(1,2)* -g# -continuous functions in Bitopological Spaces

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Abstract
In this paper, new generalizations of contra- (1,2)δ -g# -continuity by using the class of (1,2)δ -g# -closed sets called ((1,2)δ -g#, s)-continuity are proudly presented. Characterizations and properties of the class of ((1,2)δ -g#, s)-continuous functions are investigated and discussed in detail. We obtain various important results in the field of bitopological spaces. Finally, we introduce the two spaces called (1,2)δ -g# -regular spaces and (1,2)δ -g# -normal spaces in bitopological spaces. We obtain several bitopological characterizations of (1,2)δ -g# -regular and (1,2)δ -g# -normal spaces and some preservation theorems for (1,2)δ -g# -regular and (1,2)δ -g# -normal spaces.

Key words: (1,2)δ -g# -T₁, (1,2)δ -g# -T₂, (1,2)δ -g# -regular spaces and (1,2)δ -g# -normal spaces.

1. Introduction and Preliminaries

The systematic study of contra-continuity was due to Dontchev[2]. Several different types of contra-continuous functions have been introduced and investigated over the years by many authors[1, 4]. In this chapter, new generalizations of contra-(1,2)δ -continuity by using the class of (1,2)δ -g# -closed sets called ((1,2)δ -g#, s)-continuity are proudly presented. Characterizations and properties of the class of ((1,2)δ -g#, s)-continuous functions are investigated and discussed in detail. We obtain various important results in the field of bitopological spaces.

Using the class of g -closed sets, Munchi [6] introduced the classes of g -regular and g -normal spaces in topological spaces. In a similar way, Sheik John [13] introduced the classes of ω-regular and ω-normal spaces using the class of ω-closed sets in topological spaces. Finally, we introduce the two spaces called (1,2)δ -g# -regular spaces and (1,2)δ -g# -normal spaces in bitopological spaces. We obtain several bitopological characterizations of (1,2)δ -g# -regular and (1,2)δ -g# -normal spaces and some preservation theorems for (1,2)δ -g# -regular and (1,2)δ -g# -normal spaces.
The \((1,2)^\delta\)-\(\delta\)-interior of a subset \(H\) of \(P\) is the union of all \((1,2)^\delta\)-regular open sets of \(P\) contained in \(H\) and it is denoted by \((1,2)^\delta\)-\(\delta\)-int(\(H\)). A subset \(H\) is said to be \((1,2)^\delta\)-\(\delta\)-open if \(H = (1,2)^\delta\)-\(\delta\)-int(\(H\)). The complement of a \((1,2)^\delta\)-\(\delta\)-open set is said to be \((1,2)^\delta\)-\(\delta\)-closed. The \((1,2)^\delta\)-\(\delta\)-closure of a set \(H\) in a space \(P\) is defined by \((1,2)^\delta\)-\(\delta\)-cl(\(H\)) = \(a \in P\) : \(H \cap \tau_{1,2}^{-}\text{int}(\tau_{1,2}^{-}\text{cl}(G)) \neq \phi\), \(G \in (1,2)^\delta\)-O(\(P\)) and \(a \in G\) and it is denoted by \((1,2)^\delta\)-\(\delta\)-cl(\(H\)).

A subset \(H\) of a bitopological space \(P\) is said to be \((1,2)^\delta\)-\(\pi\)-open [12] if it is the finite union of \((1,2)^\delta\)-regular open sets. The complement of a \((1,2)^\delta\)-\(\pi\)-open set is said to be \((1,2)^\delta\)-\(\pi\)-closed. A subset \(H\) of a bitopological space \(P\) is said to be \((1,2)^\delta\)-\(\pi\)-g-closed [12] if \(\tau_{1,2}^{-}\text{-cl}(H) \subseteq G\) whenever \(H \subseteq G\) and \(G\) is \((1,2)^\delta\)-\(\pi\)-open in \(P\). The complement of a \((1,2)^\delta\)-\(\pi\)-g-closed set is said to be a \((1,2)^\delta\)-\(\pi\)-open set.

The collection of all \((1,2)^\delta\)-\(g\)-open (resp. \((1,2)^\delta\)-\(g\)-closed, \((1,2)^\delta\)-semi-open, \(\tau_{1,2}^{-}\text{-closed}\)) sets of \(P\) containing a point \(a \in P\) is denoted by \((1,2)^\delta\)-G\(O\)(\(P\), \(a\)) (resp. \((1,2)^\delta\)-RC(\(P\), \(a\)), \((1,2)^\delta\)-SO(\(P\), \(a\)), \((1,2)^\delta\)-C(\(P\), \(a\))). The collection of all \((1,2)^\delta\)-\(\pi\)-g-open (resp. \((1,2)^\delta\)-\(\pi\)-g-closed) sets of \(P\) is denoted by \((1,2)^\delta\)-GO(\(P\)) (resp. \((1,2)^\delta\)-\(\pi\)G(\(P\))). A bitopological space \(P\) is said to be (i) \((1,2)^\delta\)-Urysohn [8] if for each pair of distinct points \(a\) and \(b\) in \(P\), there exist \(M \in (1,2)^\delta\)-SO(\(P\), \(a\)) and \(N \in (1,2)^\delta\)-SO(\(P\), \(b\)) such that \(\tau_{1,2}^{-}\text{-cl}(M) \cap \tau_{1,2}^{-}\text{-cl}(N) = \phi\). (ii) Weakly \((1,2)^\delta\)-Hausdorff [8] if each element of \(P\) is an intersection of \((1,2)^\delta\)-regular closed sets.

Let \(G\) be a subset of a bitopological space \(P\). The set \(\cap H \in (1,2)^\delta\)-RO(\(P\)) \(G \subseteq H\) is called the \((1,2)^\delta\)-r-kernel of \(G\) and is denoted by \((1,2)^\delta\)-r-ker(\(G\)) where \((1,2)^\delta\)-RO(\(P\)) denote the family of all \((1,2)^\delta\)-regular open sets of \(P\). A bitopological space \(P\) is said to be (i) \((1,2)^\delta\)-S-closed [8] if every \((1,2)^\delta\)-regular closed cover of \(P\) has a finite subcover. (ii) Countably \((1,2)^\delta\)-S-closed [8] if every \((1,2)^\delta\)-countable cover of \(P\) by \((1,2)^\delta\)-regular closed sets has a finite subcover. (iii) \((1,2)^\delta\)-Lindelöf [8] if every cover of \(P\) by \((1,2)^\delta\)-regular closed sets has a \((1,2)^\delta\)-countable subcover. A function \(f : P \rightarrow Q\) is said to be weakly \((1,2)^\delta\)-continuous [9] if for each point \(a \in P\) and each \(\sigma_{1,2}\)-open set \(N\) in \(Q\) containing \(f(a)\), there exists an \(\tau_{1,2}\)-open set \(M\) containing \(a\) such that \(f(M) \subseteq \sigma_{1,2}\)-cl(\(N\)). A bitopological space \(P\) will be termed \((1,2)^\delta\)-symmetric [9] if and only if for \(a\) and \(b\) in \(P\), \(a \in \tau_{1,2}\)-cl(\(b\)) implies that \(b \in \tau_{1,2}\)-cl(\(a\)).

