(1,2)⋆ -g# -continuous functions in Bitopological Spaces

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Abstract

In this paper, new generalizations of contra-$(1,2)^{å}$-continuity by using the class of $(1,2)^{å}$-$g^{\#}$-closed sets called ($(1,2)^{å}$-$g^{\#}$, s)-continuity are proudly presented. Characterizations and properties of the class of ($(1,2)^{å}$-$g^{\#}$, s)-continuous functions are investigated and discussed in detail. We obtain various important results in the field of bitopological spaces. Finally, we introduce the two spaces called $(1,2)^{å}$-$g^{\#}$-regular spaces and $(1,2)^{å}$-$g^{\#}$-normal spaces in bitopological spaces. We obtain several bitopological characterizations of $(1,2)^{å}$-$g^{\#}$-regular and $(1,2)^{å}$-$g^{\#}$-normal spaces and some preservation theorems for $(1,2)^{å}$-$g^{\#}$-regular and $(1,2)^{å}$-$g^{\#}$-normal spaces.

Key words : $(1,2)^{å}$-$g^{\#}$-T_1, $(1,2)^{å}$-$g^{\#}$-T_2, $(1,2)^{å}$-$g^{\#}$-regular spaces and $(1,2)^{å}$-$g^{\#}$-normal spaces.

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1. Introduction and Preliminaries

The systematic study of contra-continuity was due to Dontchev[2] . Several different types of contra-continuous functions have been introduced and investigated over the years by many authors[1, 4]. In this chapter, new generalizations of contra-$(1,2)^{å}$-continuity by using the class of $(1,2)^{å}$-$g^{\#}$-closed sets called ($(1,2)^{å}$-$g^{\#}$, s)-continuity are proudly presented. Characterizations and properties of the class of ($(1,2)^{å}$-$g^{\#}$, s)-continuous functions are investigated and discussed in detail. We obtain various important results in the field of bitopological spaces.

Using the class of g-closed sets, Munchi [6] introduced the classes of g-regular and g-normal spaces in topological spaces. In a similar way, Sheik John [13] introduced the classes of ω-regular and ω-normal spaces using the class of ω-closed sets in topological spaces. Finally, we introduce the two spaces called $(1,2)^{å}$-$g^{\#}$-regular spaces and $(1,2)^{å}$-$g^{\#}$-normal spaces in bitopological spaces. We obtain several bitopological characterizations of $(1,2)^{å}$-$g^{\#}$-regular and $(1,2)^{å}$-$g^{\#}$-normal spaces and some preservation theorems for $(1,2)^{å}$-$g^{\#}$-regular and $(1,2)^{å}$-$g^{\#}$-normal spaces.

The $(1,2)^{å}$-δ-interior of a subset H of P is the union of all $(1,2)^{å}$-regular open sets of P contained in H and it is denoted by $(1,2)^{å}$-δ-int(H). A subset H is said to be $(1,2)^{å}$-δ-open if H = $(1,2)^{å}$-δ-int(H). The complement of a $(1,2)^{å}$-δ-open set is said to be $(1,2)^{å}$-δ-closed. The $(1,2)^{å}$-δ-closure of a set H in a space P is defined by $(1,2)^{å}$-δ-cl(H) = a $\in$ P: H $\cap$ $\tau_{1,2}$-int($\tau_{1,2}$-cl(G)) $\neq$ ϕ, G $\in$ $(1,2)^{å}$-O(P) and a $\in$ G and it is denoted by $(1,2)^{å}$-δ-cl(H). A subset H of a bitopological space P is said to be a $(1,2)^{å}$-π-open [12] if it is the finite union of $(1,2)^{å}$-regular open sets. The complement of a $(1,2)^{å}$-π-open set is said to be $(1,2)^{å}$-π-closed set. A subset H of a bitopological space P is said to be $(1,2)^{å}$-πg-closed [12] if $\tau_{1,2}$-cl(H) $\subseteq$ G whenever H $\subseteq$ G and G is $(1,2)^{å}$-π-open in P. The complement of a $(1,2)^{å}$-πg-closed set is said to be a $(1,2)^{å}$-πg-open set. The collection of all $(1,2)^{å}$-$g^{\#}$-open (resp. $(1,2)^{å}$-regular closed, $(1,2)^{å}$-semi-open, $\tau_{1,2}$-closed) sets of P containing a point a $\in$ P is denoted by $(1,2)^{å}$-G$^{\#}$O(P, a) (resp. $(1,2)^{å}$-RC(P, a), $(1,2)^{å}$-SO(P, a), $(1,2)^{å}$-C(P, a)). The collection of all $(1,2)^{å}$-πg-open (resp. $(1,2)^{å}$-πg-closed) sets of P is denoted by $(1,2)^{å}$-πGO(P) (resp. $(1,2)^{å}$-πGC(P)). A bitopological space P is said to be (i) $(1,2)^{å}$-s-Urysohn [8] if for each pair of distinct points a and b in P, there exist M $\in$ $(1,2)^{å}$-SO(P, a) and N $\in$ $(1,2)^{å}$-SO(P, b) such that $\tau_{1,2}$-cl(M) $\cap$ $\tau_{1,2}$-cl(N) = ϕ. (ii) Weakly $(1,2)^{å}$-Hausdorff [8] if each element of P is an intersection of $(1,2)^{å}$-regular closed sets. Let G be a subset of a bitopological space P. The set $\cap$ H $\in$ $(1,2)^{å}$-RO(P) : G $\subseteq$ H is called the $(1,2)^{å}$-r-kernel of G and is denoted by $(1,2)^{å}$-r-ker (G) where $(1,2)^{å}$-RO(P) denote the family of all $(1,2)^{å}$-regular open sets of P. A bitopological space P is said to be (i) $(1,2)^{å}$-S-closed [8] if every $(1,2)^{å}$-regular closed cover of P has a finite subcover. (ii) Countably $(1,2)^{å}$-S-closed [8] if every $(1,2)^{å}$-countable cover of P by $(1,2)^{å}$-regular closed sets has a finite subcover. (iii) $(1,2)^{å}$-S-Lindelof [8] if every cover of P by $(1,2)^{å}$-regular closed sets has a $(1,2)^{å}$-countable subcover. A function f : P $\rightarrow$ Q is said to be weakly $(1,2)^{å}$-continuous [9] if for each point a $\in$ P and each $\sigma_{1,2}$-open set N in Q containing f(a), there exists an $\tau_{1,2}$-open set M containing a such that f(M) $\subseteq$ $\sigma_{1,2}$-cl(N). A bitopological space P will be termed $(1,2)^{å}$-symmetric [9] if and only if for a and b in P, a $\in$ $\tau_{1,2}$-cl(b) implies that b $\in$ $\tau_{1,2}$-cl(a). For a subset H of a bitopological space P, $\tau_{1,2}$-cl$_{\theta}$(H) = a $\in$ P: $\tau_{1,2}$-cl(G) $\cap$ H $\neq$ ϕ, G $\in$ $(1,2)^{å}$-O(P) and a $\in$ G. The following properties hold for subsets M, N of a bitopological space P: (i) a $\in$ $(1,2)^{å}$-r-ker(M) if and only if M $\cap$ L $\neq$ ϕ for any $(1,2)^{å}$-regular closed set L containing a. (ii) M $\subseteq$ $(1,2)^{å}$-r-ker(M) and M = $(1,2)^{å}$-r-ker(M) if M is $(1,2)^{å}$-regular open in P. (iii) If M $\subseteq$ N, then $(1,2)^{å}$-r-ker(M) $\subseteq$ $(1,2)^{å}$-r-ker(N). The bitopological space X is $(1,2)^{å}$-symmetric [9] if and only if a is $(1,2)^{å}$-g-closed in P for each point a of P.

