T^{å}}(A,r,s),r,s)$ and (r,s)-fuzzy regular closed (briefly, (r,s)- frc) if if $A=C_{T,T^{å}}(I_{T,T^{å}}(A,r,s),r,s)$.*

Definition 3.2 *Let $(X,T,T^{å})$ be a dfts. Then for each $A\in I^X, r\in I_0$ and $s\in I_1$, we define operators $\delta C_{T,T^{å}}$ and $\delta I_{T,T^{å}}: I^X\times I_0\times I_1\to I^X$ as follows: $\delta C_{T,T^{å}}(A,r,s)=\wedge\{B\in I^X : A\leq B, B \text{ is } (r,s)-frc\}$ and $\delta I_{T,T^{å}}(A,r,s)=\vee\{B\in I^X : B\leq A, B \text{ is } (r,s)-fro\}$.*

Definition 3.3 *Let $(X,T,T^{å})$ be a dfts. Then for each $A\in I^X, r\in I_0$ and $s\in I_1$: a fuzzy set A is called an*

(i) (r,s)-fuzzy δ semiopen (briefly, (r,s)- $f\delta so$) if $A\leq C_{T,T^{å}}(\delta I_{T,T^{å}}(A,r,s),r,s)$ and (r,s)-fuzzy δ semi closed (briefly, (r,s)- $f\delta sc$) if $A\geq I_{T,T^{å}}(\delta C_{T,T^{å}}(A,r,s),r,s)$.

(ii) (r,s)-fuzzy δ pre open (briefly, (r,s)- $f\delta po$) if $A\leq I_{T,T^{å}}(\delta C_{T,T^{å}}(A,r,s),r,s)$ and (r,s)-fuzzy δ pre closed (briefly, (r,s)- $f\delta pc$) if $A\geq C_{T,T^{å}}(\delta I_{T,T^{å}}(A,r,s),r,s)$.

(iii) (r,s)-fuzzy β open (briefly, (r,s)- $f\beta o$) if $A\leq C_{T,T^{å}}(I_{T,T^{å}}(C_{T,T^{å}}(A,r,s), r,s)r,s)$ and (r,s)-fuzzy β closed (briefly, (r,s)- $f\beta c$) if $A\geq I_{T,T^{å}}(C_{T,T^{å}}(I_{T,T^{å}}(A,r,s),r,s)r,s)$.

(iv) (r,s)-fuzzy e-open (briefly, (r,s)- feo) if $A\leq C_{T,T^{å}}(\delta I_{T,T^{å}}(A,r,s),r,s)\vee I_{T,T^{å}}(\delta C_{T,T^{å}}(A,r,s),r,s)$ and (r,s)-fuzzy e-closed (briefly, (r,s)- fec) if $A\geq I_{T,T^{å}}(\delta C_{T,T^{å}}(A,r,s),r,s)\wedge C_{T,T^{å}}(\delta I_{T,T^{å}}(A,r,s),r,s)$.

Definition 3.4 *Let $(X,T,T^{å})$ be a dfts. Then for each $A\in I^X, r\in I_0$ and $s\in I_1$, we define operators $eC_{T,T^{å}}$ (resp. $\delta SC_{T,T^{å}}$, $\delta PC_{T,T^{å}}$ and $\beta C_{T,T^{å}}$) and $eI_{T,T^{å}}$ (resp. $\delta SI_{T,T^{å}}$, $\delta PI_{T,T^{å}}$ and $\beta I_{T,T^{å}}$): $I^X\times I_0\times I_1\to I^X$ as follows: $eC_{T,T^{å}}$ (resp. $\delta SC_{T,T^{å}}$, $\delta PC_{T,T^{å}}$ and $\beta C_{T,T^{å}}$)$(A,r,s)=$ $\wedge\{B\in I^X : A\leq B, B \text{ is}(r,s)-fec \text{ (resp. } f\delta sc,\ f\delta pc \text{ and } f\beta c)\}$ and*

$eI_{T,T^{å}}$ (resp. $\delta SI_{T,T^{å}}$, $\delta PI_{T,T^{å}}$ and $\beta I_{T,T^{å}}$) $(A,r,s) =$

$\bigvee\{B \in I^X : B \le A, B \text{ is(r,s)} - feo \text{ (resp. } f\delta so,\ f\delta po \text{ and } f\beta o)\}$.