A subset \(H\) of a bitopological space \(P\), \(\tau_{1,2}\)-cl\(_{g}\)(\(H\)) = \(a \in P\) : \(\tau_{1,2}\)-cl(\(G\)) \(\cap H \neq \phi\), \(G \in (1,2)^\delta\)-O(\(P\)) and \(a \in G\). The following properties hold for subsets \(M\), \(N\) of a bitopological space \(P\): (i) \(a \in (1,2)^\delta\)-r-ker(\(M\)) if and only if \(M \subseteq L \neq \phi\) for any \((1,2)^\delta\)-regular closed set \(L\) containing \(a\). (ii) \(M \subseteq (1,2)^\delta\)-r-ker(\(M\)) and \(M = (1,2)^\delta\)-r-ker(\(M\)) if \(M\) is \((1,2)^\delta\)-regular open in \(P\). (iii) If \(M \subseteq N\), then \((1,2)^\delta\)-r-ker(\(M\)) \(\subseteq (1,2)^\delta\)-r-ker(\(N\)). The bitopological space \(X\) is \((1,2)^\delta\)-symmetric [9] if and only if for \(a\) is \((1,2)^\delta\)-\(g\)-closed in \(P\) for each point \(a\) of \(P\).

**Remark 1.1** [7, 9, 10] We have the following relations:

\(\tau_{1,2}\)-closed \(\rightarrow (1,2)^\delta\)-\(g\)-closed \(\rightarrow (1,2)^\delta\)-\(\pi\)-closed \(\rightarrow (1,2)^\delta\)-\(g\)-closed \(\rightarrow (1,2)^\delta\)-\(\pi\)-g-closed.

None of these implications are reversible.
2. Characterizations of \((1, 2)^{\hat{a}} - g^{\#}\)-open sets

**Lemma 2.1** For any subset \(G\) of a bitopological space \(P, P \setminus (1, 2)^{\hat{a}} - g^{\#}\)-cl\((G) = (1, 2)^{\hat{a}} - g^{\#}\)-int \((P \setminus G)\).

**Theorem 2.1** Suppose that \(G\) is \((1, 2)^{\hat{a}} - g^{\#}\)-open in \(P\) and that \(H\) is \((1, 2)^{\hat{a}} - g^{\#}\)-open in \(Q\). Then \(G \times H\) is \((1, 2)^{\hat{a}} - g^{\#}\)-open in \(P \times Q\).

**Proof.** Suppose that \(E\) is \(\tau_{1, 2}\)-closed and hence \((1, 2)^{\hat{a}} - \alpha g\)-closed in \(P \times Q\) and that \(E \subseteq G \times H\). It suffices to show that \(E \subseteq \tau_{1, 2}\)-int\((G \times H)\).

Let \((1, 2) \in E\). Then, for each \((1, 2) \in E\), \(\tau_{1, 2}\)-cl\((1) \times \tau_{1, 2}\)-cl\((2) = \tau_{1, 2}\)-cl\((1 \times 2) = \tau_{1, 2}\)-cl\((1, 2) \subseteq \tau_{1, 2}\)-cl\((E) \in E \subseteq G \times H\). Two \(\tau_{1, 2}\)-closed sets \(\text{cl}(1)\) and \(\text{cl}(2)\) are contained in \(G\) and \(H\) respectively. It follows from the assumption that \(\tau_{1, 2}\)-cl\((1) \subseteq \tau_{1, 2}\)-int\((G)\) and that \(\tau_{1, 2}\)-cl\((2) \subseteq \tau_{1, 2}\)-int\((H)\). Thus \((1, 2) \in \tau_{1, 2}\)-cl\((1) \times \tau_{1, 2}\)-cl\((2) \subseteq \tau_{1, 2}\)-int\((G) \times \tau_{1, 2}\)-int\((H) \subseteq \tau_{1, 2}\)-int\((G \times H)\). It means that, for each \((1, 2) \in E\), \((1, 2) \in \tau_{1, 2}\)-int\((G \times H)\) and hence \(E \subseteq \tau_{1, 2}\)-int\((G \times H)\). Therefore \(G \times H\) is \((1, 2)^{\hat{a}} - g^{\#}\)-open in \(P \times Q\).

3. Properties of \((1, 2)^{\hat{a}} - g^{\#}, s\) - continuous functions

**Definition 3.1** A function \(f: P \rightarrow Q\) is said to be \((1, 2)^{\hat{a}} - g^{\#}, s\)-continuous if the inverse image of each \((1, 2)^{\hat{a}}\)-regular open set of \(Q\) is \((1, 2)^{\hat{a}}\)-g\(^{\#}\)-closed in \(P\).

**Theorem 3.1** The following are equivalent for a function \(f: P \rightarrow Q\):

1. \(f\) is \((1, 2)^{\hat{a}} - g^{\#}, s\)-continuous.
2. The inverse image of a \((1, 2)^{\hat{a}}\)-regular closed set of \(Q\) is \((1, 2)^{\hat{a}} - g^{\#}\)-open in \(P\).
3. \(f^{-1}(\tau_{1, 2}\)-int\((\sigma_{1, 2}\)-cl\((K))\) is \((1, 2)^{\hat{a}} - g^{\#}\)-closed in \(P\) for every \(\sigma_{1, 2}\)-open subset \(K\) of \(Q\).
4. \(f^{-1}(\tau_{1, 2}\)-cl\((\sigma_{1, 2}\)-int\((K))\) is \((1, 2)^{\hat{a}} - g^{\#}\)-open in \(P\) for every \(\sigma_{1, 2}\)-closed subset \(K\) of \(Q\).
5. \(f^{-1}(\sigma_{1, 2}\)-cl\((K))\) is \((1, 2)^{\hat{a}} - g^{\#}\)-open in \(P\) for every \(K \in (1, 2)^{\hat{a}} - \beta O(Q)\).
6. \(f^{-1}(\sigma_{1, 2}\)-cl\((K))\) is \((1, 2)^{\hat{a}} - g^{\#}\)-open in \(P\) for every \(K \in (1, 2)^{\hat{a}} - \beta SO(Q)\).
7. \(f^{-1}(\sigma_{1, 2}\)-int\((\sigma_{1, 2}\)-cl\((K))\) is \((1, 2)^{\hat{a}} - g^{\#}\)-closed in \(P\) for every \(K \in (1, 2)^{\hat{a}} - \beta PO(Q)\).

**Proof.**

(1) \(\iff\) (2). Obvious.