Remark 1.1 *[7, 9, 10] We have the following relations:*
$\tau_{1,2}$-closed $\rightarrow$ $(1,2)^{å}$-$g^{\#}$-closed $\rightarrow$ $(1,2)^{å}$-$\hat{g}$-closed $\rightarrow$ $(1,2)^{å}$-g-closed $\rightarrow$ $(1,2)^{å}$-πg-closed.
None of these implications are reversible.

2.Characterizations of $(1,2)^{å}$ - $g^{\#}$ -open sets

Lemma 2.1 *For any subset G of a bitopological space P, P\ $(1,2)^{å}$ - $g^{\#}$ -cl(G) = $(1,2)^{å}$ - $g^{\#}$ -int (P \ G).*

Theorem 2.1 *Suppose that G is $(1,2)^{å}$ - $g^{\#}$ -open in P and that H is $(1,2)^{å}$ - $g^{\#}$ -open in Q. Then G x H is $(1,2)^{å}$ - $g^{\#}$ -open in P x Q.*

Proof. Suppose that E is $\tau_{1,2}$-closed and hence $(1,2)^{å}$ - αg -closed in P x Q and that E $\subseteq$ G x H. It suffices to show that E $\subseteq$ $\tau_{1,2}$-int(G x H).

Let (1, 2) $\in$ E. Then, for each (1, 2) $\in$ E, $\tau_{1,2}$-cl(1) x $\tau_{1,2}$-cl(2) = $\tau_{1,2}$-cl(1 x 2) = $\tau_{1,2}$-cl(1, 2) $\subseteq$ $\tau_{1,2}$-cl(E) = E $\subseteq$ G x H. Two $\tau_{1,2}$-closed sets $\tau_{1,2}$-cl(1) and $\tau_{1,2}$-cl(2) are contained in G and H respectively. It follows from the assumption that $\tau_{1,2}$-cl(1) $\subseteq$ $\tau_{1,2}$-int(G) and that $\tau_{1,2}$-cl(2) $\subseteq$ $\tau_{1,2}$-int(H). Thus (1, 2) $\in$ $\tau_{1,2}$-cl(1) x $\tau_{1,2}$-cl(2) $\subseteq$ $\tau_{1,2}$-int(G) x $\tau_{1,2}$-int(H) $\subseteq$ $\tau_{1,2}$-int(G x H). It means that, for each (1, 2) $\in$ E, (1, 2) $\in$ $\tau_{1,2}$-int(G x H) and hence E $\subseteq$ $\tau_{1,2}$-int(G x H). Therefore G x H is $(1,2)^{å}$ - $g^{\#}$ -open in P x Q.

3.Properties of ($(1,2)^{å}$ - $g^{\#}$, s) - continuous functions

Definition 3.1 *A function f : P $\rightarrow$ Q is said to be ($(1,2)^{å}$ - $g^{\#}$, s)-continuous if the inverse image of each $(1,2)^{å}$ -regular open set of Q is $(1,2)^{å}$ - $g^{\#}$ -closed in P.*

Theorem 3.1 *The following are equivalent for a function f : P $\rightarrow$ Q:*

1. f is ($(1,2)^{å}$ - $g^{\#}$, s)-continuous.
2. The inverse image of a $(1,2)^{å}$ -regular closed set of Q is $(1,2)^{å}$ - $g^{\#}$ -open in P.
3. f^{-1} ($\tau_{1,2}$ -int($\sigma_{1,2}$ -cl(K))) is $(1,2)^{å}$ - $g^{\#}$ -closed in P for every $\sigma_{1,2}$ -open subset K of Q.
4. f^{-1} ($\tau_{1,2}$ -cl($\sigma_{1,2}$ -int(K))) is $(1,2)^{å}$ - $g^{\#}$ -open in P for every $\sigma_{1,2}$ -closed subset K of Q.
5. f^{-1} ($\sigma_{1,2}$ -cl(K)) is $(1,2)^{å}$ - $g^{\#}$ -open in P for every K $\in$ $(1,2)^{å}$ - β O(Q).
6. f^{-1} ($\sigma_{1,2}$ -cl(K)) is $(1,2)^{å}$ - $g^{\#}$ -open in P for every K $\in$ $(1,2)^{å}$ -SO(Q).
7. f^{-1} ($\sigma_{1,2}$ -int($\sigma_{1,2}$ -cl(K))) is $(1,2)^{å}$ - $g^{\#}$ -closed in P for every K $\in$ $(1,2)^{å}$ -PO(Q).

Proof. (1) $\Leftrightarrow$ (2). Obvious.

(1) $\Leftrightarrow$ (3). Let K be an $\sigma_{1,2}$-open subset of Q. Since $\sigma_{1,2}$ -int($\sigma_{1,2}$ -cl(K)) is $(1,2)^{å}$ -regular open, f^{-1} ($\sigma_{1,2}$ -int($\sigma_{1,2}$ -cl(K))) is $(1,2)^{å}$ - $g^{\#}$ -closed. The converse is similar.

(2) $\Leftrightarrow$ (4). Similar to (1) $\Leftrightarrow$ (3).