Definition 3.5 *Let $(X,T,T^{å})$ be a dfts, $A \in I^X, r \in I_0$ and $s \in I_1$, A is called an (r,s)-fuzzy e-Q-neighborhood of $x_t \in P_t(X)$ if there exists an (r,s)-feo set $B \in I^X$ such that $x_t qB$ and $B \le A$.*

The family of all (r,s)-fuzzy e-Q-neighborhood of x_t denoted by e-$Q(x_t,r,s)$.

Theorem 3.1 *Let $(X,T,T^{å})$ be a dfts. Then for each $A \in I^X, r \in I_0$ and $s \in I_1$, the operator $eC_{T,T^{å}}$ satisfies the following statements:*

(i) $eC_{T,T^{å}}(\underline{0},r,s) = \underline{0}$, $eC_{T,T^{å}}(\underline{1},r,s) = \underline{1}$.

(ii) $A \le eC_{T,T^{å}}(A,r,s)$.

(iii) If $A \le B$, then $eC_{T,T^{å}}(A,r,s) \le eC_{T,T^{å}}(B,r,s)$.

(iv) If A is an (r,s)-fec, then $A = eC_{T,T^{å}}(A,r,s)$.

(v) If A is an (r,s)-feo, then BqA iff $BqeC_{T,T^{å}}(A,r,s)$.

(vi) $eC_{T,T^{å}}(eC_{T,T^{å}}(A,r,s),r,s) = eC_{T,T^{å}}(A,r,s)$.

(vii) $eC_{T,T^{å}}(A,r,s) \vee eC_{T,T^{å}}(B,r,s) \le eC_{T,T^{å}}(A \vee B,r,s)$.

(viii) $eC_{T,T^{å}}(A,r,s) \wedge eC_{T,T^{å}}(B,r,s) \ge eC_{T,T^{å}}(A \wedge B,r,s)$.

Proof. (i), (ii), (iii) and (iv) are proved easily.

(v) Let $B\bar{q}A$ and B is an (r,s)-feo set, then $A \le \underline{1} - B$. But we have, BqA iff $BqeC_{T,T^{å}}(A,r,s)$ and $eC_{T,T^{å}}(A,r,s) \le eC_{T,T^{å}}(\underline{1}-B,r,s) = \underline{1}-B$, so $B\bar{q}eC_{T,T^{å}}(A,r,s)$, which is contradiction. Then BqA iff $BqeC_{T,T^{å}}(A,r,s)$.

(vi) Let x_t be a fuzzy point such that x_t ' $eC_{T,T^{å}}(A,r,s)$. Then there is an (r,s)-fuzzy e-Q-neighborhood B of x_t such that $B\bar{q}A$. But by (v), we have an (r,s)-fuzzy e-Q-neighborhood B of x_t such that $B\bar{q}eC_{T,T^{å}}(A,r,s)$. Also, x_t Ž $eC_{T,T^{å}}(eC_{T,T^{å}}(A,r,s),r,s)$. Then $eC_{T,T^{å}}(eC_{T,T^{å}}(A,r,s),r,s) \le eC_{T,T^{å}}(A,r,s)$. But we have, $eC_{T,T^{å}}(eC_{T,T^{å}}(A,r,s),r,s) \ge eC_{T,T^{å}}(A,r,s)$. Therefore $eC_{T,T^{å}}(eC_{T,T^{å}}(A,r,s),r,s) = eC_{T,T^{å}}(A,r,s)$.

(vii) and (viii) are obvious.

Similarly the other operators (i.e) $\delta SC_{T,T^{å}}$, $\delta PC_{T,T^{å}}$ and $\beta C_{T,T^{å}}$ satisfies the above conditions.

Theorem 3.2 *Let $(X,T,T^{å})$ be a dfts. Then for each $A \in I^X, r \in I_0$ and $s \in I_1$, the operator $eI_{T,T^{å}}$ satisfies the following statements:*

(i) $eI_{T,T^{å}}(\underline{1}-A,r,s) = \underline{1} - eC_{T,T^{å}}(A,r,s)$, $eC_{T,T^{å}}(\underline{1}-A,r,s) = \underline{1} - eI_{T,T^{å}}(A,r,s)$.

(ii) $eI_{T,T^{å}}(\underline{0},r,s) = \underline{0}$, $eI_{T,T^{å}}(\underline{1},r,s) = \underline{1}$.

(iii) $eI_{T,T^{å}}(A,r,s) \le A$.

(iv) If A is an (r,s)-feo, then $A = eI_{T,T^{å}}(A,r,s)$.