(1) \(\iff\) (3). Let \(K\) be an \(\sigma_{1, 2}\)-open subset of \(Q\). Since \(\sigma_{1, 2}\)-int\((\sigma_{1, 2}\)-cl\((K))\) is \((1, 2)^{\hat{a}}\)-regular open, \(f^{-1}(\sigma_{1, 2}\)-int\((\sigma_{1, 2}\)-cl\((K))\)) is \((1, 2)^{\hat{a}} - g^{\#}\)-closed. The converse is similar.

(2) \(\iff\) (4). Similar to (1) \(\iff\) (3).

(2) \(\implies\) (5). Let \(L\) be any \((1, 2)^{\hat{a}}\)-\(\beta\)-open set of \(Q\). We know that \(\sigma_{1, 2}\)-cl\((L)\) is \((1, 2)^{\hat{a}}\)-regular closed. Then by (2) \(f^{-1}(\sigma_{1, 2}\)-cl\((L))\) is \((1, 2)^{\hat{a}} - g^{\#}\)-open in \(P\).

(5) \(\implies\) (6). Obvious from the fact that \((1, 2)^{\hat{a}}\)-\(\beta\)-SO\((Q) \subseteq (1, 2)^{\hat{a}}\)-\(\beta\) O\((Q)\).

(6) \(\implies\) (7). Let \(L \in (1, 2)^{\hat{a}} - \beta PO(Q)\). Then \(Q\) \(\setminus \sigma_{1, 2}\)-int\((\sigma_{1, 2}\)-cl\((L))\) is \((1, 2)^{\hat{a}}\)-regular closed and hence it is \((1, 2)^{\hat{a}}\)-semi-open. Then, we have \(P \setminus f^{-1}(\sigma_{1, 2}\)-int\((\sigma_{1, 2}\)-cl\((L))\) \(= f^{-1}(Q \setminus \sigma_{1, 2}\)-int\((\sigma_{1, 2}\)-cl\((L))) = f^{-1}(\sigma_{1, 2}\)-cl\((Q \setminus \sigma_{1, 2}\)-int\((\sigma_{1, 2}\)-cl\((L))))\) is \((1, 2)^{\hat{a}} - g^{\#}\)-open in \(P\). Hence \(f^{-1}(\sigma_{1, 2}\)-int\((\sigma_{1, 2}\)-cl\((L)))\) is \((1, 2)^{\hat{a}} - g^{\#}\)-closed in \(P\).
-cl(L)) is $(1,2)^\delta - g^\#$-closed in P.

(7) $\Rightarrow$ (1). Let $L$ be any $(1,2)^\delta$-regular open set of $Q$. Then $L \subset (1,2)^\delta$-PO($Q$) and hence $f^{-1}(L) = f^{-1}([\sigma_{1,2} - \text{int}(\sigma_{1,2} - \text{cl}(L))]$ is $(1,2)^\delta - g^\#$-closed in P.

**Lemma 3.1** For a subset $H$ of a bitopological space $Q$, the following properties hold:
1. $(1,2)^\delta - \alpha \text{cl}(H) = \sigma_{1,2} - \text{cl}(H)$ for every $H \subset (1,2)^\delta - \beta O(Q)$.
2. $(1,2)^\delta - pcl(H) = \sigma_{1,2} - \text{cl}(H)$ for every $H \subset (1,2)^\delta - SO(Q)$.
3. $(1,2)^\delta - scl(H) = \sigma_{1,2} - \text{int}(\sigma_{1,2} - \text{cl}(H))$ for every $H \subset (1,2)^\delta - PO(Q)$.

**Corollary 3.1** The following are equivalent for a function $f : P \rightarrow Q$:
1. $f$ is $(1,2)^\delta - g^\#$, s)-continuous.
2. $f^{-1}((1,2)^\delta - \alpha \text{cl}(H))$ is $(1,2)^\delta - g^\#$-open in $P$ for every $H \subset (1,2)^\delta - \beta O(Q)$.
3. $f^{-1}((1,2)^\delta - pcl(H))$ is $(1,2)^\delta - g^\#$-open in $P$ for every $H \subset (1,2)^\delta - SO(Q)$.
4. $f^{-1}((1,2)^\delta - scl(H))$ is $(1,2)^\delta - g^\#$-closed in $P$ for every $H \subset (1,2)^\delta - PO(Q)$.

**Proof.** It follows from Theorem (3.1) and Lemma (3.1)

4. The related functions with $(1,2)^\delta - g^\#$, s)-continuous functions

**Definition 4.1** A function $f : P \rightarrow Q$ is said to be
1. perfectly $(1,2)^\delta$-continuous [3] if $f^{-1}(K)$ is $(1,2)^\delta$-clopen in $P$ for every $\sigma_{1,2}$-open set $K$ of $Q$.
2. regular $(1,2)^\delta$-set-connected [3] if $f^{-1}(K)$ is $(1,2)^\delta$-clopen in $P$ for every $K \subset (1,2)^\delta$-RO($Q$).
3. almost $(1,2)^\delta$-s-continuous [3] if for each $a \subset P$ and each $K \subset (1,2)^\delta$-SO($Q$, $f(a)$), there exists an $\tau_{1,2}$-open set $V$ in $P$ containing a such that $f(V) \subset (1,2)^\delta$-scl($K$).
4. strongly $(1,2)^\delta$-continuous [5] if the inverse image of every set in $Q$ is $(1,2)^\delta$-clopen in $P$.
5. $(1,2)^\delta$-RC-continuous [5] if $f^{-1}(K)$ is $(1,2)^\delta$-regular closed in $P$ for each $\sigma_{1,2}$-open set $K$ of $Q$.
6. contra $(1,2)^\delta$-R-map [5] if $f^{-1}(K)$ is $(1,2)^\delta$-regular closed in $P$ for each $(1,2)^\delta$-regular open set $K$ of $Q$.
7. contra-$(1,2)^\delta$-super-continuous [8] if for each $a \subset P$ and for each $G \subset (1,2)^\delta$-C($Q$, $f(a)$), there exists a $(1,2)^\delta$-regular open set $H$ in $P$ containing a such that $f(H) \subset G$.
8. almost $(1,2)^\delta$-contra-super-continuous [8] if $f^{-1}(K)$ is $(1,2)^\delta$-s-connected in $P$ for every $(1,2)^\delta$-regular open set $K$ of $Q$.
9. contra $(1,2)^\delta$-continuous [10] if $f^{-1}(K)$ is $\tau_{1,2}$-closed in $P$ for every $\sigma_{1,2}$-open set $K$ of $Q$.
10. contra $(1,2)^\delta - g$-continuous [10] if $f^{-1}(K)$ is $(1,2)^\delta - g$-closed in $P$ for every $\sigma_{1,2}$-open set $K$ of $Q$.
11. $(1,2)^\delta - \theta$, s)-continuous [11] if for each $a \subset P$ and each $N \subset (1,2)^\delta$-SO($Q$, $f(a)$),
there exists an $\tau_{1,2}$-open set $M$ in $P$ containing a such that $f(M) \subseteq \tau_{1,2}\text{-cl}(N)$.

