On e-connectedness in Intuitionistic Fuzzy Topological Space in Ŝostak's Sense

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Abstract

In this paper, we introduce various types of fuzzy e-connectedness in intuitionistic fuzzy topological spaces in view of $\hat{S}$ostak's sense. The interrelationship between different notions of intuitionistic fuzzy e-connectedness are investigate. Also, we inspect some interrelations between these types of intuitionistic fuzzy e-connectedness together with the preservation properties under intuitionistic fuzzy e-continuous maps.

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1. Introduction Zadeh [33] introduced the fundamental concept of a fuzzy set. Later Chang [7] defined fuzzy topological spaces. $\hat{S}$ostak [29] introduced the fundamental concept of a fuzzy topological structure, as an extension of both crisp topology and Chang's fuzzy topology. The fuzzy topology in $\hat{S}$ostak's sense were rediscovered by Chattopadhyay et. al [8]. In the same year, Ramadan [21] gave a similar definition of a fuzzy topology under the name "smooth topology".

On the other hand, Atanssove and his collegues [2]-[6] introduced the fundamental concept of an intuitionistic fuzzy set. Coker [11],[13] used this type of generalized fuzzy set to define "intuitionistic fuzzy topological spaces". Also Coker and Demirci [12] introduced the basic definition and properties of "intuitionistic fuzzy topological spaces in $\hat{S}$ostak's sense" which is a generalized form of "fuzzy topological spaces" developed by $\hat{S}$ostak [29],[30]. In this sense many work have been launched [14],[16]-[19],[24],[31]. Recently, Sobana et.al [28] were introduced the concept of fuzzy e-open sets, fuzzy e-continuity and fuzzy e-compactness in intuitionistic fuzzy topological spaces.

Connectedness of fuzzy sets is an important subject in fuzzy topology, it won the attention of many researchers [1],[7],[15],[20],[23],[25]-[27],[32].

In this paper, many different notions of e-connectedness of fuzzy sets are extended to intuitionistic fuzzy topological spaces in $\hat{S}$ostak's sense and interrelationship between them are studied. Also, we inspect some interrelations between these types of intuitionistic fuzzy e-connectedness together with the preservation properties under intuitionistic fuzzy e-continuous maps.

Preliminaries

Definition 2.1 *[2] Let X be a nonempty fixed set. An intuitionistic fuzzy set (briefly, IFS) A is an object having the form*

$$A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}.$$

where the map $\mu_A : X\to I$ and $\gamma_A : X\to I$ denote the degree of membership (namely, $\mu_A(x)$) and the degree of nonmembership (namely, $\gamma_A(x)$) of each element $x\in X$ to the set A, respectively, and $0\le \mu_A(x)+\gamma_A(x)\le 1$ for each $x\in X$. Obviously, every fuzzy set A on a nonempty set X is an IFS having the form

$$A=\{\langle x,\mu_A(x),1-\mu_A(x)\rangle : x\in X\}.$$

For the sake of simplicity, we shall use the symbol $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$. For a given nonempty set X, let us denote the family of all IFSs in X by the symbol ζ^X.

Definition 2.2*[2],[4] Let X be a nonempty set, and the IFSs A and B in X be the form $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}, B=\{\langle x,\mu_B(x),\gamma_B(x)\rangle : x\in X\}$ Furthermore, let $\{A_i : i\in J\}$ be an arbitrary family of IFSs in X. Then*

1. $A\subseteq B$ iff $\mu_A(x)\le\mu_B(x)$ and $\gamma_A(x)\ge\gamma_B(x)$, for all $x\in X$,
2. $A=B$ iff $A\subseteq B$ and $B\subseteq A$,
3. $\overline{A}=\{\langle x,\gamma_A(x),\mu_A(x)\rangle : x\in X\}$,
4. $A-B=A\cap\overline{B}$,
5. $\bigcap A_i=\{\langle x,\wedge\mu_A(x),\vee\gamma_A(x)\rangle : x\in X\}$,
6. $\bigcup A_i=\{\langle x,\vee\mu_A(x),\wedge\gamma_A(x)\rangle : x\in X\}$,
7. $0_{\sim}=\{\langle x,0,1\rangle : x\in X\}$ and $1_{\sim}=\{\langle x,1,0\rangle : x\in X\}$.

Definition 2.3*[10] Let a and b be two real numbers in $[0,1]$ satisfying the inequality $a+b\le 1$. Then the pair $\langle a,b\rangle$ is called an intuitionistic fuzzy pair. Let $\langle a_1,b_1\rangle,\langle a_2,b_2\rangle$ be two intuitionistic fuzzy pairs. Then we define*

1. $\langle a_1,b_1\rangle\le\langle a_2,b_2\rangle\Leftrightarrow a_1\le a_2$ and $b_1\ge b_2$,
2. $\langle a_1,b_1\rangle=\langle a_2,b_2\rangle\Leftrightarrow a_1=a_2$ and $b_1=b_2$,
3. If $\{\langle a_i,b_i\rangle : i\in J\}$ is a family of intuitonistic fuzzy pairs, then $\vee\langle a_i,b_i\rangle=\langle\vee a_i,\wedge b_i\rangle$ and $\wedge\langle a_i,b_i\rangle=\langle\wedge a_i,\vee b_i\rangle$,
4. The complement of an intuitionistic fuzzy pair $\langle a,b\rangle$ is the intuitionistic fuzzy pair defined by $\overline{\langle a,b\rangle}=\langle b,a\rangle$,
5. $\tilde{1}=\langle 1,0\rangle$ and $\tilde{0}=\langle 0,1\rangle$.

Definition 2.4 *[13] Let X and Y be two nonempty sets and $f:X\to Y$ be a map.*

1. If $B=\{\langle y,\mu_B(y),\gamma_B(y)\rangle : y\in Y\}$ is an IFS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by $f^{-1}(B)=\{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) : x\in X\}$.
2. If $A=\{\langle y,\mu_A(y),\gamma_A(y)\rangle : x\in X\}$ is an IFS in X, then the image of A under f, denoted by $f(A)$, is the IFS in Y defined by $f(A)=\{\langle y, f(\mu_A)(y), f(\gamma_A)(y) : y\in Y\}$, where $\underline{f}(\gamma_A)=1-f(1-\gamma_A)$.

Definition 2.5*[9] Let $A,B\in\zeta^X$. Then, A and B are said to be quasi-coincident, denoted by AqB iff there exists an element $x\in X$ such that $\mu_A(x)>\gamma_B(x)$ or $\gamma_A(x)<\mu_B(x)$, otherwise $A\bar{q}B$***.**

Theorem 2.1*[9],[31] Let $A,B\in\zeta^X$. Then*

1. $A\bar{q}\overline{B}$ iff $A\subseteq B$,
2. AqB iff $A\not\subseteq\overline{B}$,
3. if $A\cap B=0_{\sim}$, then $A\subseteq\overline{B}$,
4. if $A\not\subset\overline{B}$, then $A\cap B\ne 0_{\sim}$.

Definition 2.6*[10] An IFS ξ on the set ζ^X is called an intuitionistic fuzzy family (IFF for short) on X. In symbols, denote such an IFF in form $\xi=\langle\mu_\xi,\gamma_\xi\rangle$.* Let ξ be an IFF on X. Then the complemented IFF of ξ on X is defined by $\xi^* = \langle \mu_{\xi^*}, \gamma_{\xi^*} \rangle$, where $\mu_{\xi^*}(A) = \mu_\xi(\overline{A})$ and γ_{ξ^*}(A)=γ_ξ($\overline{A}$), for each A$\in \zeta^X$.