(2) $\Rightarrow$ (5). Let L be any $(1,2)^{å}$ - β -open set of Q. We know that $\sigma_{1,2}$ -cl(L) is $(1,2)^{å}$ -regular closed. Then by (2) f^{-1} ($\sigma_{1,2}$ -cl(L)) is $(1,2)^{å}$ - $g^{\#}$ -open in P.

(5) $\Rightarrow$ (6). Obvious from the fact that $(1,2)^{å}$ -SO(Q) $\subseteq$ $(1,2)^{å}$ - β O(Q).

(6) $\Rightarrow$ (7). Let L $\in$ $(1,2)^{å}$ -PO(Q). Then Q\ $\sigma_{1,2}$ -int($\sigma_{1,2}$ -cl(L)) is $(1,2)^{å}$ -regular closed and hence it is $(1,2)^{å}$ -semi-open. Then, we have P\ f^{-1} ($\sigma_{1,2}$ -int($\sigma_{1,2}$ -cl(L))) = f^{-1} (Q\ $\sigma_{1,2}$ -int($\sigma_{1,2}$ -cl(L))) = f^{-1} ($\sigma_{1,2}$ -cl(Q\ $\sigma_{1,2}$ -int($\sigma_{1,2}$ -cl(L)))) is $(1,2)^{å}$ - $g^{\#}$ -open in P. Hence f^{-1} ($\sigma_{1,2}$ -int($\sigma_{1,2}$

-cl(L))) is $(1,2)^{å}$ - $g^{\#}$-closed in P.

(7) $\Rightarrow$ (1). Let L be any $(1,2)^{å}$-regular open set of Q. Then L $\in$ $(1,2)^{å}$-PO(Q) and hence f^{-1}(L) = f^{-1}($\sigma_{1,2}$-int($\sigma_{1,2}$-cl(L))) is $(1,2)^{å}$ - $g^{\#}$-closed in P.

Lemma 3.1 *For a subset H of a bitopological space Q, the following properties hold:*

1. $(1,2)^{å}$ - α cl(H) = $\sigma_{1,2}$-cl(H) for every H $\in$ $(1,2)^{å}$ - β O(Q).
2. $(1,2)^{å}$-pcl(H) = $\sigma_{1,2}$-cl(H) for every H $\in$ $(1,2)^{å}$-SO(Q).
3. $(1,2)^{å}$-scl(H) = $\sigma_{1,2}$-int($\sigma_{1,2}$-cl(H)) for every H $\in$ $(1,2)^{å}$-PO(Q).

Corollary 3.1 *The following are equivalent for a function f : P $\rightarrow$ Q:*

1. f is ($(1,2)^{å}$ - $g^{\#}$, s)-continuous.
2. f^{-1}($(1,2)^{å}$ - α cl(H)) is $(1,2)^{å}$ - $g^{\#}$-open in P for every H $\in$ $(1,2)^{å}$ - β O(Q).
3. f^{-1}($(1,2)^{å}$-pcl(H)) is $(1,2)^{å}$ - $g^{\#}$-open in P for every H $\in$ $(1,2)^{å}$-SO(Q).
4. f^{-1}($(1,2)^{å}$-scl(H)) is $(1,2)^{å}$ - $g^{\#}$-closed in P for every H $\in$ $(1,2)^{å}$-PO(Q).

Proof. It follows from Theorem (3.1) and Lemma (3.1)

4.The related functions with ($(1,2)^{å}$ - $g^{\#}$, s)-continuous functions

Definition 4.1 *A function f : P $\rightarrow$ Q is said to be*

1. perfectly $(1,2)^{å}$-continuous [3] if f^{-1}(K) is $(1,2)^{å}$-clopen in P for every $\sigma_{1,2}$-open set K of Q.
2. regular $(1,2)^{å}$-set-connected [3] if f^{-1}(K) is $(1,2)^{å}$-clopen in P for every K $\in$ $(1,2)^{å}$-RO(Q).
3. almost $(1,2)^{å}$-s-continuous [3] if for each a $\in$ P and each K $\in$ $(1,2)^{å}$-SO(Q, f(a)), there exists an $\tau_{1,2}$-open set V in P containing a such that f(V) $\subseteq$ $(1,2)^{å}$-scl(K).
4. strongly $(1,2)^{å}$-continuous 5] if the inverse image of every set in Q is $(1,2)^{å}$-clopen in P.
5. $(1,2)^{å}$-RC-continuous [5] if f^{-1}(K) is $(1,2)^{å}$-regular closed in P for each $\sigma_{1,2}$-open set K of Q.
6. contra $(1,2)^{å}$-R-map [5] if f^{-1}(K) is $(1,2)^{å}$-regular closed in P for each $(1,2)^{å}$-regular open set K of Q.
7. contra-$(1,2)^{å}$-super-continuous [8] if for each a $\in$ P and for each G $\in$ $(1,2)^{å}$-C(Q, f(a)), there exists a $(1,2)^{å}$-regular open set H in P containing a such that f(H) $\subseteq$ G.
8. almost $(1,2)^{å}$-contra-super-continuous [8] if f^{-1}(K) is $(1,2)^{å}$ - δ-closed in P for every $(1,2)^{å}$-regular open set K of Q.
9. contra $(1,2)^{å}$-continuous [10] if f^{-1}(K) is $\tau_{1,2}$-closed in P for every $\sigma_{1,2}$-open set K of Q.
10. contra $(1,2)^{å}$ - g-continuous [10] if f^{-1}(K) is $(1,2)^{å}$ - g-closed in P for every $\sigma_{1,2}$-open set K of Q.
11. ($(1,2)^{å}$ - θ, s)-continuous [11] if for each a $\in$ P and each N $\in$ $(1,2)^{å}$-SO(Q, f(a)),

there exists an $\tau_{1,2}$-open set M in P containing a such that f(M) $\subseteq$ $\tau_{1,2}$-cl(N).

12. contra $(1,2)^{å}$ - πg -continuous [12] if f^{-1}(K) is $(1,2)^{å}$ - πg -closed in P for each $\sigma_{1,2}$ -open set K of Q.

13. ($(1,2)^{å}$ - g , s)-continuous [11] if f^{-1}(K) is $(1,2)^{å}$ - g -closed in P for each $(1,2)^{å}$ -regular open set K of Q.

14. ($(1,2)^{å}$ - $\hat{g}$, s)-continuous [12] if f^{-1}(K) is $(1,2)^{å}$ - $\hat{g}$ -closed in P for each $(1,2)^{å}$ -regular open set K of Q.