12. contra $(1, 2)^{\hat{a}} - \pi g$ -continuous [12] if $f^{-1}(K)$ is $(1, 2)^{\hat{a}} - \pi g$ -closed in $P$ for each $\sigma_{1,2}$-open set $K$ of $Q$.

13. $(1, 2)^{\hat{a}} - g$, $s$)-continuous [11] if $f^{-1}(K)$ is $(1, 2)^{\hat{a}} - g$ -closed in $P$ for each $(1, 2)^{\hat{a}}$-regular open set $K$ of $Q$.

14. $(1, 2)^{\hat{a}} - \hat{g}$, $s$)-continuous [12] if $f^{-1}(K)$ is $(1, 2)^{\hat{a}} - \hat{g}$ -closed in $P$ for each $(1, 2)^{\hat{a}}$-regular open set $K$ of $Q$.

15. $(1, 2)^{\hat{a}} - \pi g$, $s$)-continuous [12] if $f^{-1}(K)$ is $(1, 2)^{\hat{a}} - \pi g$ -closed in $P$ for each $(1, 2)^{\hat{a}}$-regular open set $K$ of $Q$.

Definition 4.2 A function $f : P \to Q$ is said to be contra $(1, 2)^{\hat{a}} - \hat{g}$ -continuous (resp. contra $(1, 2)^{\hat{a}} - g^s$ -continuous) if $f^{-1}(K)$ is $\hat{g}$ -closed (resp. $(1, 2)^{\hat{a}} - g^s$ -closed) in $P$ for each $\sigma_{1,2}$-open set $K$ of $Q$.

Remark 4.1 The following diagram holds for a function $f : P \to Q$:
None of these implications is reversible as shown in the following examples and in the related paper[3, 5, 8, 10, 11, 12].

**Example 4.1** Let $P = 1, 2, 3$ with the topologies $\tau_1 = \phi, 2, P$ and $\tau_2 = \phi, 1, 3, P$. Then the subsets in $\phi, 2, 1, 3, P$ are called $\tau_{1,2}$-open and the subsets in $\phi, 2, 1, 3, P$ are called $\tau_{1,2}$-closed. Let $Q = 1, 2, 3$ with the topologies $\sigma_1 = \phi, Q$ and $\sigma_2 = \phi, 1, 2, Q$. Then the subsets in $\phi, 1, 2, Q$ are called $\sigma_{1,2}$-open and the subsets in $\phi, 3, Q$ are called $\sigma_{1,2}$-closed. Then the identity function $f : P \to Q$ is $((1,2)^{\downarrow} - g^{\uparrow}, s)$-continuous but not contra $(1,2)^{\downarrow} - g^{\uparrow}$-continuous.

**Example 4.2** Let $P = 1, 2, 3$ with the topologies $\tau_1 = \phi, 1, P$ and $\tau_2 = \phi, 2, 3, P$. Then the subsets in $\phi, 1, 2, 3, P$ are called $\tau_{1,2}$-open and the subsets in $\phi, 1, 2, 3, P$ are called $\tau_{1,2}$-closed. Let $Q = 1, 2, 3$ with the topologies $\sigma_1 = \phi, Q$ and $\sigma_2 = \phi, 1, 2, Q$. Then the subsets in $\phi, 1, 2, Q$ are called $\sigma_{1,2}$-open and the subsets in $\phi, 3, Q$ are called $\sigma_{1,2}$-closed. Then the identity function $f : P \to Q$ is contra $(1,2)^{\downarrow} - g^{\uparrow}$-continuous but not contra $(1,2)^{\downarrow} - g^{\uparrow}$-continuous.

**Example 4.3** Let $P = 1, 2, 3, 4$ with the topologies $\tau_1 = \phi, 2, P$ and $\tau_2 = \phi, 1, 3, 4, P$. Then the subsets in $\phi, 2, 1, 3, 4, P$ are called $\tau_{1,2}$-open and the subsets in $\phi, 2, 1, 3, 4, P$ are called $\tau_{1,2}$-closed. Let $Q = 1, 2, 3, 4$ with the topologies $\sigma_1 = \phi, 2, Q$ and $\sigma_2 = \phi, 1, 3, 4, Q$. Then the subsets in $\phi, 2, 1, 3, 4, Q$ are called $\sigma_{1,2}$-open and the subsets in $\phi, 2, 1, 3, 4, Q$ are called $\sigma_{1,2}$-closed. Then the function $f : P \to Q$ which is defined as $f(1) = 2, f(2) = 3, f(3) = 4, f(4) = 1$ is $((1,2)^{\downarrow} - \theta, s)$-continuous but not $((1,2)^{\downarrow} - g^{\uparrow}, s)$-continuous.

**Example 4.4** Let $P = 1, 2, 3, 4$ with the topologies $\tau_1 = \phi, 3, 1, 3, 4, P$ and $\tau_2 = \phi, 1, 4, P$. Then the subsets in $\phi, 3, 1, 3, 4, P$ are called $\tau_{1,2}$-open and the subsets in $\phi, 2, 2, 3, 1, 2, 4, P$ are called $\tau_{1,2}$-closed. Let $Q = 1, 2, 3, 4$ with the topologies $\sigma_1 = \phi, 3, 1, 3, 4, Q$ and $\sigma_2 = \phi, 1, 4, Q$. Then the subsets in $\phi, 3, 1, 3, 4, Q$ are called $\sigma_{1,2}$-open and the subsets in $\phi, 2, 3, 1, 2, 4, Q$ are called $\sigma_{1,2}$-closed. Then the function $f : P \to Q$ which is defined as $f(1) = 3, f(2) = 3, f(3) = 2, f(4) = 2$ is $((1,2)^{\downarrow} - g^{\uparrow}, s)$-continuous but not $((1,2)^{\downarrow} - \theta, s)$-continuous.

**Example 4.5** Let $P = 1, 2, 3$ with the topologies $\tau_1 = \phi, P$ and $\tau_2 = \phi, 1, 2, P$. Then the subsets in $\phi, 1, 2, P$ are called $\tau_{1,2}$-open and the subsets in $\phi, 3, P$ are called $\tau_{1,2}$-closed. Let $Q = 1, 2, 3$ with the topologies $\sigma_1 = \phi, 3, Q$ and $\sigma_2 = \phi, 1, 3, Q$. Then the subsets in $\phi, 3, 1, 3, Q$ are called $\sigma_{1,2}$-open and the subsets in $\phi, 2, 1, 2, Q$ are called $\sigma_{1,2}$-closed. Then the identity function $f : P \to Q$ is contra $(1,2)^{\downarrow} - g^{\uparrow}$-continuous but not contra $(1,2)^{\downarrow}$-continuous.