(v) If $A \le B$, then $eI_{T,T^{å}}(A,r,s) \le eI_{T,T^{å}}(B,r,s)$.

(vi) $eI_{T,T^{å}}(eI_{T,T^{å}}(A,r,s),r,s) = eI_{T,T^{å}}(A,r,s)$.

(vii) $eI_{T,T^{å}}(A \vee B,r,s) \ge eI_{T,T^{å}}(A,r,s) \vee eI_{T,T^{å}}(B,r,s)$.

(viii) $eI_{T,T^{å}}(A \vee B, r, s) \le eI_{T,T^{å}}(A, r, s) \wedge eI_{T,T^{å}}(B, r, s)$.

Proof. It is similar to Theorem 3.1.

Similarly the other operators (i.e) $\delta SI_{T,T^{å}}, \delta PI_{T,T^{å}}$ and $\beta I_{T,T^{å}}$ satisfies the above conditions.

Definition 3.6 *Let* $(X, T, T^{å})$ *be a dfts. Then for each* $A \in I^X, r \in I_0$ *and* $s \in I_1$*: a fuzzy set* A *is called*

(i) (r,s)-generalized fuzzy δ semiopen (briefly, (r,s)-gf δs o) if $B \le \delta SI_{T,T^{å}}(A, r, s)$ whenever $B \le A$ and $T(\underline{1} - B) \ge r, T^{å}(\underline{1} - B) \le s$.

(ii) (r,s)-generalized fuzzy δ preopen (briefly, (r,s)-gf δp o) if $B \le \delta PI_{T,T^{å}}(A, r, s)$ whenever $B \le A$ and $T(\underline{1} - B) \ge r, T^{å}(\underline{1} - B) \le s$.

(iii) (r,s)-generalized fuzzy β-open (briefly, (r,s)-gf β o) if $B \le \beta I_{T,T^{å}}(A, r, s)$ whenever $B \le A$ and $T(\underline{1} - B) \ge r, T^{å}(\underline{1} - B) \le s$.

(iv) (r,s)-generalized fuzzy e-open (briefly, (r,s)-gf e o) if $B \le eI_{T,T^{å}}(A, r, s)$ whenever $B \le A$ and $T(\underline{1} - B) \ge r, T^{å}(\underline{1} - B) \le s$.

(v) (r,s)-generalized fuzzy δ semiclosed (briefly, (r,s)-gf δs c) if $\delta SC_{T,T^{å}}(A, r, s) \le B$ whenever $A \le B$ and $T(B) \ge r, T^{å}(B) \le s$.

(vi) (r,s)-generalized fuzzy δ preclosed (briefly, (r,s)-gf δp c) if $\delta PC_{T,T^{å}}(A, r, s) \le B$ whenever $A \le B$ and $T(B) \ge r, T^{å}(B) \le s$.

(vii) (r,s)-generalized fuzzy β-closed (briefly, (r,s)-gf β c) if $\beta C_{T,T^{å}}(A, r, s) \le B$ whenever $A \le B$ and $T(B) \ge r, T^{å}(B) \le s$.

(viii) (r,s)-generalized fuzzy e-closed (briefly, (r,s)-gf e c) if $eC_{T,T^{å}}(A, r, s) \le B$ whenever $A \le B$ and $T(B) \ge r, T^{å}(B) \le s$.

Example 3.1 *Let* $X = \{x, y\}$. *Defined* B, C, D *and* E *by* $B(x) = (0.3), B(y) = (0.4); C(x) = (0.4), C(y) = (0.5); D(x) = (0.8), D(y) = (0.8); E(x) = (0.4), E(y) = (0.6); F(x) = 0.4, F(y) = 0.4$.

$$T(A) = \begin{cases} 1, & \text{if } A \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } A \in \{B, C\}, \\ 0, & \text{otherwise.} \end{cases} \quad T^{å}(A) = \begin{cases} 0, & \text{if } A \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } A \in \{B, C\}, \\ 1, & \text{otherwise.} \end{cases}$$

(i) C is an $(\frac{1}{2}, \frac{1}{2})$-f δ sc (resp. $(\frac{1}{2}, \frac{1}{2})$-gf δ sc) but not an $(\frac{1}{2}, \frac{1}{2})$-fc (resp. $(\frac{1}{2}, \frac{1}{2})$-gfc).

(ii) C is an $(\frac{1}{2}, \frac{1}{2})$-fec (resp. $(\frac{1}{2}, \frac{1}{2})$-gfec) but not an $(\frac{1}{2}, \frac{1}{2})$-f δ pc (resp. $(\frac{1}{2}, \frac{1}{2})$-gf δ pc).