15. ($(1,2)^{å}$ - πg , s)-continuous [12] if f^{-1}(K) is $(1,2)^{å}$ - πg -closed in P for each $(1,2)^{å}$ -regular open set K of Q.

Definition 4.2 *A function $f : P \to Q$ is said to be contra $(1,2)^{å}$ - $\hat{g}$ -continuous* ***Error! Reference source not found.*** *(resp. contra $(1,2)^{å}$ - $g^{\#}$-continuous) if f^{-1}(K) is $\hat{g}$ -closed (resp. $(1,2)^{å}$ - $g^{\#}$-closed) in P for each $\sigma_{1,2}$ -open set K of Q.*

Remark 4.1 *The following diagram holds for a function $f : P \to Q$:*

$(1,2)^{å}$ -strongly continuous	$\to$	$(1,2)^{å}$ -almost s-continuous
$\downarrow$		$\downarrow$
$(1,2)^{å}$ -perfectly continuous	$\to$	$(1,2)^{å}$ -regular set-connected
$\downarrow$		$\downarrow$
$(1,2)^{å}$ -RC continuous	$\to$	contra $(1,2)^{å}$ -R-continuous
$\downarrow$		$\downarrow$
contra $(1,2)^{å}$ -super-continuous	$\to$	almost contra $(1,2)^{å}$ -super-continuous
$\downarrow$		$\downarrow$
contra $(1,2)^{å}$ -continuous	$\to$	$(1,2)^{å}$ -(θ , s)-continuous
$\downarrow$		$\downarrow$
contra $(1,2)^{å}$ - $g^{\#}$-continuous	$\to$	($(1,2)^{å}$ - $g^{\#}$, s)-continuous
$\downarrow$		$\downarrow$
contra $(1,2)^{å}$ - $\hat{g}$ -continuous	$\to$	($(1,2)^{å}$ - $\hat{g}$, s)-continuous
$\downarrow$		$\downarrow$
contra $(1,2)^{å}$ -g-continuous	$\to$	($(1,2)^{å}$ -g, s)-continuous
$\downarrow$		$\downarrow$
contra $(1,2)^{å}$ - πg -continuous	$\to$	($(1,2)^{å}$ - πg , s)-continuous

None of these implications is reversible as shown in the following examples and in the related paper[3, 5, 8, 10, 11, 12].

Example 4.1 *Let P = 1, 2, 3 with the topologies* $\tau_1 = \phi$, *2, P and* $\tau_2 = \phi$, *1, 3, P. Then the subsets in* ϕ, *2, 1, 3, P are called* $\tau_{1,2}$*-open and the subsets in* ϕ, *2, 1, 3, P are called* $\tau_{1,2}$ *-closed. Let Q = 1, 2, 3 with the topologies* $\sigma_1 = \phi$, *Q and* $\sigma_2 = \phi$, *1, 2, Q. Then the subsets in* ϕ, *1, 2, Q are called* $\sigma_{1,2}$*-open and the subsets in* ϕ, *3, Q are called* $\sigma_{1,2}$*-closed. Then the identity function f : P* $\rightarrow$ *Q is (*$(1,2)^{å}$ *-* $g^{\#}$*, s)-continuous but not contra* $(1,2)^{å}$ *-* $g^{\#}$*-continuous.*

Example 4.2 *Let P = 1, 2, 3 with the topologies* $\tau_1 = \phi$, *1, P and* $\tau_2 = \phi$, *2, 3, P. Then the subsets in* ϕ, *1, 2, 3, P are called* $\tau_{1,2}$*-open and the subsets in* ϕ, *1, 2, 3, P are called* $\tau_{1,2}$ *-closed. Let Q = 1, 2, 3 with the topologies* $\sigma_1 = \phi$, *Q and* $\sigma_2 = \phi$, *1, 2, Q. Then the subsets in* ϕ, *1, 2, Q are called* $\sigma_{1,2}$*-open and the subsets in* ϕ, *3, Q are called* $\sigma_{1,2}$*-closed. Then the identity function f : P* $\rightarrow$ *Q is contra* $(1,2)^{å}$ *-* $\hat{g}$ *-continuous but not contra* $(1,2)^{å}$ *-* $g^{\#}$ *-continuous.*

Example 4.3 *Let P = 1, 2, 3, 4 with the topologies* $\tau_1 = \phi$, *2, P and* $\tau_2 = \phi$, *1, 3, 4, P. Then the subsets in* ϕ, *2, 1, 3, 4, P are called* $\tau_{1,2}$*-open and the subsets in* ϕ, *2, 1, 3, 4, P are called* $\tau_{1,2}$*-closed. Let Q = 1, 2, 3, 4 with the topologies* $\sigma_1 = \phi$, *2, Q and* $\sigma_2 = \phi$, *1, 3, 4, Q. Then the subsets in* ϕ, *2, 1, 3, 4, Q are called* $\sigma_{1,2}$*-open and the subsets in* ϕ, *2, 1, 3, 4, Q are called* $\sigma_{1,2}$*-closed. Then the function f : P* $\rightarrow$ *Q which is defined as f(1) = 2, f(2) = 3, f(3) = 4, f(4) = 1 is (*$(1,2)^{å}$ *-* θ*, s)-continuous but not (*$(1,2)^{å}$ *-* $g^{\#}$*, s)-continuous.*

Example 4.4 *Let P = 1, 2, 3, 4 with the topologies* $\tau_1 = \phi$, *3, 1, 3, 4, P and* $\tau_2 = \phi$, *1, 4, P. Then the subsets in* ϕ, *3, 1, 4, 1, 3, 4, P are called* $\tau_{1,2}$*-open and the subsets in* ϕ, *2, 2, 3, 1, 2, 4, P are called* $\tau_{1,2}$*-closed. Let Q = 1, 2, 3, 4 with the topologies* $\sigma_1 = \phi$, *3, 1, 3, 4, Q and* $\sigma_2 = \phi$, *1, 4, Q. Then the subsets in* ϕ, *3, 1, 4, 1, 3, 4, Q are called* $\sigma_{1,2}$*-open and the subsets in* ϕ, *2, 2, 3, 1, 2, 4, Q are called* $\sigma_{1,2}$*-closed. Then the function f : P* $\rightarrow$ *Q which is defined as f(1) = 3, f(2)= 3, f(3) = 2, f(4) = 2 is (*$(1,2)^{å}$ *-* $g^{\#}$*, s)-continuous but not (*$(1,2)^{å}$ *-* θ*, s)-continuous.*