If T is an IFF on X, then for any $A \in \zeta^X$, construct the intuitionistic fuzzy pair $\langle \mu_T$(A), γ_T(A)$\rangle$ and use the symbol T(A)=$\langle \mu_T$(A), γ_T(A)$\rangle$.

Definition 2.7.*[12] An intuitionistic fuzzy topology in $\hat{S}$ostak's sense (IFT for short) on a nonempty set X is an IFF T on X satisfying the following axioms:*

1. $T(0_{\sim}) = T(1_{\sim}) = \tilde{1}$,
2. $T(A \cap B) \geq T(A) \wedge T(B)$, for any $A, B \in \zeta^X$,
3. $T(\bigcup A_i) \geq \bigwedge T(A_i)$, for any $\{A_i : j \in J\} \subseteq \zeta^X$.

In this case, the pair (X, T) is called an intuitionistic fuzzy topological space in $\hat{S}$ostak's sense (IFTS for short). For any $A \in \zeta^X$, the number $\mu_T(A)$ is called the openness degree of A, while $\gamma_T(A)$ is called the nonopenness degree of A.

Definition 2.8*[12] Let (X, T) be an IFTS on X. Then, the IFF T^* of complemented IFSs on X is defined by: $T^*(A) = T(\overline{A})$. The number $\mu_{T^*}(A) = \mu_T(\overline{A})$ is called the closedness degree of A, while $\gamma_{T^*}(A) = \gamma_T(\overline{A})$ is called the nonclosedness degree of A.*

Theorem 2.2*[12] The IFF T^*on X satisfies the following properties:*

1. $T^*(0_{\sim}) = T^*(1_{\sim}) = \tilde{1}$,
2. $T^*(A \cup B) \geq T^*(A) \wedge T^*(B)$, for any $A, B \in \zeta^X$,
3. $T^*(\bigcap A_i) \geq \bigwedge T^*(A_i)$, for any $\{A_i : i \in J\} \subseteq \zeta^X$.

Definition 2.9*[12] Let (X, T) be an IFTS and A be an IFS in X. Then the fuzzy closure and fuzzy interior of A are defined by*

$cl_{\alpha,\beta}(A) = \cap\{K \in \zeta^X : A \subseteq K, T^*(K) \geq \langle \alpha, \beta \rangle\}$. $int_{\alpha,\beta}(A) = \cup\{G \in \zeta^X : G \subseteq A, T(G) \geq \langle \alpha, \beta \rangle\}$.

where $\alpha \in I_0 = (0,1], \beta \in I_1 = [0,1)$ with $\alpha + \beta \leq 1$.

Theorem 2.3*[12] The closure and interior operators satisfy the following properties:*

1. $A \subseteq cl_{\alpha,\beta}(A)$,
2. $int_{\alpha,\beta}(A) \subseteq A$,
3. $cl_{\alpha,\beta}(cl_{\alpha,\beta}(A)) = cl_{\alpha,\beta}(A)$,
4. $int_{\alpha,\beta}(int_{\alpha,\beta}(A)) = int_{\alpha,\beta}(A)$,
5. $cl_{\alpha,\beta}(A \cup B) = cl_{\alpha,\beta}(A) \cup cl_{\alpha,\beta}(B)$,
6. $int_{\alpha,\beta}(A \cap B) = int_{\alpha,\beta}(A) \cap int_{\alpha,\beta}(B)$,
7. $\overline{cl_{\alpha,\beta}(A)} = int_{\alpha,\beta}(\overline{A})$,
8. $\overline{int_{\alpha,\beta}(A)} = cl_{\alpha,\beta}(\overline{A})$.

Definition 2.10*[12] Let (X, T_1) and (Y, T_2) be two IFTSs and $f : X \to Y$ be a map. Then f is said to be intuitionistic fuzzy continuous iff*

$$T_1(f^{-1}(B)) \geq T_2(B)$$

for each $B \in \zeta^Y$.

Theorem 2.4*[12] The following properties are equivalent*

1. $f : (X, T_1) \to (Y, T_2)$ is an intuitionistic fuzzy continuous,
2. $T_1^*(f^{-1}(B)) \geq T_2^*(B)$, for each $B \in \zeta^Y$.

Definition 2.11*[22] Let A be an IFS in an IFTS (X,T). For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, A is called:*

1. an (α,β)-intuitionistic fuzzy regular open (briefly, (α,β)-ifro) set of X, if $int_{\alpha,\beta}(cl_{\alpha,\beta}(A)) = A$,
2. an (α,β)-intuitionistic fuzzy regular closed (briefly, (α,β)-ifrc) set of X, if $cl_{\alpha,\beta}(int_{\alpha,\beta}(A)) = A$.

Theorem 2.5*[22] Let A be an IFS in an IFTS (X,T). Then, for $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$.*

1. If A is an (α,β)-ifro (resp. (α,β)-ifrc) set then, $T(A) \geq \langle \alpha,\beta \rangle$(resp. $T^*(A) \geq \langle \alpha,\beta \rangle$),
2. A is an (α,β)-ifro set iff $\overline{A}$ is an (α,β)-ifrc set.

3. Different notions of e-connectedness of intuitionistic fuzzy sets

Definition 3.1. *Let (X,T) be an IFTS and $A = < x, \mu_A(x), \nu_A(x) >$ be a IFS in X. Then the fuzzy δ closure of A are denoted and defined by $\delta cl_{\alpha,\beta}(A) = \cap\{K : K$ is an (α,β) ifrcs in X and $A \subseteq K\}$ and $\delta int_{\alpha,\beta}(A) = \cup\{G : G$ is an (α,β) ifros in X and $G \subseteq A\}$.*

Definition 3.2 *Let A be an IFS in an IFTS (X,T). A is called an (α,β) intuitionistic fuzzy e-open set (resp. e-closed set) ((α,β)-ifeos, for short)(resp. (α,β)-ifecs, for short) in X if $A \leq cl_{\alpha,\beta}\delta int_{\alpha,\beta}(A) \vee int_{\alpha,\beta}\delta cl_{\alpha,\beta}(A)$ (resp. $A \geq int_{\alpha,\beta}\delta cl_{\alpha,\beta}(A) \wedge cl_{\alpha,\beta}\delta int_{\alpha,\beta}(A)$).*

Definition 3.3 *Let (X,T) be an IFTS and $A = < x, \mu_A, \nu_A >$ be an IFS in X. Then the intuitionistic fuzzy e-interior and intuitionistic fuzzy e-closure are defined and denoted by:*

$$ecl_{\alpha,\beta}(A) = \cap\{K : K \text{ is an } (\alpha,\beta) \text{ ifecs in } X \text{ and } A \subseteq K\}$$

and

$$eint_{\alpha,\beta}(A) = \cup\{G : G \text{ is an } (\alpha,\beta) \text{ ifeos in } X \text{ and } G \subseteq A\}.$$

It is clear that A is an (α,β)-ifecs ((α,β)-ifeos) in X iff $A = ecl_{\alpha,\beta}(A)(A = int_e(A))$.