(iii) D is an $(\frac{1}{2}, \frac{1}{2})$-f δ pc, $(\frac{1}{2}, \frac{1}{2})$-$e$ c but not an $(\frac{1}{2}, \frac{1}{2})$-fc, $(\frac{1}{2}, \frac{1}{2})$-$\delta$ sc.

(iv) E is an $(\frac{1}{2}, \frac{1}{2})$-f β c but not an $(\frac{1}{2}, \frac{1}{2})$-fec.

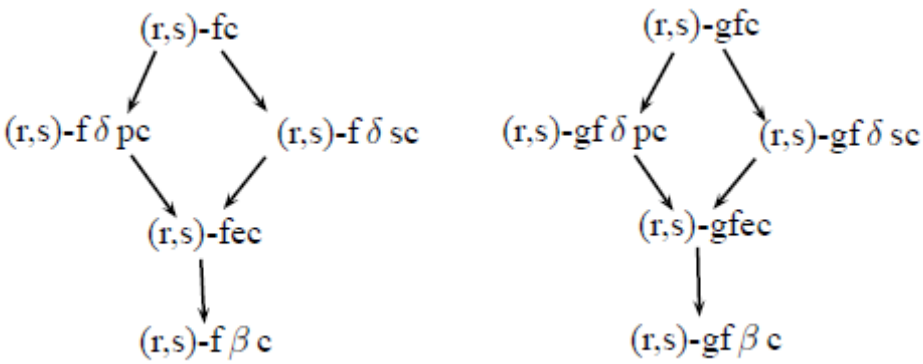
(v) F is an $(\frac{1}{2}, \frac{1}{2})$-gf e c, $(\frac{1}{2}, \frac{1}{2})$-g δ pc but not an $(\frac{1}{2}, \frac{1}{2})$-gf δ sc, $(\frac{1}{2}, \frac{1}{2})$-gfc.

Example 3.2 *Let* $X=\{x,y\}$ *. Defined* G,H,I,J,K *and* L *by*
$G(x)=(0.1), G(y)=(0.3); H(x)=(0.3), H(y)=(0.2); I(x)=(0.1), I(y)=(0.2);$
$J(x)=(0.3), J(y)=(0.3); K(x)=0.7,$
$K(y)=0.6; L(x)=(0.4), L(y)=(0.3).$

$$T(A)=\begin{cases}1, & \text{if } A\in\{\underline{0},\underline{1}\},\\ \frac{1}{2}, & \text{if } A\in\{G,H,I,J,K\},\\ 0, & \text{otherwise.}\end{cases}$$

$$T^{å}(A)=\begin{cases}0, & \text{if } A\in\{\underline{0},\underline{1}\},\\ \frac{1}{2}, & \text{if } A\in\{G,H,I,J,K\},\\ 1, & \text{otherwise.}\end{cases}$$

(i) L is an $(\frac{1}{2},\frac{1}{2})$-gf β c but not an $(\frac{1}{2},\frac{1}{2})$-gf e sc.



Theorem 3.3 *Let* $(X,T,T^{å})$ *be a dfts,* $A\in I^X$*, is* (r,s)*-gfeo set* $r\in I_0$ *and* $s\in I_1$*, if and only if* $B\le eI_{T,T^{å}}(A,r,s)$ *whenever* $B\le A$*,* $T(\underline{1}-B)\ge r$ *and* $T^{å}(\underline{1}-B)\le s$*.*

Proof. Suppose that A is an (r,s)-gfeo set in I^X, and let $T(\underline{1}-B)\ge r$ and $T^{å}(\underline{1}-B)\le s$ such that $B\le A$. By the definition, $\underline{1}-A$ is an (r,s)-gfec set in I^X. So, $eC_{T,T^{å}}(\underline{1}-A,r,s)\le\underline{1}-B$. Also, $\underline{1}-eI_{T,T^{å}}(A,r,s)\le\underline{1}-B$. And then, $B\le eI_{T,T^{å}}(A,r,s)$. Conversely, let $B\le A, T(\underline{1}-B)\ge r$ and $T^{å}(\underline{1}-B)\le s, r\in I_0$ and $s\in I_1$, such that $B\le eI_{T,T^{å}}(A,r,s)$. Now $\underline{1}-eI_{T,T^{å}}(A,r,s)\le\underline{1}-B$, Thus $eC_{T,T^{å}}(\underline{1}-A,r,s)\le\underline{1}-B$. That is, $\underline{1}-A$ is an (r,s)-gfec set, then A is an (r,s)-gfeo set.