**Definition 4.3** A function $f : P \to Q$ is said to be almost $(1,2)^{\downarrow} - g^{\uparrow}$-continuous if $f^{-1}(G)$ is $(1,2)^{\downarrow} - g^{\uparrow}$-open in $P$ for every $(1,2)^{\downarrow}$-regular open set $G$ of $Q$.

**Definition 4.4** A bitopological space $P$ is said to be $(1,2)^{\downarrow} - P_{\alpha}$ if for any $\tau_{1,2}$-open set $K$ of $P$ and each $a \in K$, there exists $L \in (1,2)^{\downarrow} - RC(P, a)$ such that $a \in L \subseteq K$. 
Theorem 4.1 Let \( f : P \rightarrow Q \) be a function from a \( (1,2)^\# -T^g_\# \) -space \( P \) to a bitopological space \( Q \). Then the following are equivalent.

1. \( f \) is \( (1,2)^\# -\theta, s \)-continuous,
2. \( f \) is \( (1,2)^\# -g^#, s \)-continuous.

Theorem 4.2 Let \( f : P \rightarrow Q \) be a function. Then, if \( f \) is \( (1,2)^\# -g^#, s \)-continuous, \( P \) is \( (1,2)^\# -T^g_\# \) and \( Q \) is \( (1,2)^\# -P_\# \), then \( f \) is \( (1,2)^\# -\text{continuous} \).

Proof. Let \( F \) be any \( \sigma_{1,2} \) -open set of \( Q \). Since \( Q \) is \( (1,2)^\# -P_\# \), there exists a subfamily \( \Phi \) of \( (1,2)^\# -RC(Q) \) such that \( G = \bigcup H : H \in \Phi \). Since \( P \) is \( (1,2)^\# -T^g_\# \) and \( f \) is \( (1,2)^\# -g^#, s \)-continuous, \( f^{-1}(H) \) is \( \tau_{1,2} \)-open in \( P \) for each \( H \in \Phi \) and \( f^{-1}(F) \) is \( \tau_{1,2} \)-open in \( P \). Thus, \( f \) is \( (1,2)^\# -\text{continuous} \).

Theorem 4.3 Let \( f : P \rightarrow Q \) and \( g : Q \rightarrow R \) be functions. Then, the following properties hold:

1. If \( f \) is \( (1,2)^\# -g^# \)-irresolute and \( g \) is \( (1,2)^\# -g^#, s \)-continuous, then \( g \ o f : P \rightarrow R \) is \( (1,2)^\# -g^#, s \)-continuous.
2. If \( f \) is \( (1,2)^\# -g^# \), \( s \)-continuous and \( g \) is contra \( (1,2)^\# -R \)-map, then \( g \ o f : P \rightarrow R \) is almost \( (1,2)^\# -g^# \)-continuous.
3. If \( f \) is \( (1,2)^\# -g^# \)-continuous and \( g \) is \( (1,2)^\# -\theta, s \)-continuous, then \( g \ o f : P \rightarrow R \) is \( (1,2)^\# -g^#, s \)-continuous.
4. If \( f \) is \( (1,2)^\# -g^#, s \)-continuous and \( g \) is \( (1,2)^\# -RC \)-continuous, then \( g \ o f : P \rightarrow R \) is \( (1,2)^\# -g^# \)-continuous.
5. If \( f \) is almost \( (1,2)^\# -g^# \)-continuous and \( g \) is contra \( (1,2)^\# -R \)-map, then \( g \ o f : P \rightarrow R \) is \( (1,2)^\# -g^#, s \)-continuous.

5. Bitopological Fundamental properties

Definition 5.1 A bitopological space \( P \) is said to be

1. \( (1,2)^\# -g^# -T_2 \) if for each pair of distinct points \( a \) and \( b \) in \( P \), there exist \( M \in (1,2)^\# -G^# O(P, a) \) and \( N \in (1,2)^\# -G^# O(P, b) \) such that \( M \cap N = \emptyset \).
2. \( (1,2)^\# -g^# -T_1 \) if for each pair of distinct points \( a \) and \( b \) in \( P \), there exist \( (1,2)^\# -g^# \) -open sets \( M \) and \( N \) containing \( a \) and \( b \), respectively, such that \( b \not\in M \) and \( a \not\in N \).

Remark 5.1 The following implications are hold for a bitopological space \( P \).

1. \( (1,2)^\# -T_2 \Rightarrow (1,2)^\# -g^# -T_2 \).
2. \( (1,2)^\# -T_1 \Rightarrow (1,2)^\# -g^# -T_1 \).

These implications are not reversible.

Example 5.1 Let \( P = 1, 2, 3 \) with the topologies \( \tau_1 = \emptyset, 1, 2, 1, 2, P \) and \( \tau_2 = \emptyset, 3, P \). Then
the subsets in \( \phi, 1, 2, 3, 1, 2, 1, 3, 2, 3, P \) are called \( \tau_{1,2} \)-open and the subsets in \( \phi, 1, 2, 3, 1, 2, 1, 3, 2, 3, P \) are called \( \tau_{1,2} \)-closed. Then \( P \) is both \((1,2)^{\hat{\alpha}} - g^{\#} - T_2\) and \((1,2)^{\hat{\alpha}} - g^{\#} - T_1\) but neither \((1,2)^{\hat{\alpha}} - T_1\) nor \((1,2)^{\hat{\alpha}} - T_2\).

Lemma 5.1 In the \((1,2)^{\hat{\alpha}} - g^{\#} - T_2\) space, the \((1,2)^{\hat{\alpha}} - g^{\#}\)-closure of every \((1,2)^{\hat{\alpha}} - g^{\#}\)-open set is \((1,2)^{\hat{\alpha}} - g^{\#}\)-open.

Proof. Every \((1,2)^{\hat{\alpha}} - g^{\#}\)-regular open subset is \( \tau_{1,2} \)-open and every \( \tau_{1,2} \)-open set is \((1,2)^{\hat{\alpha}} - g^{\#}\)-open. Thus, every \((1,2)^{\hat{\alpha}} - g^{\#}\)-regular closed subset is \((1,2)^{\hat{\alpha}} - g^{\#}\)-closed. Now let \( H \) be any \((1,2)^{\hat{\alpha}} - g^{\#}\)-open set \( G \) such that \( G \subseteq H \subseteq \tau_{1,2} \)-cl(\(G\)). Hence, we have \( G \subseteq (1,2)^{\hat{\alpha}} - g^{\#}\)-cl(\(G\)) \subseteq (1,2)^{\hat{\alpha}} - g^{\#}\)-cl(H) \subseteq (1,2)^{\hat{\alpha}} - g^{\#}\)-cl(\(\tau_{1,2} \)-cl(\(G\))) = \( \tau_{1,2} \)-cl(\(G\)) since \( \tau_{1,2} \)-cl(\(G\)) is \((1,2)^{\hat{\alpha}} - g^{\#}\)-regular closed. Therefore, \((1,2)^{\hat{\alpha}} - g^{\#}\)-cl(H) is \((1,2)^{\hat{\alpha}} - g^{\#}\)-open.