Definition 3.4 *Let (X,T) be an IFTS and $N \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$*

1. If there exist the IFSs $U, V \in \zeta^X$ with U and V are $\langle \alpha,\beta \rangle$-intuitionistic fuzzy e-open sets satisfying the following properties, then N is called an intuitionistic fuzzy $ec_i^{\alpha,\beta}$-disconnected (briefly, $IFec_i^{\alpha,\beta}$-disconnected), ($i = 1,2,3,4$):
   $IFec_1^{\alpha,\beta} : N \subseteq U \cup V, U \cap V \subseteq \overline{N}, N \cap U \neq 0_{:}, N \cap V \neq 0_{:}$,
   $IFec_2^{\alpha,\beta} : N \subseteq U \cup V, N \cap U \cap V = 0_{:}, N \cap U \neq 0_{:}, N \cap V \neq 0_{:}$,
   $IFec_3^{\alpha,\beta} : N \subseteq U \cup V, U \cap V \subseteq \overline{N}, U \not\subseteq \overline{N}, V \not\subset \overline{N}$,
   $IFec_4^{\alpha,\beta} : N \subseteq U \cup V, N \cap U \cap V = 0_{:}, U \not\subseteq \overline{N}, V \not\subset \overline{N}$.
2. N is said to be intuitionistic fuzzy $ec_i^{\alpha,\beta}$-connected (briefly, $IFec_i^{\alpha,\beta}$-connected) if N is not an $IFec_i^{\alpha,\beta}$-disconnected, ($i = 1,2,3,4$).

$$\begin{tikzcd}
\text{IF } ec_1^{\alpha,\beta}\text{-connectedness} \arrow[r] \arrow[d] \arrow[dr] & \text{IF } ec_2^{\alpha,\beta}\text{-connectedness} \arrow[d] \\
\text{IF } ec_3^{\alpha,\beta}\text{-connectedness} \arrow[r] & \text{IF } ec_4^{\alpha,\beta}\text{-connectedness}
\end{tikzcd}$$

From Definition 3.4., we have the following implication between $IFec_i^{\alpha,\beta}$ *-connectedness (* $i=1,2,3,4$*).*

But the reciprocal implication are not true in general as shown by the following examples.

Example 3.1 *Let* $X=\{a,b,c\}$ *and* $N_1, N_2, G_i \in \zeta^X$ $(i=1,2,3,4)$ *defined as follows:*

$N_1=\langle x,(\frac{a}{0.1},\frac{b}{0.2},\frac{c}{0.1}),(\frac{a}{0.3},\frac{b}{0.5},\frac{c}{0.3})\rangle$; $N_2=\langle x,(\frac{a}{0.4},\frac{b}{0.3},\frac{c}{0.3}),(\frac{a}{0.3},\frac{b}{0.4},\frac{c}{0.2})\rangle$;

$G_1=\langle x,(\frac{a}{0.4},\frac{b}{0.4},\frac{c}{0.3}),(\frac{a}{0.1},\frac{b}{0.3},\frac{c}{0.3})\rangle$; $G_2=\langle x,(\frac{a}{0.3},\frac{b}{0.5},\frac{c}{0.1}),(\frac{a}{0.4},\frac{b}{0.2},\frac{c}{0.2})\rangle$;

$G_3=\langle x,(\frac{a}{0.4},\frac{b}{0.5},\frac{c}{0.3}),(\frac{a}{0.1},\frac{b}{0.2},\frac{c}{0.2})\rangle$; $G_4=\langle x,(\frac{a}{0.3},\frac{b}{0.4},\frac{c}{0.1}),(\frac{a}{0.4},\frac{b}{0.3},\frac{c}{0.3})\rangle$.

Let $T:\zeta^X \to I\times I$ defined as follows:

$$T(A)=\begin{cases}\tilde{1}, & \text{if } A\in\{0_{\sim},1_{\sim}\}\\ \langle 0.5,0.2\rangle, & \text{if } A=G_i (i=1,2,3,4)\\ \tilde{0}, & \text{otherwise.}\end{cases}$$

Let $\alpha=0.3,\beta=0.4$. Then, N_1 is both an $IFec_2^{\alpha,\beta}$ -connected and $IFec_3^{\alpha,\beta}$ -connected but not an $IFec_1^{\alpha,\beta}$ -connected (i.e., N_1 is an $IFec_2^{\alpha,\beta}$ -connected since, for every $G_i\in\zeta^X$ with G_i is (α,β) -ifeos, $i=1,2,3,4$, and satisfies $N_1\subseteq G_1\cup G_2, N_1\subseteq G_1\cup G_3, N_1\subseteq G_1\cup G_4, N_1\subseteq G_3\cup G_2, N_1\subseteq G_3\cup G_4$, we have $N_1\cap G_1\cap G_2\neq 0_{\sim}$,
$N_1\cap G_1\cap G_3\neq 0, N_1\cap G_1\cap G_4\neq 0_{\sim}, N_1\cap G_3\cap G_2\neq 0_{\sim}, N_1\cap G_3\cap G_4\neq 0_{\sim}$. Similarly, N_1 is an $IFec_3^{\alpha,\beta}$ -connected. N_1 is not an $IFec_1^{\alpha,\beta}$ -connected since, there exist $G_1,G_2 \in\zeta^X$ with G_i is (α,β) -ifeos, $i=1,2$ and satisfies $N_1\subseteq G_1\cup G_2, G_1\cap G_2\subseteq \overline{N_1}, N_1\cap G_1\neq 0_{\sim}$ and $N_1\cap G_2\neq 0_{\sim}$. By the same technique we have, N_2 is an $IFec_4^{\alpha,\beta}$ -connected but not an $IFec_3^{\alpha,\beta}$ -connected.

Example 3.2 *Let* $X=\{a,b,c\}$ *and* $N, G_i\in\zeta^X$ $(i=1,2,3,4)$ *defined as follows:*

$N_1=\langle x,(\frac{a}{0.4},\frac{b}{0.2},\frac{c}{0.0}),(\frac{a}{0.3},\frac{b}{0.5},\frac{c}{1.0})\rangle$; $G_1=\langle x,(\frac{a}{0.0},\frac{b}{0.3},\frac{c}{0.2}),(\frac{a}{1.0},\frac{b}{0.4},\frac{c}{0.3})\rangle$;

$G_2=\langle x,(\frac{a}{0.5},\frac{b}{0.0},\frac{c}{0.1}),(\frac{a}{0.2},\frac{b}{1.0},\frac{c}{0.3})\rangle$; $G_3=\langle x,(\frac{a}{0.5},\frac{b}{0.3},\frac{c}{0.2}),(\frac{a}{0.2},\frac{b}{0.4},\frac{c}{0.3})\rangle$;

$G_4=\langle x,(\frac{a}{0.0},\frac{b}{0.0},\frac{c}{0.1}),(\frac{a}{1.0},\frac{b}{1.0},\frac{c}{0.3})\rangle$. Let $T:\zeta^X\to I\times I$ defined as follows:

$$T(A)=\begin{cases}\tilde{1}, & \text{if } A\in\{0_{\sim},1_{\sim}\}\\ \langle 0.4,0.1\rangle, & \text{if } A=\{G_1,G_2\}\\ \langle 0.6,0.1\rangle, & \text{if } A\in\{G_3,G_4\}\\ \tilde{0}, & \text{otherwise.}\end{cases}$$

Let $\alpha=0.2,\beta=0.5$. Then, N is an $IFec_4^{\alpha,\beta}$ -connected but not an $IFec_2^{\alpha,\beta}$ (resp. $IFec_2^{\alpha,\beta}$) - connected.