Theorem 3.4 *Let* $(X,T,T^{å})$ *be a dfts,* $A\in I^X$*, is* (r,s)*-gfeo set* $r\in I_0$ *and* $s\in I_1$*. If* A *is an* (r,s) *-gfec set, then*

(i) $eC_{T,T^{å}}(A,r,s)-A$ does not contain any non-zero (r,s) -fc sets.

(ii) A is an (r,s)-fec iff $eC_{T,T^{å}}(A,r,s)-A$ is (r,s)-fc.

(iii) B is (r,s)-gfec set for each set $B\in I^X$ such that $A\le B\le eC_{T,T^{å}}(A,r,s)$.

(iv) For each (r,s)-fo set $B\in I^X$ such that $B\le A$, B is an (r,s)-gfec relative to A if and only if B is an (r,s)-gfec in I^X.

(v) For each (r,s)-feo set $B\in I^X$ such that $eC_{T,T^{å}}(A,r,s)\overline{q}B$ iff $A\overline{q}B$.

Proof. (i) Suppose that $T(\underline{1}-B)\geq r$ and $T^{å}(\underline{1}-B)\leq s, r\in I_0$ and $s\in I_1$ such that $B\leq eC_{T,T^{å}}(A,r,s)-A$ whenever $A\in I^X$ is an (r,s)-gfec set. Since $\underline{1}-B$ is an (r,s)-fo set,

$$A\leq(\underline{1}-B)\Rightarrow eC_{T,T^{å}}(A,r,s)\leq(\underline{1}-B)$$
$$\Rightarrow B\leq(\underline{1}-eC_{T,T^{å}}(A,r,s))$$
$$\Rightarrow B\leq(\underline{1}-eC_{T,T^{å}}(A,r,s))\wedge(eC_{T,T^{å}}(A,r,s)-A)$$
$$=\underline{0}$$

and hence $B=\underline{0}$ which is a contradiction. Then $eC_{T,T^{å}}(A,rs)-A$ does not contain any non-zero (r,s)-fc sets.

(ii) Let A be an (r,s)-gfec set. So, for each $r\in I_0$ and $s\in I_1$ if A is an (r,s)-fec set then, $eC_{T,T^{å}}(A,r,s)-A=\underline{0}$ which is an (r,s)-fc set.

Conversely, suppose that $eC_{T,T^{å}}(A,r,s)-A$ is an (r,s)-fc set. Then by (i), $eC_{T,T^{å}}(A,r,s)-A$ does not contain any non-zero an (r,s) -fc set. But $eC_{T,T^{å}}(A,r,s)-A$ is an (r,s) -fc set, then $eC_{T,T^{å}}(A,r,s)-A=\underline{0}\Rightarrow A=eC_{T,T^{å}}(A,r,s)$. So, A is an (r,s)-fec set

(iii) Suppose that $T(C)\geq r$ and $T^{å}(C)\leq s$ where $r\in I_0$ and $s\in I_1$ such that $B\leq C$ and let A be an (r,s)-gfec set such that $A\leq C$. Then $eC_{T,T^{å}}(A,r,s)\leq C$. So, $eC_{T,T^{å}}(A,r,s)=eC_{T,T^{å}}(B,r,s)$, Therefore $eC_{T,T^{å}}(B,r,s)\leq C$. So, B is an (r,s)-gfec set.

(iv) Let A be an (r,s) -gfec and $T(A)\geq r$ and $T^{å}(A)\leq s$, where $r\in I_0$ and $s\in I_1$. Then $eC_{T,T^{å}}(A,r,s)\leq A$. But, $B\leq A$ so, $eC_{T,T^{å}}(B,r,s)\leq eC_{T,T^{å}}(A,r,s)\leq A$. Also, since B is an (r,s)-gfec relative to A, then $A\wedge eC_{T,T^{å}(A)}(B,r,s)=eC_{T,T^{å}}(B,r,s)$ so $eC_{T,T^{å}}(B,r,s)=eC_{T,T^{å}}(B,r,s)\leq A$.

Now, if B is an (r,s)-gfec relative to A and $T(C)\geq r$ and $T^{å}(C)\leq s$ where $r\in I_0$ and $s\in I_1$ such that $B\leq C$, then for each an (r,s)-fo set $C\wedge A, B=B\wedge A\leq C\wedge A$. Hence B is an (r,s)-gfec relative to $A, eC_{T,T^{å}}(B,r,s)=eC_{T,T^{å}(A)}(B,r,s)\leq(C\wedge A)\leq C$. Therefore, B is an (r,s)-gfec in I^X.