Theorem 5.1 A bitopological space \( P \) is \((1,2)^{\hat{\alpha}} - g^{\#} - T_2\) if and only if for any pair of distinct points \( a, b \) of \( P \) there exist \((1,2)^{\hat{\alpha}} - g^{\#}\)-open sets \( M \) and \( N \) such that \( a \in M \) and \( b \in N \) and \((1,2)^{\hat{\alpha}} - g^{\#}\)-cl(\(M\)) \( \cap \) \((1,2)^{\hat{\alpha}} - g^{\#}\)-cl(\(N\)) = \( \phi \).

Proof. Necessity. Suppose that \( P \) is \((1,2)^{\hat{\alpha}} - g^{\#} - T_2\). Let \( a \) and \( b \) be distinct points of \( P \). There exist \((1,2)^{\hat{\alpha}} - g^{\#}\)-open sets \( M \) and \( N \) such that \( a \in M \), \( b \in N \) and \( M \cap N = \phi \). Hence \((1,2)^{\hat{\alpha}} - g^{\#}\)-cl(\(M\)) \( \cap \) \((1,2)^{\hat{\alpha}} - g^{\#}\)-cl(\(N\)) = \( \phi \) and \((1,2)^{\hat{\alpha}} - g^{\#}\)-cl(\(M\)) is \((1,2)^{\hat{\alpha}} - g^{\#}\)-open. Therefore, we obtain \((1,2)^{\hat{\alpha}} - g^{\#}\)-cl(\(M\)) \( \cap \) \((1,2)^{\hat{\alpha}} - g^{\#}\)-cl(\(N\)) = \( \phi \). Sufficiency. This is obvious.

Definition 5.2 A bitopological space \( P \) is said to be
1. countably \((1,2)^{\hat{\alpha}} - g^{\#}\)-compact if every \((1,2)^{\hat{\alpha}}\)-countable cover of \( P \) by \((1,2)^{\hat{\alpha}} - g^{\#}\)-open sets has a finite subcover.
2. \((1,2)^{\hat{\alpha}} - g^{\#}\)-Lindelof if every \((1,2)^{\hat{\alpha}} - g^{\#}\)-open cover of \( P \) has a \((1,2)^{\hat{\alpha}}\)-countable subcover.

Theorem 5.2 Let \( f : P \rightarrow Q \) be a \((1,2)^{\hat{\alpha}} - g^{\#}, s\)-continuous surjection. Then the following statements hold:
1. if \( P \) is \((1,2)^{\hat{\alpha}} - g^{\#}\)-Lindelof, then \( Q \) is \((1,2)^{\hat{\alpha}} - S\)-Lindelof.
2. if \( P \) is countably \((1,2)^{\hat{\alpha}} - g^{\#}\)-compact, then \( Q \) is countably \((1,2)^{\hat{\alpha}} - S\)-closed.

Definition 5.3 A bitopological space \( P \) is said to be \((1,2)^{\hat{\alpha}} - g^{\#}\)-connected if \( P \) is not the union of two disjoint nonempty \((1,2)^{\hat{\alpha}} - g^{\#}\)-open sets.

Example 5.2 Let \( P = 1, 2, 3 \) with the topologies \( \tau_1 = \phi, 1, P \) and \( \tau_2 = \phi, 2, 3, P \). Then the subsets in \( \phi, 1, 2, 3, P \) are called \( \tau_{1,2}\)-open and the subsets in \( \phi, 1, 2, 3, P \) are called \( \tau_{1,2}\)-closed. Then \( P \) is not \((1,2)^{\hat{\alpha}} - g^{\#}\)-connected.
Example 5.3 Let $P = 1, 2, 3$ with the topologies $\tau_1 = \phi, 1, P$ and $\tau_2 = \phi, 1, 2, P$. Then the subsets in $\phi, 1, 2, P$ are called $\tau_{1,2}$-open and the subsets in $\phi, 3, 2, 3, P$ are called $\tau_{1,2}$-closed. Then $P$ is $(1,2)^{\tilde{\alpha}}$-connected.

Theorem 5.3 Let $f : P \to Q$ be a $(1,2)^{\tilde{\alpha}}$-continuous surjection. If $P$ is $(1,2)^{\tilde{\alpha}}$-closed, then $Q$ is $(1,2)^{\tilde{\alpha}}$-connected.

Proof. Assume that $Q$ is not $(1,2)^{\tilde{\alpha}}$-connected space. Then there exist nonempty disjoint $\tau_{1,2}$-open sets $M$ and $N$ such that $Q = M \cup N$. Also $M$ and $N$ are $(1,2)^{\tilde{\alpha}}$-clopen in $Q$. Since $f$ is $(1,2)^{\tilde{\alpha}}$-continuous, $f^{-1}(M)$ and $f^{-1}(N)$ are $(1,2)^{\tilde{\alpha}}$-open in $P$. Moreover $f^{-1}(M)$ and $f^{-1}(N)$ are nonempty disjoint and $P = f^{-1}(M) \cup f^{-1}(N)$. This shows that $P$ is not $(1,2)^{\tilde{\alpha}}$-connected. This contradicts the assumption that $Q$ is not $(1,2)^{\tilde{\alpha}}$-connected.

6. $(1,2)^{\tilde{\alpha}}$-regular and $(1,2)^{\tilde{\alpha}}$-normal spaces

Definition 6.1 A bitopological space $P$ is said to be $(1,2)^{\tilde{\alpha}}$-regular if for every $(1,2)^{\tilde{\alpha}}$-closed set $G$ and each point $a \notin G$, there exist disjoint $\tau_{1,2}$-open sets $M$ and $N$ such that $G \subseteq M$ and $a \in N$.

Theorem 6.1 Let $P$ be a bitopological space. Then the following statements are equivalent:

1. $P$ is a $(1,2)^{\tilde{\alpha}}$-regular space.
2. For each $a \in P$ and $(1,2)^{\tilde{\alpha}}$-neighbourhood $G$ of $a$ there exists an $\tau_{1,2}$-open neighbourhood $H$ of $a$ such that $\tau_{1,2}$-cl($H$) $\subseteq$ $G$.