Definition 3.5 *Let* X *be a nonempty set and* $A,B\in\zeta^X$*. For* $\alpha\in I_0,\beta\in I_1$ *with* $\alpha+\beta\leq 1$, A *and* B *are said to be*

1. (α,β) -intuitionistic fuzzy e -weakly separated (briefly, (α,β) - $IFeWS$) if there exist

IFSs $U,V \in \zeta^X$ with U and V are $\langle \alpha,\beta \rangle$- ifeos such that $A \subseteq U$, $B \subseteq V$, $A\bar{q}V$, $B\bar{q}U$,

2. (α,β)-intuitionistic fuzzy eq-separated (briefly, (α,β)-$IFeqS$) if $ecl_{\alpha,\beta}(A) \cap B = 0_{\sim}$ and $A \cap ecl_{\alpha,\beta}(B) = 0_{\sim}$.

Theorem 3.1 *Let (X,T) be an IFTS and $A,B \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha+\beta \le 1$, A and B are (α,β)-IFeWS iff $ecl_{\alpha,\beta}A \subseteq \overline{B}$ and $ecl_{\alpha,\beta}B \subseteq \overline{A}$.*

Proof. Suppose that A,B are (α,β)-IFeWS. Then, there exists (α,β)-ifeo sets $U,V \in \zeta^X$ such that $A \subseteq U$, $B \subseteq V$, $A\bar{q}V$, $B\bar{q}U$. By Theorem 2.1, $A \subseteq \overline{V}$, since $\overline{V}$ is (α,β)-ifeo-set then $ecl_{\alpha,\beta}A \subseteq \overline{V} \subseteq \overline{B}$. Similarly, $ecl_{\alpha,\beta}B \subseteq \overline{A}$.

Conversely, suppose that $ecl_{\alpha,\beta}A \subseteq \overline{B}$ and $ecl_{\alpha,\beta}B \subseteq \overline{A}$. Then, $B \subseteq \overline{ecl_{\alpha,\beta}A} = V$, $A \subseteq \overline{ecl_{\alpha,\beta}B} = U$ which implies that, $U = \overline{ecl_{\alpha,\beta}B} = ecl_{\alpha,\beta}B$ is (α,β)-ifeos and $V = \overline{ecl_{\alpha,\beta}A} = ecl_{\alpha,\beta}A$ is ifeos. Also, $A \subseteq ecl_{\alpha,\beta}A = \overline{V}$ and $B \subseteq ecl_{\alpha,\beta}B = \overline{U}$, which implies that $A\bar{q}V$, $B\bar{q}U$. Hence, A,B are (α,β)-IFeWS.

Definition 3.6 *Let (X,T) be an IFTS and $N \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha+\beta \le 1$,*

1. N is called an intuitionistic fuzzy $ec_s^{\alpha,\beta}$-disconnected (briefly, $IFec_s^{\alpha,\beta}$-disconnected) if there exist an (α,β)-$IFeWS$ sets $A,B \in \zeta^X$ such that $A \cup B = N$ and $A \neq 0_{\sim}$, $B \neq 0_{\sim}$,

2. N is called an intuitionistic fuzzy $ec_M^{\alpha,\beta}$-disconnected (briefly, $IFec_M^{\alpha,\beta}$-disconnected) if there exist an (α,β)-$IFeqS$ sets $A,B \in \zeta^X$ such that $A \cup B = N$ and $A \neq 0_{\sim}$, $B \neq 0_{\sim}$,

3. N called an $IFec_s^{\alpha,\beta}$-connected if N is not an $IFec_s^{\alpha,\beta}$-disconnected,

4. N called an $IFec_M^{\alpha,\beta}$-connected if N is not an $IFec_M^{\alpha,\beta}$-disconnected.

Theorem 3.2 *Let (X,T) be an IFTS and $N \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha+\beta \le 1$, if N is an $IFec_s^{\alpha,\beta}$-connected then, N is an $IFec_M^{\alpha,\beta}$-connected.*

Proof. Suppose for a contradiction that N is an $IFec_M^{\alpha,\beta}$-disconnected. Then, there exist $A,B \in \zeta^X$ such that $A \cup B = N$, $(ecl_{\alpha,\beta}A) \cap B = 0_{\sim}$, $(ecl_{\alpha,\beta}B) \cap A = 0_{\sim}$, $A \neq 0_{\sim}$, $B \neq 0_{\sim}$. By Theorem 2(`)@, we have $ecl_{\alpha,\beta}A \subseteq \overline{B}$, $ecl_{\alpha,\beta}B \subseteq \overline{A}$. Then by Theorem 3.1, N is an $IFec_s^{\alpha,\beta}$-disconnected which is a contradiction. Hence, N is an $IFec_M^{\alpha,\beta}$-connected.

Theorem 3.3. *Let (X,T) be an IFTS and $N \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha+\beta \le 1$, if N is an $IFec_1^{\alpha,\beta}$-connected then, N is an $IFec_s^{\alpha,\beta}$-connected.*

Proof. Suppose that N is an $IFec_s^{\alpha,\beta}$-disconnected. Then, there exist $A,B \in \zeta^X$ such that $A \cup B = N$, $(ecl_{\alpha,\beta}A) \subseteq \overline{B}$, $(ecl_{\alpha,\beta}B) \subseteq \overline{A}$, $A \neq 0_{\sim}$, $B \neq 0_{\sim}$. By Theorem 3.1, there exist $U,V \in \zeta^X$ with, U and V are $\langle \alpha,\beta \rangle$- ifeos such that, $A \subseteq U$, $B \subseteq V$, $A\bar{q}V$, $B\bar{q}U$. Then, $N = A \cup B \subseteq U \cup V$. Also, $N \cap U \neq 0_{\sim}$. For, if $N \cap U = 0_{\sim}$, then $N \cap A = 0_{\sim}$. Also, $U \cap V \subseteq \overline{N}$. For, if $U \cap V \not\subseteq \overline{N}$ then, $(U \cap V)qN$ which implies that $(U \cap V)qA$ or $(U \cap V)qB$. Then (UqA and VqA) or (UqB and VqB) a contradiction with $A\bar{q}V$ and $B\bar{q}U$. Thus, N is an $IFec_1^{\alpha,\beta}$-disconnected, which is a contradiction. Hence, N is an

$IFec_s^{\alpha,\beta}$ -connected.