Example 4.5 *Let P = 1, 2, 3 with the topologies* $\tau_1 = \phi$, *P and* $\tau_2 = \phi$, *1, 2, P. Then the subsets in* ϕ, *1, 2, P are called* $\tau_{1,2}$*-open and the subsets in* ϕ, *3, P are called* $\tau_{1,2}$*-closed. Let Q = 1, 2, 3 with the topologies* $\sigma_1 = \phi$, *3, Q and* $\sigma_2 = \phi$, *1, 3, Q. Then the subsets in* ϕ, *3, 1, 3, Q are called* $\sigma_{1,2}$*-open and the subsets in* ϕ, *2, 1, 2, Q are called* $\sigma_{1,2}$*-closed. Then the identity function f : P* $\rightarrow$ *Q is contra* $(1,2)^{å}$ *-* $g^{\#}$*-continuous but not contra* $(1,2)^{å}$ *-continuous.*

Definition 4.3 *A function f : P* $\rightarrow$ *Q is said to be almost* $(1,2)^{å}$ *-* $g^{\#}$*-continuous if* $f^{-1}(G)$ *is* $(1,2)^{å}$ *-* $g^{\#}$*-open in P for every* $(1,2)^{å}$ *-regular open set G of Q.*

Definition 4.4 *A bitopological space P is said to be* $(1,2)^{å}$ *-*P_{Σ} *if for any* $\tau_{1,2}$*-open set K of P and each a* $\in$ *K, there exists L* $\in$ $(1,2)^{å}$ *-RC(P, a) such that a* $\in$ *L* $\subseteq$ *K.*

Theorem 4.1 *Let f : P $\rightarrow$ Q be a function from a $(1,2)^{å}$-$T_g^{\#}$ -space P to a bitopological space Q. Then the following are equivalent.*

1. f is ($(1,2)^{å}$ - θ, s)-continuous,
2. f is ($(1,2)^{å}$ - $g^{\#}$, s)-continuous.

Theorem 4.2 *Let f : P $\rightarrow$ Q be a function. Then, if f is ($(1,2)^{å}$ - $g^{\#}$, s)-continuous, P is $(1,2)^{å}$-$T_g^{\#}$ and Q is $(1,2)^{å}$-P_{Σ}, then f is $(1,2)^{å}$-continuous.*

Proof. Let F be any $\sigma_{1,2}$-open set of Q. Since Q is $(1,2)^{å}$-P_{Σ}, there exists a subfamily Φ of $(1,2)^{å}$-RC(Q) such that G = $\cup$H: H$\in$ Φ. Since P is $(1,2)^{å}$-$T_g^{\#}$ and f is ($(1,2)^{å}$ - $g^{\#}$, s)-continuous, f^{-1}(H) is $\tau_{1,2}$-open in P for each H $\in$ Φ and f^{-1}(F) is $\tau_{1,2}$-open in P. Thus, f is $(1,2)^{å}$-continuous.

Theorem 4.3 *Let f : P $\rightarrow$ Q and g : Q $\rightarrow$ R be functions. Then, the following properties hold:*

1. If f is $(1,2)^{å}$ - $g^{\#}$-irresolute and g is ($(1,2)^{å}$ - $g^{\#}$, s)-continuous, then g o f : P $\rightarrow$ R is ($(1,2)^{å}$ - $g^{\#}$, s)-continuous.
2. If f is ($(1,2)^{å}$ - $g^{\#}$, s)-continuous and g is contra $(1,2)^{å}$-R-map, then g o f : P $\rightarrow$ R is almost $(1,2)^{å}$ - $g^{\#}$-continuous.
3. If f is $(1,2)^{å}$ - $g^{\#}$-continuous and g is ($(1,2)^{å}$ - θ, s)-continuous, then g o f : P $\rightarrow$ R is ($(1,2)^{å}$ - $g^{\#}$, s)-continuous.
4. If f is ($(1,2)^{å}$ - $g^{\#}$, s)-continuous and g is $(1,2)^{å}$-RC-continuous, then g o f : P $\rightarrow$ R is $(1,2)^{å}$ - $g^{\#}$-continuous.
5. If f is almost $(1,2)^{å}$ - $g^{\#}$-continuous and g is contra $(1,2)^{å}$-R-map, then g o f : P $\rightarrow$ R is ($(1,2)^{å}$ - $g^{\#}$, s)-continuous.

5.Bitopological Fundamental properties

Definition 5.1 *A bitopological space P is said to be*

1. $(1,2)^{å}$ - $g^{\#}$-T_2 if for each pair of distinct points a and b in P, there exist M $\in$ $(1,2)^{å}$-$G^{\#}$O(P, a) and N $\in$ $(1,2)^{å}$-$G^{\#}$O(P, b) such that M $\cap$ N = ϕ.
2. $(1,2)^{å}$ - $g^{\#}$-T_1 if for each pair of distinct points a and b in P, there exist $(1,2)^{å}$ - $g^{\#}$ -open sets M and N containing a and b, respectively, such that b $\not\in$ M and a $\not\in$ N.

Remark 5.1 *The following implications are hold for a bitopological space P.*

1. $(1,2)^{å}$-T_2 $\Rightarrow$ $(1,2)^{å}$ - $g^{\#}$-T_2.
2. $(1,2)^{å}$-T_1 $\Rightarrow$ $(1,2)^{å}$ - $g^{\#}$-T_1.

These implications are not reversible.

Example 5.1 *Let P = 1, 2, 3 with the topologies τ_1 = ϕ, 1, 2, 1, 2, P and τ_2 = ϕ, 3, P. Then*

the subsets in ϕ, 1, 2, 3, 1, 2, 1, 3, 2, 3, P are called $\tau_{1,2}$-open and the subsets in ϕ, 1, 2, 3, 1, 2, 1, 3, 2, 3, P are called $\tau_{1,2}$-closed. Then P is both $(1,2)^{å}$-$g^{\#}$-T_2 and $(1,2)^{å}$-$g^{\#}$-T_1 but neither $(1,2)^{å}$-T_1 nor $(1,2)^{å}$-T_2.