Proof. (1) $\Rightarrow$ (2). Let $G$ be any $(1,2)^{\tilde{\alpha}}$-neighbourhood of $a$. Then there exist a $(1,2)^{\tilde{\alpha}}$-open set $K$ such that $a \in K \subseteq G$. Since $K^c$ is $(1,2)^{\tilde{\alpha}}$-closed and $a \notin K^c$, by hypothesis there exist $\tau_{1,2}$-open sets $M$ and $N$ such that $K^c \subseteq M$, $a \in N$ and $M \cap N = \phi$ and so $N \subseteq M^c$. Now, $\tau_{1,2}$-cl($N$) $\subseteq$ $\tau_{1,2}$-cl($M^c$) = $M^c$ and $K^c \subseteq M$ implies $M^c \subseteq K \subseteq G$. Therefore $\tau_{1,2}$-cl($N$) $\subseteq$ $G$.

(2) $\Rightarrow$ (1). Let $K$ be any $(1,2)^{\tilde{\alpha}}$-closed set and $a \notin K$. Then $a \in K^c$ and $K^c$ is $(1,2)^{\tilde{\alpha}}$-open and so $K^c$ is an $(1,2)^{\tilde{\alpha}}$-neighbourhood of $a$. By hypothesis, there exists an $\tau_{1,2}$-open neighbourhood $L$ of $a$ such that $a \in L$ and $\tau_{1,2}$-cl($L$) $\subseteq$ $K^c$, which implies $K \subseteq (\tau_{1,2}$-cl($L$))$^c$. Then $(\tau_{1,2}$-cl($L$))$^c$ is an open set containing $K$ and $L \cap (\tau_{1,2}$-cl($L$))$^c$ = $\phi$. Therefore, $P$ is $(1,2)^{\tilde{\alpha}}$-regular.

Theorem 6.2 For a bitopological space $P$ the following are equivalent:

1. $P$ is $(1,2)^{\tilde{\alpha}}$-normal.
2. For every pair of disjoint $\tau_{1,2}$-closed sets $M$ and $N$, there exist $(1,2)^{\tilde{\alpha}}$-open sets $S$ and $T$ such that $M \subseteq S$, $N \subseteq T$ and $S \cap T = \phi$.

Proof. (1) $\Rightarrow$ (2). Let $M$ and $N$ be disjoint $\tau_{1,2}$-closed subsets of $P$. By hypothesis, there exist disjoint $\tau_{1,2}$-open sets (and hence $(1,2)^{\tilde{\alpha}}$-open sets) $S$ and $T$ such that $M \subseteq S$ and $N \subseteq T$. Therefore, $P$ is $(1,2)^{\tilde{\alpha}}$-normal.
T.
(2) \( \Rightarrow \) (1). Let M and N be \( \tau_{1,2} \)-closed subsets of P. Then by assumption, M \( \subseteq E, N \subseteq F \) and \( E \cap F = \phi \), where E and F are disjoint \( (1,2)^\# \)-open sets. Since M and N are \( (1,2)^\# - \alpha g \)-closed, by Theorem Error! Reference source not found., \( M \subseteq \tau_{1,2} \)-int(E) and N \( \subseteq \tau_{1,2} \)-int(F). Further, \( \tau_{1,2} \)-int(E) \( \cap \tau_{1,2} \)-int(F) = \( \tau_{1,2} \)-int(E \( \cap \) F) = \( \phi \).

A bitopological space \( P \) is \( (1,2)^\# - g^\# \)-regular if and only if for each \( (1,2)^\# - g^\# \)-closed set K of \( P \) and each a \( \in K^c \) there exist \( \tau_{1,2} \)-open sets M and N of \( P \) such that a \( \in M, K \subseteq N \) and \( \tau_{1,2} \)-cl(M) \( \cap \tau_{1,2} \)-cl(N) = \( \phi \).

**Proof.** Let K be a \( (1,2)^\# - g^\# \)-closed set of \( P \) and a \( \not\in K \). Then there exist \( \tau_{1,2} \)-open sets \( S_0 \) and \( T \) of \( P \) such that x \( \in S_0, K \subseteq T \) and \( S_0 \cap T = \phi \), which implies \( S_0 \cap \tau_{1,2} \)-cl(T) = \( \phi \).

Since \( \tau_{1,2} \)-cl(T) is \( \tau_{1,2} \)-closed, it is \( (1,2)^\# - g^\# \)-closed and a \( \not\in \tau_{1,2} \)-cl(T). Since P is \( (1,2)^\# - g^\# \)-regular, there exist \( \tau_{1,2} \)-open sets E and F of \( P \) such that a \( \in E, \tau_{1,2} \)-cl(T) \( \subseteq \) F and \( E \cap F = \phi \), which implies \( \tau_{1,2} \)-cl(E) \( \cap \) F = \( \phi \). Let \( D = S_0 \cap E, \) then D and T are \( \tau_{1,2} \)-open sets of \( P \) such that a \( \in D, K \subseteq T \) and \( \tau_{1,2} \)-cl(D) \( \cap \tau_{1,2} \)-cl(T) = \( \phi \).

Converse part is trivial.

**Theorem 6.3** A bitopological space \( T \) is said to be \( (1,2)^\# - g^\# \)-normal if for any pair of disjoint \( (1,2)^\# - g^\# \)-closed sets M and N, there exist disjoint \( \tau_{1,2} \)-open sets G and H such that M \( \subseteq G \) and N \( \subseteq H \).

We characterize \( (1,2)^\# - g^\# \)-normal space.

**Theorem 6.4** Let \( P \) be a bitopological space. Then the following statements are equivalent:

1. P is \( (1,2)^\# - g^\# \)-normal.
2. For each \( (1,2)^\# - g^\# \)-closed set K and for each \( (1,2)^\# - g^\# \)-open set G containing K, there exists an \( \tau_{1,2} \)-open set H containing K such that \( \tau_{1,2} \)-cl(G) \( \subseteq \) H.
3. For each pair of disjoint \( (1,2)^\# - g^\# \)-closed sets M and N in P, there exists an \( \tau_{1,2} \)-open set G containing M such that \( \tau_{1,2} \)-cl(G) \( \cap \) N = \( \phi \).
4. For each pair of disjoint \( (1,2)^\# - g^\# \)-closed sets M and N in P, there exist \( \tau_{1,2} \)-open sets G containing M and H containing N such that \( \tau_{1,2} \)-cl(G) \( \cap \tau_{1,2} \)-cl(H) = \( \phi \).

**Proof.** (1) \( \Rightarrow \) (2). Let K be a \( (1,2)^\# - g^\# \)-closed set and G be a \( (1,2)^\# - g^\# \)-open set such that K \( \subseteq \) G. Then K \( \cap \) G\(^c\) = \( \phi \). By assumption, there exist \( \tau_{1,2} \)-open sets H and L such that K \( \subseteq \) H, G\(^c\) \( \subseteq \) L and H \( \cap \) L = \( \phi \), which implies \( \tau_{1,2} \)-cl(H) \( \cap \) L = \( \phi \). Now, \( \tau_{1,2} \)-cl(H) \( \subseteq \) G\(^c\) \( \subseteq \tau_{1,2} \)-cl(H) \( \cap \) L = \( \phi \) and so \( \tau_{1,2} \)-cl(H) \( \subseteq \) G.