Theorem 3.4 *Let* (X,T) *be an IFTS and* $N\in\zeta^X$. *For* $\alpha\in I_0,\beta\in I_1$ *with* $\alpha+\beta\le 1$, *if* N *is an* $IFec_s^{\alpha,\beta}$ *-connected then,* N *is an* $IFec_2^{\alpha,\beta}$ *-connected.*

Proof. Suppose that N is an $IFec_2^{\alpha,\beta}$ -disconnected. Then, there exist (α,β)-ifeo sets $U,V\in\zeta^X$ such that, $N\subseteq U\cup V$, $N\cap U\cap V=0_{\sim}$, $N\cap U\neq 0_{\sim}$ $N\cap V\neq 0_{\sim}$. Put $A=N\cap U\subseteq U$ and $B=N\cap V\subseteq V$. Then, $A\cup B=(N\cap U)\cup(N\cap V)=N\cap(U\cup V)=N$. Now, if AqV then, there exists $x\in X$ such that $\mu_A(x)>\gamma_V(x)$ or $\gamma_A(x)<\mu_V(x)$. First if $\mu_A(x)>\gamma_V(x)$ then, $\mu_A(x)>0_{\sim}$. Since $N=A\vee B$ and $A\le U$ then, $\mu_N(x)>0_{\sim}$ and $\mu_U(x)>0_{\sim}$. $\mu_V(x)\neq 0_{\sim}$. (since, if $\mu_V(x)=0_{\sim}$, then $\gamma_V(x)=1_{\sim}$, a contradiction with $\mu_A(x)>\gamma_V(x)$). Thus $(\mu_N(x)\wedge\mu_U(x)\wedge\mu_V(x))>0_{\sim}$. So, $N\cap U\cap V\neq 0_{\sim}$, a contradiction with $N\cap U\cap V=0_{\sim}$. Thus $A\bar{q}V$. Second: if $\gamma_A(x)<\mu_V(x)$ then, $\mu_V(x)>0_{\sim}$, $\mu_N(x)>0_{\sim}$ (for, if $\mu_N(x)=0_{\sim}$ then, $N=A\cup B$ implies that, $\mu_A(x)=\mu_B(x)=0_{\sim}$ so $\gamma_A(x)=1$ a contradicts with $\gamma_A(x)<\mu_V(x)$). Since, $N\subseteq U\cup V\neq 0_{\sim}$, $\mu_V(x)>0$, $\mu_N(x)>0$ then, $(\mu_N\wedge\mu_U\wedge\mu_V)(x)\neq 0_{\sim}$ and so, $N\cap U\cap V\neq 0$ a contradiction, then $A\bar{q}V$. Similarly, $B\bar{q}U$. Then, N is an $IFec_s^{\alpha,\beta}$ -disconnected, a contradiction. Thus N is an $IFec_2^{\alpha,\beta}$ -connected.

Theorem 3.5 *Let* (X,T) *be an IFTS and* $N\in\zeta^X$. *For* $\alpha\in I_0,\beta\in I_1$ *with* $\alpha+\beta\le 1$, *if* N *is an* $IFec_s^{\alpha,\beta}$ *-connected then,* N *is an* $IFec_3^{\alpha,\beta}$ *-connected.*

Proof. Suppose that N is an $IFec_2^{\alpha,\beta}$ -disconnected. Then, there exist $U,V\in\zeta^X$ with, U and V are $\langle\alpha,\beta\rangle$ -ifeo-sets such that, $N\subseteq U\cup V$, $U\cap V\subseteq\overline{N}$, $U\not\subseteq\overline{N}$ $V\not\subseteq\overline{N}$. Put $A=N\cap U\subseteq U$ and $B=N\cap V\subseteq V$. Then, $A\cup B=N$. Let $R=\langle x,\mu_R(x),\gamma_R(x)\rangle$ and $S=\langle x,\mu_S(x),\gamma_S(x)\rangle$. Where

$$\mu_R(x)=\begin{cases}\mu_A(x), & \text{if } \mu_U(x)\ge\mu_V(x)\\ 0, & \text{otherwise}\end{cases}$$

$$\gamma_R(x)=\begin{cases}\gamma_A(x), & \text{if } \gamma_U(x)\le\gamma_V(x)\\ 1, & \text{otherwise}\end{cases}$$

$$\mu_S(x)=\begin{cases}\mu_B(x), & \text{if } \mu_U(x)<\mu_V(x)\\ 0, & \text{otherwise}\end{cases}$$

$$\gamma_S(x)=\begin{cases}\gamma_B(x), & \text{if } \gamma_U(x)>\gamma_V(x)\\ 1, & \text{otherwise.}\end{cases}$$

Then, $N=R\cup S$ Now, $R\neq 0_{\sim}$ (since, if $R=0_{\sim}$, then $U\subset V$ which implies that, $U=U\cap V\subseteq\overline{N}$, a contradiction). Similarly, $S\neq 0_{\sim}$. Also, $R\subseteq A\subseteq U$ and $S\subseteq B\subseteq V$. Now, $R\bar{q}V$. For, if RqV then, there exists, $x\in X$ such that, $\mu_R(x)>\gamma_V(x)$ or $\gamma_R(x)<\mu_V(x)$. First, if $\mu_R(x)>\gamma_V(x)$ then, $\mu_R(x)>0_{\sim}$ which implies that, $\mu_U(x)\ge\mu_V(x)$. Since $N=R\cup S$ then, $\mu_N(x)\ge\mu_R(x)>\gamma_V(x)$. Since, $U\cap V\subseteq\overline{N}$ then, $\mu_N(x)\le\gamma_U(x)\vee\gamma_V(x)$ but, $\mu_N(x)>\gamma_V(x)$ this implies that $\gamma_U(x)>\gamma_V(x)$ then, $\gamma_R(x)=1$ this implies that $\mu_R(x)=0_{\sim}$, a contradiction. Then, $R\bar{q}V$. Second, if $\gamma_R(x)<\mu_V(x)$ then, $\gamma_R(x)<1_{\sim}$ which implies that $\gamma_U(x)\le\gamma_V(x)$. Since $N=R\cup S$ then, $\gamma_N(x)=(\gamma_R\wedge\gamma_S)(x)$

implies that $\gamma_N(x)\le\gamma_R(x)$ then, $\gamma_N(x)\le\mu_V(x)$. Since, $U\cap V\subseteq\overline{N}$ then, $\gamma_N(x)\ge(\mu_U\wedge\mu_V)(x)$ but, $\gamma_N(x)<\mu_V(x)$ this implies that $\mu_V(x)>\mu_U(x)$ then, $\mu_R(x)=0$ implies $\gamma_R(x)=1$ a contradiction. Then, $R\bar{q}V$. Similarly, $S\bar{q}U$. Then N is an $IFec_s^{\alpha,\beta}$-disconnected, which is a contradiction. Thus, N is an $IFec_3^{\alpha,\beta}$-connected.

Theorem 3.6 *Let (X,T) be an IFTS and $N\in\zeta^X$. For $\alpha\in I_0,\beta\in I_1$ with $\alpha+\beta\le 1$, if N is an $IFec_3^{\alpha,\beta}$-connected then, N is an $IFec_M^{\alpha,\beta}$-connected.*

Proof. Suppose that N is an $IFec_M^{\alpha,\beta}$-disconnected. Then, there exist $A,B\in\zeta^X$ such that, $A\cup B=N,(ecl_{\alpha,\beta}A)\cap B=0_{\sim},(ecl_{\alpha,\beta}B)\cap A=0_{\sim}$, $A\ne 0_{\sim}$, $B\ne 0_{\sim}$. Let $U=\overline{ecl_{\alpha,\beta}A}$ and $V=\overline{ecl_{\alpha,\beta}B}$. Then, $U=\overline{U}^c=(ecl_{\alpha,\beta}A)^c$ and $V=\overline{V}^c=(ecl_{\alpha,\beta}B)^c$ are (α,β)-ifeo sets.