Lemma 5.1 *In the $(1,2)^{å}$-$g^{\#}$-T_2 space, the $(1,2)^{å}$-$g^{\#}$-closure of every $(1,2)^{å}$-$g^{\#}$-open set is $(1,2)^{å}$-$g^{\#}$-open.*

Proof. Every $(1,2)^{å}$-regular open subset is $\tau_{1,2}$-open and every $\tau_{1,2}$-open set is $(1,2)^{å}$-$g^{\#}$-open. Thus, every $(1,2)^{å}$-regular closed subset is $(1,2)^{å}$-$g^{\#}$-closed. Now let H be any $(1,2)^{å}$-$g^{\#}$-open set in $(1,2)^{å}$-$g^{\#}$-T_2 space. There exists a $(1,2)^{å}$-$g^{\#}$-open set G such that G $\subseteq$ H $\subseteq$ $\tau_{1,2}$-cl(G). Hence, we have G $\subseteq$ $(1,2)^{å}$-$g^{\#}$-cl(G) $\subseteq$ $(1,2)^{å}$-$g^{\#}$-cl(H) $\subseteq$ $(1,2)^{å}$-$g^{\#}$-cl($\tau_{1,2}$-cl(G)) = $\tau_{1,2}$-cl(G) since $\tau_{1,2}$-cl(G) is $(1,2)^{å}$-regular closed. Therefore, $(1,2)^{å}$-$g^{\#}$-cl(H) is $(1,2)^{å}$-$g^{\#}$-open.

Theorem 5.1 *A bitopological space P is $(1,2)^{å}$-$g^{\#}$-T_2 if and only if for any pair of distinct points a, b of P there exist $(1,2)^{å}$-$g^{\#}$-open sets M and N such that a $\in$ M and b $\in$ N and $(1,2)^{å}$-$g^{\#}$-cl(M) $\cap$ $(1,2)^{å}$-$g^{\#}$-cl(N) = ϕ.*

Proof. Necessity. Suppose that P is $(1,2)^{å}$-$g^{\#}$-T_2. Let a and b be distinct points of P. There exist $(1,2)^{å}$-$g^{\#}$-open sets M and N such that a $\in$ M, b $\in$ N and M $\cap$ N = ϕ. Hence $(1,2)^{å}$-$g^{\#}$-cl(M) $\cap$ $(1,2)^{å}$-$g^{\#}$-cl(N) = ϕ and $(1,2)^{å}$-$g^{\#}$-cl(M) is $(1,2)^{å}$-$g^{\#}$-open. Therefore, we obtain $(1,2)^{å}$-$g^{\#}$-cl(M) $\cap$ $(1,2)^{å}$-$g^{\#}$-cl(N) = ϕ.
Sufficiency. This is obvious.

Definition 5.2 *A bitopological space P is said to be*

1. countably $(1,2)^{å}$-$g^{\#}$-compact if every $(1,2)^{å}$-countable cover of P by $(1,2)^{å}$-$g^{\#}$-open sets has a finite subcover.
2. $(1,2)^{å}$-$g^{\#}$-Lindelof if every $(1,2)^{å}$-$g^{\#}$-open cover of P has a $(1,2)^{å}$-countable subcover.

Theorem 5.2 *Let $f : P \to Q$ be a ($(1,2)^{å}$-$g^{\#}$, s)-continuous surjection. Then the following statements hold:*

1. if P is $(1,2)^{å}$-$g^{\#}$-Lindelof, then Q is $(1,2)^{å}$-S-Lindelof.
2. if P is countably $(1,2)^{å}$-$g^{\#}$-compact, then Q is countably $(1,2)^{å}$-S-closed.

Definition 5.3 *A bitopological space P is said to be $(1,2)^{å}$-$g^{\#}$-connected if P is not the union of two disjoint nonempty $(1,2)^{å}$-$g^{\#}$-open sets.*

Example 5.2 *Let P = 1, 2, 3 with the topologies τ_1 = ϕ, 1, P and τ_2 = ϕ, 2, 3, P. Then the subsets in ϕ, 1, 2, 3, P are called $\tau_{1,2}$-open and the subsets in ϕ, 1, 2, 3, P are called $\tau_{1,2}$-closed. Then P is not $(1,2)^{å}$-$g^{\#}$-connected.*

Example 5.3 *Let P = 1, 2, 3 with the topologies* $\tau_1 = \phi$, *1, P and* $\tau_2 = \phi$, *1, 2, P. Then the subsets in* ϕ, *1, 1, 2, P are called* $\tau_{1,2}$*-open and the subsets in* ϕ, *3, 2, 3, P are called* $\tau_{1,2}$ *-closed. Then P is* $(1,2)^{å}$ *-* $g^{\#}$ *-connected.*

Theorem 5.3 *Let f : P* $\rightarrow$ *Q be a (*$(1,2)^{å}$ *-* $g^{\#}$*, s)-continuous surjection. If P is* $(1,2)^{å}$ *-* $g^{\#}$ *-connected, then Q is* $(1,2)^{å}$ *-connected.*

Proof. Assume that Q is not $(1,2)^{å}$ -connected space. Then there exist nonempty disjoint $\tau_{1,2}$ -open sets M and N such that Q = M $\cup$ N. Also M and N are $(1,2)^{å}$ -clopen in Q. Since f is ($(1,2)^{å}$ - $g^{\#}$, s)-continuous, f^{-1}(M) and f^{-1}(N) are $(1,2)^{å}$ - $g^{\#}$-open in P. Moreover f^{-1}(M) and f^{-1} (N) are nonempty disjoint and P = f^{-1}(M) $\cup$ f^{-1}(N). This shows that P is not $(1,2)^{å}$ - $g^{\#}$ -connected. This contradicts the assumption that Q is not $(1,2)^{å}$ -connected.

6. $(1,2)^{å}$ - $g^{\#}$ -regular and $(1,2)^{å}$ - $g^{\#}$ -normal spaces

Definition 6.1 *A bitopological space P is said to be* $(1,2)^{å}$ *-* $g^{\#}$ *-regular if for every* $(1,2)^{å}$ *-* $g^{\#}$ *-closed set G and each point a* $\notin$ *G, there exist disjoint* $\tau_{1,2}$*-open sets M and N such that G* $\subseteq$ *M and a* $\in$ *N.*

Theorem 6.1 *Let P be a bitopological space. Then the following statements are equivalent:*

1. P is a $(1,2)^{å}$ - $g^{\#}$ -regular space.
2. For each a $\in$ P and $(1,2)^{å}$ - $g^{\#}$ -neighbourhood G of a there exists an $\tau_{1,2}$ -open neighbourhood H of a such that $\tau_{1,2}$ -cl(H) $\subseteq$ G.