(2) \( \Rightarrow \) (3). Let M and N be disjoint \( (1,2)^\# - g^\# \)-closed sets of P. Since M \( \cap \) N = \( \phi \), M \( \subseteq \) N\(^c\) and N\(^c\) is \( (1,2)^\# - g^\# \)-open. By assumption, there exists an \( \tau_{1,2} \)-open set G containing M such that \( \tau_{1,2} \)-cl(G) \( \subseteq \) N\(^c\) and so \( \tau_{1,2} \)-cl(G) \( \cap \) N = \( \phi \).

(3) \( \Rightarrow \) (4). Let M and N be any two disjoint \( (1,2)^\# - g^\# \)-closed sets of P. Then by assumption, there exists an \( \tau_{1,2} \)-open set G containing M such that \( \tau_{1,2} \)-cl(G) \( \cap \) N = \( \phi \). Since \( \tau_{1,2} \)-cl(G) is
τ_{1,2} \text{-closed}, it is (1, 2)^{\#} - g^\# \text{-closed and so } N \text{ and } τ_{1,2} - \text{cl}(G) \text{ are disjoint (1, 2)^{\#} - g^\# \text{-closed sets in}}
\text{P. Therefore again by assumption, there exists an } τ_{1,2} \text{-open set } H \text{ containing } N \text{ such that } τ_{1,2} - \text{cl}(H) \bigcap τ_{1,2} - \text{cl}(G) = \phi.

(4) ⇒ (1). Let M and N be any two disjoint (1, 2)^{\#} - g^\# \text{-closed sets of } P. By assumption, there exist τ_{1,2} \text{-open sets } G \text{ containing } M \text{ and } H \text{ containing } N \text{ such that } τ_{1,2} - \text{cl}(G) \bigcap τ_{1,2} - \text{cl}(H) = \phi, \text{ we have } G \bigcap H = \phi \text{ and thus } P \text{ is (1, 2)^{\#} - g^\# \text{-normal}.}

**Theorem 6.5** \text{If } f : P \to Q \text{ is bijective, pre-(1, 2)^{\#} - } αg \text{-open, (1, 2)^{\#} - } g^\# \text{-continuous and}
\text{(1, 2)^{\#} - open and } P \text{ is (1, 2)^{\#} - } g^\# \text{-normal, then } Q \text{ is (1, 2)^{\#} - } g^\# \text{-normal.}

**Proof.** Let M and N be any disjoint (1, 2)^{\#} - g^\# \text{-closed sets of } Q. The function f is (1, 2)^{\#} - g^\# \text{-irresolute and so } f^{-1}(M) \text{ and } f^{-1}(N) \text{ are disjoint (1, 2)^{\#} - } g^\# \text{-closed sets of } P. \text{Since } P \text{ is (1, 2)^{\#} - } g^\# \text{-normal, there exist disjoint (1, 2)^{\#} - open sets } S \text{ and } T \text{ such that } f^{-1}(M) \subseteq S \text{ and } f^{-1}(N) \subseteq T. \text{Since } f \text{ is (1, 2)^{\#} - open and bijective, we have } f(S) \text{ and } f(T) \text{ are } σ_{1,2} \text{-open in } Q \text{ such that } M \subseteq f(S), N \subseteq f(T) \text{ and } f(S) \bigcap f(T) = \phi. \text{Therefore, } Q \text{ is (1, 2)^{\#} - } g^\# \text{-normal.}

**Theorem 6.6** \text{If } f : P \to Q \text{ is (1, 2)^{\#} - } αg \text{-irresolute (1, 2)^{\#} - } g^\# \text{-closed (1, 2)^{\#} - continuous}
\text{injection and } Q \text{ is (1, 2)^{\#} - } g^\# \text{-normal, then } P \text{ is (1, 2)^{\#} - } g^\# \text{-normal.}

**Proof.** Let M and N be any disjoint (1, 2)^{\#} - g^\# \text{-closed subsets of } P. \text{Since } f \text{ is (1, 2)^{\#} - } αg \text{-irresolute (1, 2)^{\#} - } g^\# \text{-closed, } f(M) \text{ and } f(N) \text{ are disjoint (1, 2)^{\#} - } g^\# \text{-closed sets of } Q. \text{Since } Q \text{ is (1, 2)^{\#} - } g^\# \text{-normal, there exist disjoint } τ_{1,2} \text{-open sets } S \text{ and } T \text{ such that } f(M) \subseteq S \text{ and } f(N) \subseteq T. \text{i.e., } M \subseteq f^{-1}(S), N \subseteq f^{-1}(T) \text{ and } f^{-1}(S) \bigcap f^{-1}(T) = \phi. \text{Since } f \text{ is (1, 2)^{\#} - continuous, } f^{-1}(S) \text{ and } f^{-1}(T) \text{ are } τ_{1,2} \text{-open in } P, \text{we have } P \text{ is (1, 2)^{\#} - } g^\# \text{-normal.}

**Theorem 6.7** \text{If } f : P \to Q \text{ is weakly (1, 2)^{\#} - continuous (1, 2)^{\#} - } g^\# \text{-closed injection and } Q \text{ is}
\text{(1, 2)^{\#} - } g^\# \text{-normal, then } P \text{ is (1, 2)^{\#} - normal.}

**Proof.** Let M and N be any two disjoint τ_{1,2} \text{-closed sets of } P. \text{Since } f \text{ is injective and (1, 2)^{\#} - } g^\# \text{-closed, } f(M) \text{ and } f(N) \text{ are disjoint (1, 2)^{\#} - } g^\# \text{-closed sets of } Q. \text{Since } Q \text{ is (1, 2)^{\#} - } g^\# \text{-normal, there exist } σ_{1,2} \text{-open sets } S \text{ and } T \text{ such that } f(M) \subseteq S, f(N) \subseteq T \text{ and } σ_{1,2} - \text{cl}(S) \bigcap σ_{1,2} - \text{cl}(T) = \phi. \text{Since } f \text{ is weakly (1, 2)^{\#} - continuous, } M \subseteq f^{-1}(S) \subseteq τ_{1,2} - \text{int}(f^{-1}(σ_{1,2} - \text{cl}(S))), N \subseteq f^{-1}(T) \subseteq τ_{1,2} - \text{int}(f^{-1}(σ_{1,2} - \text{cl}(T))) \text{ and } τ_{1,2} - \text{int}(f^{-1}(σ_{1,2} - \text{cl}(S))) \bigcap τ_{1,2} - \text{int}(f^{-1}(σ_{1,2} - \text{cl}(T))) = \phi. \text{Therefore, } P \text{ is (1, 2)^{\#} - normal.}
References

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