Now, $U\cap V=\overline{ecl_{\alpha,\beta}A}\cap\overline{ecl_{\alpha,\beta}B}=\overline{ecl_{\alpha,\beta}A\cup ecl_{\alpha,\beta}B}=\overline{ecl_{\alpha,\beta}(A\cup B)}=\overline{ecl_{\alpha,\beta}N}\subseteq\overline{N}$. Also, $U\cup V=\overline{ecl_{\alpha,\beta}A}\cup\overline{ecl_{\alpha,\beta}B}=\overline{ecl_{\alpha,\beta}A\cap ecl_{\alpha,\beta}B}=\overline{ecl_{\alpha,\beta}(A\cap B)}=\overline{ecl_{\alpha,\beta}0_{\sim}}=\overline{0_{\sim}}=1_{\sim}$. Then, $N\subseteq U\cup V$ Also $U\not\subseteq\overline{N}$. For if $U\subseteq\overline{N}$ then, $N\subseteq\overline{U}=ecl_{\alpha,\beta}A$ this implies that, $ecl_{\alpha,\beta}A\supseteq A\cup B$ implies $ecl_{\alpha,\beta}\supseteq B$ implies $B\cap ecl_{\alpha,\beta}A\ne 0_{\sim}$, a contradiction. Similary, $V\not\subseteq\overline{N}$. Therefore, N is an $IFec_3^{\alpha,\beta}$-disconnected which is a contradiction, then N is an $IFec_M^{\alpha,\beta}$-connected.

Definition 3.7 *Let X be a nonempty set and $A,B\in\zeta^X$. For $\alpha\in I_0,\beta\in I_1$ with $\alpha+\beta\le 1$, A and B are said to be*

1. (α,β)-intuitionistic fuzzy e-separated (briefly, (α,β)-$IFes$), if there exist IFSs $U,V\in\zeta^X$ with U and V are $\langle\alpha,\beta\rangle$-ifeo-sets such that $A\subseteq U,B\subseteq V,U\cap B=0_{\sim}$ and $A\cap V=0_{\sim}$,
2. (α,β)-intuitionistic fuzzy strongly e-separated (briefly, (α,β)-$IFSeS$) if there exist IFSs $U,V\in\zeta^X$ with U and V are $\langle\alpha,\beta\rangle$-ifeo-sets such that $A\subseteq U,B\subseteq V,U\cap B=0_{\sim}$ and $A\cap V=0_{\sim}$, UqA and VqB.

Definition 3.8 *Let (X,T) be a nonempty set and $A,B\in\zeta^X$. For $\alpha\in I_0,\beta\in I_1$ with $\alpha+\beta\le 1$*

1. N is called an intuitionistic fuzzy $eO^{\alpha,\beta}$-disconnected (briefly, $IFeO^{\alpha,\beta}$-disconnected) if there exist an (α,β)-IFeS sets $A,B\in\zeta^X$ such that $A\cup B=N$, $A\ne 0_{\sim}$ and $B\ne 0_{\sim}$,
2. N is called an intuitionistic fuzzy $eO_q^{\alpha,\beta}$-disconnected (briefly, $IFeO_q^{\alpha,\beta}$-disconnected) if there exist an (α,β)-IFSeS sets $A,B\in\zeta^X$ such that $A\cup B=N$, $A\ne 0_{\sim}$ and $B\ne 0_{\sim}$,
3. N is called an $IFeO^{\alpha,\beta}$-connected if N is not an $IFeO^{\alpha,\beta}$-disconnected,
4. N is called an $IFeO_q^{\alpha,\beta}$-connected if N is not an $IFeO_q^{\alpha,\beta}$-disconnected.

Theorem 3.7 *Let (X,T) be an IFTS and $N\in\zeta^X$. For $\alpha\in I_0,\beta\in I_1$ with $\alpha+\beta\le 1$, if N is an $IFec_2^{\alpha,\beta}$-connected iff N is an $IFeO^{\alpha,\beta}$-connected.*

Proof. Suppose that N is an $IFeO^{\alpha,\beta}$-disconnected. Then, there exist $A,B\in\zeta^X$ such that, A,B are (α,β)-IFeS, $N=A\cup B$, $A\ne 0_{\sim}$, $B\ne 0_{\sim}$. Since A,B are (α,β)-IFS, there exist

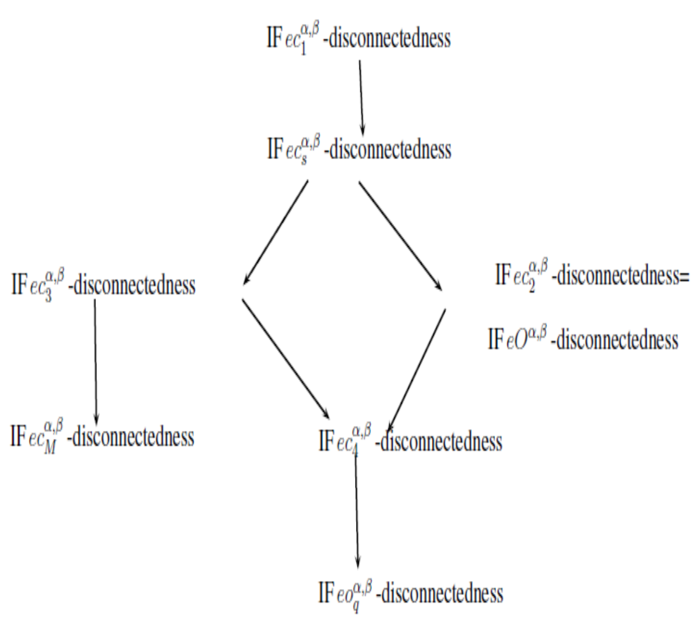
$U,V \in \zeta^X$ with U and V are $\langle\alpha,\beta\rangle$ -ifeo-sets such that, $A \subseteq U, B \subseteq V, U \cap B = 0_{\sim}, V \cap A = 0_{\sim}, N = A \cup B \subseteq U \cup V$. Now $N \cap U \cap V = (A \cup B) \cap (U \cap V) = (A \cap U \cap V) \cup (B \cap U \cap V) = 0_{\sim}$. Also, $N \cap U = (A \cup B) \cap U = (A \cap U) \cup (B \cap U) = A \cup 0_{\sim} = A \neq 0_{\sim}$. Similarly, $N \cap V \neq 0_{\sim}$. Then, N is an $IFec_2^{\alpha,\beta}$ -disconnected, which is a contradiction Hence, N is an $IFeO^{\alpha,\beta}$ -connected.

Conversely, suppose that N is an $IFec_2^{\alpha,\beta}$ -disconnected. Then, there exist $U,V \in \zeta^X$ with U and V are (α,β) -ifeo-sets such that, $N \subseteq U \cup V, N \cap U \cap V = 0_{\sim}$, $N \cap U \neq 0_{\sim}, N \cap V \neq 0_{\sim}$. Let $A = N \cap U \subseteq U$ and $B = N \cap V \subseteq V$. Then, $A \cup B = (N \cap U) \cup (N \cap V) = N \cap (U \cup V) = N$. Also, $U \cap B = U \cap N \cap V = 0_{\sim}$. Similary, $V \cap A = 0_{\sim}$. So, N is an $IFeO^{\alpha,\beta}$ -disconnected which is a contradiction. Then, N is an $IFec_2^{\alpha,\beta}$ -connected.