Proof. (1) $\Rightarrow$ (2). Let G be any $(1,2)^{å}$ - $g^{\#}$ -neighbourhood of a. Then there exist a $(1,2)^{å}$ - $g^{\#}$ -open set K such that a $\in$ K $\subseteq$ G. Since K^c is $(1,2)^{å}$ - $g^{\#}$ -closed and a $\notin$ K^c, by hypothesis there exist $\tau_{1,2}$ -open sets M and N such that K^c $\subseteq$ M, a $\in$ N and M $\cap$ N = ϕ and so N $\subseteq$ M^c. Now, $\tau_{1,2}$ -cl(N) $\subseteq$ $\tau_{1,2}$ -cl(M^c) = M^c and K^c $\subseteq$ M implies M^c $\subseteq$ K $\subseteq$ G. Therefore $\tau_{1,2}$ -cl(N) $\subseteq$ G.

(2) $\Rightarrow$ (1). Let K be any $(1,2)^{å}$ - $g^{\#}$ -closed set and a $\notin$ K. Then a $\in$ K^c and K^c is $(1,2)^{å}$ - $g^{\#}$ -open and so K^c is an $(1,2)^{å}$ - $g^{\#}$ -neighbourhood of a. By hypothesis, there exists an $\tau_{1,2}$ -open neighbourhood L of a such that a $\in$ L and $\tau_{1,2}$ -cl(L) $\subseteq$ K^c, which implies K $\subseteq$ ($\tau_{1,2}$ -cl(L))^c. Then ($\tau_{1,2}$ -cl(L))^c is an open set containing K and L $\cap$ ($\tau_{1,2}$ -cl(L))^c = ϕ. Therefore, P is $(1,2)^{å}$ - $g^{\#}$ -regular.

Theorem 6.2 *For a bitopological space P the following are equivalent:*

1. P is $(1,2)^{å}$ -normal.
2. For every pair of disjoint $\tau_{1,2}$ -closed sets M and N, there exist $(1,2)^{å}$ - $g^{\#}$ -open sets S and T such that M $\subseteq$ S, N $\subseteq$ T and S $\cap$ T = ϕ.

Proof. (1) $\Rightarrow$ (2). Let M and N be disjoint $\tau_{1,2}$ -closed subsets of P. By hypothesis, there exist disjoint $\tau_{1,2}$ -open sets (and hence $(1,2)^{å}$ - $g^{\#}$ -open sets) S and T such that M $\subseteq$ S and N $\subseteq$

T.

(2) $\Rightarrow$ (1). Let M and N be $\tau_{1,2}$-closed subsets of P. Then by assumption, M $\subseteq$ E, N $\subseteq$ F and E $\cap$ F = ϕ, where E and F are disjoint $(1,2)^{å}$-$g^{\#}$-open sets. Since M and N are $(1,2)^{å}$-αg-closed, by Theorem **Error! Reference source not found.**, M $\subseteq$ $\tau_{1,2}$-int(E) and N $\subseteq$ $\tau_{1,2}$-int(F). Further, $\tau_{1,2}$-int(E) $\cap$ $\tau_{1,2}$-int(F) = $\tau_{1,2}$-int(E $\cap$ F) = ϕ.

A bitopological space P is $(1,2)^{å}$-$g^{\#}$-regular if and only if for each $(1,2)^{å}$-$g^{\#}$-closed set K of P and each a $\in$ K^c there exist $\tau_{1,2}$-open sets M and N of P such that a $\in$ M, K $\subseteq$ N and $\tau_{1,2}$-cl(M) $\cap$ $\tau_{1,2}$-cl(N) = ϕ.

Proof. Let K be a $(1,2)^{å}$-$g^{\#}$-closed set of P and a $\notin$ K. Then there exist $\tau_{1,2}$-open sets S_0 and T of P such that x $\in$ S_0, K $\subseteq$ T and S_0 $\cap$ T = ϕ, which implies S_0 $\cap$ $\tau_{1,2}$-cl(T) = ϕ. Since $\tau_{1,2}$-cl(T) is $\tau_{1,2}$-closed, it is $(1,2)^{å}$-$g^{\#}$-closed and a $\notin$ $\tau_{1,2}$-cl(T). Since P is $(1,2)^{å}$-$g^{\#}$-regular, there exist $\tau_{1,2}$-open sets E and F of P such that a $\in$ E, $\tau_{1,2}$-cl(T) $\subseteq$ F and E $\cap$ F = ϕ, which implies $\tau_{1,2}$-cl(E) $\cap$ F = ϕ. Let D = S_0 $\cap$ E, then D and T are $\tau_{1,2}$-open sets of P such that a $\in$ D, K $\subseteq$ T and $\tau_{1,2}$-cl(D) $\cap$ $\tau_{1,2}$-cl(T) = ϕ.

Converse part is trivial.

Theorem 6.3 *A bitopological space T is said to be $(1,2)^{å}$-$g^{\#}$-normal if for any pair of disjoint $(1,2)^{å}$-$g^{\#}$-closed sets M and N, there exist disjoint $\tau_{1,2}$-open sets G and H such that M $\subseteq$ G and N $\subseteq$ H.*

We characterize $(1,2)^{å}$-$g^{\#}$-normal space.

Theorem 6.4 *Let P be a bitopological space. Then the following statements are equivalent:*

1. P is $(1,2)^{å}$-$g^{\#}$-normal.
2. For each $(1,2)^{å}$-$g^{\#}$-closed set K and for each $(1,2)^{å}$-$g^{\#}$-open set G containing K, there exists an $\tau_{1,2}$-open set H containing K such that $\tau_{1,2}$-cl(H) $\subseteq$ G.
3. For each pair of disjoint $(1,2)^{å}$-$g^{\#}$-closed sets M and N in P, there exists an $\tau_{1,2}$-open set G containing M such that $\tau_{1,2}$-cl(G) $\cap$ N = ϕ.
4. For each pair of disjoint $(1,2)^{å}$-$g^{\#}$-closed sets M and N in P, there exist $\tau_{1,2}$-open sets G containing M and H containing N such that $\tau_{1,2}$-cl(G) $\cap$ $\tau_{1,2}$-cl(H) = ϕ.