Conversely, let B be an (r,s)-gfec set in I^X and $T(C)\geq r$ and $T^{å}(C)\leq s$ whenever $C\leq A$ such that $B\leq A, r\in I_0$ and $s\in I_1$. Then for each an (r,s)-fo set $D\in I^X, C=D\cap A$. But we have, B is an (r,s) -gfec set in I^X such that $B\leq D$. $eC_{T,T^{å}}(B,r,s)\leq D\Rightarrow eC_{T,T^{å}}(B,r,s)=eC_{T,T^{å}}(B,r,s)\wedge A\leq D\wedge A=C$. That is, B is an (r,s)-gfec relative to A.

(v) Suppose B is an (r,s)-feo and $A\bar{q}B, r\in I_0$ and $s\in I_1$. Then $A\leq(\underline{1}-B)$. Since $(\underline{1}-B)$ is an (r,s) -fec set of I^X and A is an (r,s)-gfec set, then $eC_{T,T^{å}}(A,r,s)\bar{q}B$.

Conversely, let B be an (r,s)-fbc set of I^X such that $A\leq B, r\in I_0$ and $s\in I_1$. Then $A\bar{q}(\underline{1}-B)$. But $eC_{T,T^{å}}(A,r,s)\bar{q}(\underline{1}-B)\Rightarrow eC_{T,T^{å}}(A,r,s)\leq B$. Hence A is an (r,s)-gfec.

Proposition 3.1 *Let $(X,T,T^{å})$ be a dfts, $A\in I^X, r\in I_0$ and $s\in I_1$.*

(i) If A is an (r,s)-gfec and an (r,s)-feo set, then A is an (r,s)-fec set.

(ii) If A is an (r,s)-fo and an (r,s)-gfec, then $A\wedge B$ is an (r,s)-gfec set whenever $B\leq eC_{T,T^{å}}(A,r,s)$.

Proof. (i) Suppose A is an (r,s)-gfec and an (r,s)-feo set such that $A\leq B, r\in I_0$ and $s\in I_1$. Then

$eC_{T,T^{å}}(A,r,s) \le A$. But we have, $A \le eC_{T,T^{å}}(A,r,s)$. Then, $A = eC_{T,T^{å}}(A,r,s)$. Therefore, A is an (r,s) -fec set.

(ii) Suppose that A is an (r,s) -fo and an (r,s) -gfec set, $r \in I_0$ and $s \in I_1$. Then

$eC_{T,T^{å}}(A,r,s) \le A \Rightarrow A$ isan (r,s) -fec set

$\Rightarrow A \wedge B$ isan (r,s) -fec

$\Rightarrow A \wedge B$ isan (r,s) -gfec.

References

[1] S. E. Abbas, E. El-Sanousy, *Several types of double fuzzy semiclosed sets*, Journal of Fuzzy Math., **20** (2012), 89-102.

[2] K. Atanassov, *New operators defined over the intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **61** (2) (1993), 131-142.

[3] D. Coker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems, **88** (1997), 81-89.

[4] E. Ekici, *On e-open sets, DP*-sets and DPE*-sets and decompositions of continuity*, Arabian Journal for Science and Engineering, **33**(2A) (2008), 269-282.

[5] J. G. Garcia and S. E. Rodabaugh, *Order-theoretic, topological, categorial redundancies of interval-valued sets, grey sets, vague sets, intervalued ``intuitionistic'' sets, ``intuitionistic'' fuzzy sets and topologies*, Fuzzy Sets and Systems, **156** (2005), 445-484.

[6] E. P. Lee and Y. B. Im, *Mated fuzzy topological spaces*, Journal of Fuzzy Logic and Intelligent Systems, (Korea), **11**(2) (2001), 161-165.

[7] S. K. Samanta and T. K Mondal, *Intuitionistic gradation of opennes*, Intuitionistic Fuzzy Topology, Busefal, **73** (1997), 8-17.

[8] V. Seenivasan and K. Kamala, *Fuzzy e-continuity and fuzzy e-open sets*, Annals of Fuzzy Mathematics and Informatics, **8**(1) (2014), 141-148.

[9] L. A. Zadeh, *Fuzzy Sets*, Information and Control, **8**(1965), 338-353.