Theorem 3.8 *Let (X,T) be an IFTS and $N \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, if N is an $IFec_4^{\alpha,\beta}$ -connected then N is an $IFeO_q^{\alpha,\beta}$ -connected.*

Proof. Suppose that N is an $IFeO_q^{\alpha,\beta}$ -disconnected. Then, there exist $A,B \in \zeta^X$ such that, A,B are (α,β) -IFSeS, $N = A \cup B$. Since A,B are (α,β) -IFSeS, there exist $U,V \in \zeta^X$ with U and V are $\langle\alpha,\beta\rangle$ -ifeo-sets such that, $A \subseteq U, B \subseteq V, U \cap B = 0_{\sim}, V \cap A = 0_{\sim}, UqA, VqB\ N = A \cup B \subseteq U \cup V$. Now $N \cap U \cap V = (A \cup B) \cap (U \cap V) = (A \cap U \cap V) \cup (B \cap U \cap V) = 0_{\sim}$. Also, since UqA and $A \subseteq N$ there exist $x \in X$ such that $\mu_U(x) > \gamma_A(x) \geq \gamma_N(x)$ or $\gamma_U(x) < \mu_A(x) \leq \mu_N(x)$ this implies that, $U \not\subseteq \overline{N}$. Similarly, $V \not\subseteq \overline{N}$. Therefore, N is an $IFec_4^{\alpha,\beta}$ -disconnected which is a contradiction. Then, N is an $IFeO_q^{\alpha,\beta}$ -connected.

Remark 3.2 *From Remark 3.1 and Theorems 3.2-3.8, we can build the following diagram*



Examples 3.1, 3.2 and the next examples show that the reverse implications in Remark 3.2 are not true in general.

Example 3.3 *Let* $X=\{a,b\}$ *and* $N,G_i\in\zeta^X\,(i=1,2)$ *be defined as follows:*

$N=\langle x,(\frac{a}{0.3},\frac{b}{0.3}),(\frac{a}{0.4},\frac{b}{0.3})\rangle$; $G_1=\langle x,(\frac{a}{0.6},\frac{b}{0.7}),(\frac{a}{0.2},\frac{b}{0.1})\rangle$; $G_2=\langle x,(\frac{a}{0.1},\frac{b}{0.2}),(\frac{a}{0.3},\frac{b}{0.4})\rangle$.

Let $T:\zeta^X\to I\times I$ defined as follows:

$$T(A)=\begin{cases}\tilde{1}, & \text{if } A\in\{0_{:},1_{:}\}\\ \langle 0.4,0.2\rangle, & \text{if } A=G_1\\ \langle 0.6,0.1\rangle, & \text{if } A=G_2\\ \tilde{0}, & \text{otherwise.}\end{cases}$$

Let $\alpha=0.3,\beta=0.4$. Then, N is an $IFec_s^{\alpha,\beta}$-connected but not $IFec_1^{\alpha,\beta}$-connected.

***Example* 3.4** *Let* $X=\{a,b\}$ *and* $N,G_i\in\zeta^X\,(i=1,2,3,4,5,6)$ *be defined as follows:*

$N=\langle x,(\frac{a}{0.2},\frac{b}{0.2}),(\frac{a}{0.4},\frac{b}{0.5})\rangle$; $G_1=\langle x,(\frac{a}{0.2},\frac{b}{0.3}),(\frac{a}{0.4},\frac{b}{0.3})\rangle$;

$G_2=\langle x,(\frac{a}{0.3},\frac{b}{0.2}),(\frac{a}{0.2},\frac{b}{0.3})\rangle$; $G_3=\langle x,(\frac{a}{0.2},\frac{b}{0.2}),(\frac{a}{0.4},\frac{b}{0.3})\rangle$;

$G_4=\langle x,(\frac{a}{0.3},\frac{b}{0.3}),(\frac{a}{0.2},\frac{b}{0.3})\rangle$.; $G_5=\langle x,(\frac{a}{0.0},\frac{b}{0.2}),(\frac{a}{0.4},\frac{b}{0.5})\rangle$;

$G_6=\langle x,(\frac{a}{0.2},\frac{b}{0.0}),(\frac{a}{0.3},\frac{b}{0.4})\rangle$. Let $T:\zeta^X\to I\times I$ defined as follows:

$$T(A)=\begin{cases}\tilde{1}, & \text{if } A\in\{0_{:},1_{:}\}\\ \langle 0.3,0.1\rangle, & \text{if } A=\{G_1,G_2\}\\ \langle 0.4,0.3\rangle & \text{if } A=\{G_3,G_4\}\\ \tilde{0}, & \text{otherwise.}\end{cases}$$

Let $\alpha=0.3,\beta=0.4$. Then, N is an $IFec_2^{\alpha,\beta}$-connected and $IFec_3^{\alpha,\beta}$-connected but not $IFec_3^{\alpha,\beta}$-connected.

Example 3.5 *Let* $X=\{a,b,c\}$ *and* $N,G_i\in\zeta^X\,(i=1,2,3)$ *be defined as follows:*

$N=\langle x,(\frac{a}{0.5},\frac{b}{0.3},\frac{c}{0.1}),(\frac{a}{0.2},\frac{b}{0.4},\frac{c}{0.3})\rangle$; $G_1=\langle x,(\frac{a}{0.5},\frac{b}{0.0},\frac{c}{0.0}),(\frac{a}{0.2},\frac{b}{1.0},\frac{c}{1.0})\rangle$; $G_2=\langle x,(\frac{a}{0.0},\frac{b}{0.4},\frac{c}{0.2}),(\frac{a}{1.0},\frac{b}{0.1},\frac{c}{0.2})\rangle$; $G_3=\langle x,(\frac{a}{0.5},\frac{b}{0.4},\frac{c}{0.2}),(\frac{a}{0.2},\frac{b}{0.1},\frac{c}{0.2})\rangle$. Let $T:\zeta^X\to I\times I$ defined as follows:

$$T(A)=\begin{cases}\tilde{1}, & \text{if } A\in\{0_{:},1_{:}\}\\ \langle 0.2,0.4\rangle, & \text{if } A=\{G_1,G_2\}\\ \langle 0.6,0.1\rangle, & \text{if } A=G_3\\ \tilde{0}, & \text{otherwise.}\end{cases}$$

Let $\alpha=0.1,\beta=0.7$. Then, N is an $IFeO_q^{\alpha,\beta}$-connected but not $IFec_4^{\alpha,\beta}$-connected.

Example 3.6 *Let* $X=\{a,b,c\}$ *and* $N,G_i\in\zeta^X\,(i=1,2,3,4)$ *be defined as follows:*

$N=\langle x,(\frac{a}{0.3},\frac{b}{0.3},\frac{c}{0.2}),(\frac{a}{0.2},\frac{b}{0.3},\frac{c}{0.1})\rangle$; $G_1=\langle x,(\frac{a}{0.3},\frac{b}{0.3},\frac{c}{0.2}),(\frac{a}{0.1},\frac{b}{0.2},\frac{c}{0.2})\rangle$; $G_2=\langle x,(\frac{a}{0.2},\frac{b}{0.4},\frac{c}{0.1}),(\frac{a}{0.3},\frac{b}{0.1},\frac{c}{0.1})\rangle$; $G_3=\langle x,(\frac{a}{0.3},\frac{b}{0.4},\frac{c}{0.2}),(\frac{a}{0.1},\frac{b}{0.1},\frac{c}{0.1})\rangle$; $G_4=\langle x,(\frac{a}{0.2},\frac{b}{0.3},\frac{c}{0.1}),(\frac{a}{0.3},\frac{b}{0.2},\frac{c}{0.2})\rangle$. Let $T:\zeta^X\to I\times I$ defined as follows:

$$T(A)=\begin{cases}\tilde{1}, & \text{if } A\in\{0_\sim,1_\sim\}\\ \langle 0.2,0.5\rangle, & \text{if } A=\{G_1,G_2\}\\ \langle 0.4,0.3\rangle, & \text{if } A=\{G_3,G_4\}\\ \tilde{0}, & \text{otherwise.}\end{cases}$$

Let $\alpha=0.1,\beta=0.6$ Then, N is an $IFeC_M^{\alpha,\beta}$-connected but not $IFec_3^{\alpha,\beta}$-connected.