Proof. (1) $\Rightarrow$ (2). Let K be a $(1,2)^{å}$-$g^{\#}$-closed set and G be a $(1,2)^{å}$-$g^{\#}$-open set such that K $\subseteq$ G. Then K $\cap$ G^c = ϕ. By assumption, there exist $\tau_{1,2}$-open sets H and L such that K $\subseteq$ H, G^c $\subseteq$ L and H $\cap$ L = ϕ, which implies $\tau_{1,2}$-cl(H) $\cap$ L = ϕ. Now, $\tau_{1,2}$-cl(H) $\cap$ G^c $\subseteq$ $\tau_{1,2}$-cl(H) $\cap$ L = ϕ and so $\tau_{1,2}$-cl(H) $\subseteq$ G.

(2) $\Rightarrow$ (3). Let M and N be disjoint $(1,2)^{å}$-$g^{\#}$-closed sets of P. Since M $\cap$ N = ϕ, M $\subseteq$ N^c and N^c is $(1,2)^{å}$-$g^{\#}$-open. By assumption, there exists an $\tau_{1,2}$-open set G containing M such that $\tau_{1,2}$-cl(G) $\subseteq$ N^c and so $\tau_{1,2}$-cl(G) $\cap$ N = ϕ.

(3) $\Rightarrow$ (4). Let M and N be any two disjoint $(1,2)^{å}$-$g^{\#}$-closed sets of P. Then by assumption, there exists an $\tau_{1,2}$-open set G containing M such that $\tau_{1,2}$-cl(G) $\cap$ N = ϕ. Since $\tau_{1,2}$-cl(G) is

$\tau_{1,2}$-closed, it is $(1,2)^{å}$ - $g^{\#}$-closed and so N and $\tau_{1,2}$-cl(G) are disjoint $(1,2)^{å}$ - $g^{\#}$-closed sets in P. Therefore again by assumption, there exists an $\tau_{1,2}$-open set H containing N such that $\tau_{1,2}$ -cl(H) $\cap$ $\tau_{1,2}$-cl(G) = ϕ.

(4) $\Rightarrow$ (1). Let M and N be any two disjoint $(1,2)^{å}$ - $g^{\#}$-closed sets of P. By assumption, there exist $\tau_{1,2}$-open sets G containing M and H containing N such that $\tau_{1,2}$-cl(G) $\cap$ $\tau_{1,2}$-cl(H) = ϕ, we have G $\cap$ H = ϕ and thus P is $(1,2)^{å}$ - $g^{\#}$-normal.

Theorem 6.5 *If $f : P \to Q$ is bijective, pre-$(1,2)^{å}$ - αg -open, $(1,2)^{å}$ - $g^{\#}$-continuous and $(1,2)^{å}$ -open and P is $(1,2)^{å}$ - $g^{\#}$-normal, then Q is $(1,2)^{å}$ - $g^{\#}$-normal.*

Proof. Let M and N be any disjoint $(1,2)^{å}$ - $g^{\#}$-closed sets of Q. The function f is $(1,2)^{å}$ - $g^{\#}$ -irresolute and so f^{-1}(M) and f^{-1}(N) are disjoint $(1,2)^{å}$ - $g^{\#}$-closed sets of P. Since P is $(1,2)^{å}$ - $g^{\#}$-normal, there exist disjoint $(1,2)^{å}$ -open sets S and T such that f^{-1}(M) $\subseteq$ S and f^{-1}(N) $\subseteq$ T. Since f is $(1,2)^{å}$ -open and bijective, we have f(S) and f(T) are $\sigma_{1,2}$-open in Q such that M $\subseteq$ f(S), N $\subseteq$ f(T) and f(S) $\cap$ f(T) = ϕ. Therefore, Q is $(1,2)^{å}$ - $g^{\#}$-normal.

Theorem 6.6 *If $f : P \to Q$ is $(1,2)^{å}$ - αg -irresolute $(1,2)^{å}$ - $g^{\#}$-closed $(1,2)^{å}$ -continuous injection and Q is $(1,2)^{å}$ - $g^{\#}$-normal, then P is $(1,2)^{å}$ - $g^{\#}$-normal.*

Proof. Let M and N be any disjoint $(1,2)^{å}$ - $g^{\#}$-closed subsets of P. Since f is $(1,2)^{å}$ - αg -irresolute $(1,2)^{å}$ - $g^{\#}$-closed, f(M) and f(N) are disjoint $(1,2)^{å}$ - $g^{\#}$-closed sets of Q. Since Q is $(1,2)^{å}$ - $g^{\#}$-normal, there exist disjoint $\tau_{1,2}$-open sets S and T such that f(M) $\subseteq$ S and f(N) $\subseteq$ T. i.e., M $\subseteq$ f^{-1}(S), N $\subseteq$ f^{-1}(T) and f^{-1}(S) $\cap$ f^{-1}(T) = ϕ. Since f is $(1,2)^{å}$ -continuous, f^{-1}(S) and f^{-1}(T) are $\tau_{1,2}$-open in P, we have P is $(1,2)^{å}$ - $g^{\#}$-normal.

Theorem 6.7 *If $f : P \to Q$ is weakly $(1,2)^{å}$ -continuous $(1,2)^{å}$ - $g^{\#}$-closed injection and Q is $(1,2)^{å}$ - $g^{\#}$-normal, then P is $(1,2)^{å}$ -normal.*

Proof. Let M and N be any two disjoint $\tau_{1,2}$-closed sets of P. Since f is injective and $(1,2)^{å}$ - $g^{\#}$ -closed, f(M) and f(N) are disjoint $(1,2)^{å}$ - $g^{\#}$-closed sets of Q. Since Q is $(1,2)^{å}$ - $g^{\#}$-normal, there exist $\sigma_{1,2}$-open sets S and T such that f(M) $\subseteq$ S, f(N) $\subseteq$ T and $\sigma_{1,2}$-cl(S) $\cap$ $\sigma_{1,2}$-cl(T) = ϕ. Since f is weakly $(1,2)^{å}$ -continuous, M $\subseteq$ f^{-1}(S) $\subseteq$ $\tau_{1,2}$-int(f^{-1}($\sigma_{1,2}$-cl(S))), N $\subseteq$ f^{-1}(T) $\subseteq$ $\tau_{1,2}$-int(f^{-1}($\sigma_{1,2}$-jj cl(T))) and $\tau_{1,2}$-int(f^{-1}($\sigma_{1,2}$-cl(S))) $\cap$ $\tau_{1,2}$-int(f^{-1}($\sigma_{1,2}$-cl(T))) = ϕ. Therefore, P is $(1,2)^{å}$ -normal.

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