4. Intuitionistic fuzzy $ec_5^{\alpha,\beta}$ -connectedness

Definition 4.1 *Let (X,T) be an IFTS. For $\alpha\in I_0,\beta\in I_1$ with $\alpha+\beta\le 1$,*

1. X is called an intuitionistic fuzzy $ec_5^{\alpha,\beta}$-disconnected (briefly, $IFec_5^{\alpha,\beta}$-disconnected) if there exist an IFS $A\in\zeta^X$ such that A is (α,β)-ifecl-open set (that is both ifeo and ifec set) $A\neq 0_\sim$ and $A\neq 1_\sim$,
2. X is called an (α,β)-intuitionistic fuzzy e-disconnected (briefly, $(\alpha,\beta)IFe$-disconnected) if there exist an IFSs $A,B\in\zeta^X$ such that A and B are (α,β)-ifecl-open set such that $A\cup B=1_\sim$, $A\cap B=0_\sim$, $A\neq 0_\sim$ and $B\neq 0_\sim$,
3. X called an $IFec_5^{\alpha,\beta}$-connected if X is not an $IFec_5^{\alpha,\beta}$-disconnected,
4. X called an (α,β)IF e-connected if X is not an (α,β) IF e-disconnected.

Theorem 4.1 *Let (X,T) be an IFTS. For $\alpha\in I_0,\beta\in I_1$ with $\alpha,\beta\le 1$, if (X,T) is an $IFec_5^{\alpha,\beta}$-connected then, (X,T) is an $(\alpha,\beta)IFe$-connected.*

Proof. Suppose that (X,T) is an $(\alpha,\beta)IFe$-disconnected. Then, there exist (α,β)-ifeo-sets $A,B\in\zeta^X$ such that, $A\cup B=1_\sim$, $A\cap B=0_\sim$, $A\neq 0_\sim$, $B\neq 0_\sim$. This implies that, $\mu_A\vee\mu_B=1_X,\gamma_A\wedge\gamma_B=0_X,\mu_A\wedge\mu_B=0_X,\gamma_A\vee\gamma_B=1_X$. Let $C=\{x\in X:\mu_A(x)>0\}$ and $D=\{x\in X:\mu_A(x)=0\}$.

If $x\in C$ then, $\mu_A(x)>0\Rightarrow\mu_B(x)=0\Rightarrow\mu_A(x)=1\Rightarrow\gamma_A(x)=0\Rightarrow\gamma_B(x)=1$.

If $x\in D$ then, $\mu_A(x)=0\Rightarrow\gamma_A(x)=1\Rightarrow\gamma_B(x)=0\Rightarrow\mu_B(x)=1$. Then, $\mu_A=\gamma_B$ and $\gamma_A=\mu_B$; in other words, $B=\overline{A}$ then, $A^c=\overline{A}=B$ are (α,β)-ifeo sets and since $B\neq 0_\sim$, $A\neq 1_\sim$. Thus, (X,T) is an $IFec_5^{\alpha,\beta}$-disconnected which is a contradiction. Hence, (X,T) is an $(\alpha,\beta)IFe$-connected.

Theorem 4.2 *Let $(X,T_1),(Y,T_2)$ be two IFTSs and $f:(X,T_1)\to(Y,T_2)$ be an intuitionistic fuzzy continuous and surjective map. For $\alpha\in I_0,\beta\in I_1$ with $\alpha+\beta\le 1$, if (X,T_1) is an $(\alpha,\beta)IF$ -connected then so is (Y,T_2).*

Proof. Suppose that (Y,T_2) is an (α,β) IF-disconnected. Then, there exist (α,β)-ifeo-sets $U,V\in\zeta^Y$ such that, $U\cup V=1_\sim$, $U\cap V=0_\sim$, $U\neq 0_\sim$, $V\neq 0_\sim$. Since f is an intuitionistic fuzzy continuous then, (α,β)-ifeo set of $f^{-1}(U)$ in T_1 is greater than or equal to (α,β)-ifeo set of U in T_2 and (α,β)-ifeo set of $f^{-1}(V)$ in T_1 is greater than or equal to (α,β)-ifeo set of V in T_2. Let $A=f^{-1}(U),B=f^{-1}(V)$, then A and B are (α,β)-ifeo set in T_1. Since f is surjective and $U\neq 0_\sim$ then, $A=f^{-1}(U)\neq 0_\sim$. (For, if $f^{-1}(U)=0_\sim$ then, $U=f(f^{-1}(U))=f(0_\sim)=0_\sim$ a contradiction). Similarly, $B=f^{-1}(V)\neq 0_\sim$. Now, $A\cup B=f^{-1}(U)\cup f^{-1}(V)=f^{-1}(U\cup V)=f^{-1}(1_\sim)=1_\sim$. Similarly, $A\cup B=0_\sim$. Thus, (X,T_1) is an $(\alpha,\beta)IF$-disconnected which is a contradiction. Hence, (Y,T_2) is an $(\alpha,\beta)IFe$-connected.

Theorem 4.3 *Let $(X,T_1),(Y,T_2)$ be two IFTSs and $f:(X,T_1)\to(Y,T_2)$ be an intuitionistic fuzzy e-continuous and surjective map. For $\alpha\in I_0,\beta\in I_1$ with $\alpha+\beta\le 1$, if (X,T_1) is an $IFec_5^{\alpha,\beta}$-connected then so is (Y,T_2).*

Proof. It is similar to Theorem 4.2

Theorem 4.4 *Let (X,T) be an IFTS, For $\alpha\in I_0,\beta\in I_1$ with $\alpha+\beta\le 1,(X,T)$ is an $IFec_5^{\alpha,\beta}$-connected iff there is no exist IFSs $A,B\in\zeta^X$ with A and B are (α,β)-ifeo set such that $A=\overline{B}, A\neq 0_{\sim}$ and $B\neq 0_{\sim}$.*

Proof. Assume that there exist $A,B\in\zeta^X$ with, A and B are (α,β)-ifeo set such that $A=\overline{B}, A\neq 0_{\sim}$ and $B\neq 0_{\sim}$. Now, $B^c=\overline{B}=A$ is (α,β)-ifeo sets and $A\neq 0_{\sim}$ implies that $B\neq 1_{\sim}$. Then (X,T) is an $IFec_5^{\alpha,\beta}$-disconnected which is a contradiction. Conversely, assume that (X,T) is an $IFec_5^{\alpha,\beta}$-disconnected. Then there exists an IFS $A\in\zeta^X$ such that A and A^c are (α,β)-ifeo set, $A\neq 0_{\sim}$, $A\neq 1_{\sim}$. Now, take $B=\overline{A}$ then, $B=\overline{B}^c=A$ is (α,β)-ifeo set and since $A\neq 1_{\sim}$ then $B\neq 0_{\sim}$ which is a contradiction. Hence, (X,T) is an $IFec_5^{\alpha,\beta}$-connected.

Theorem 4.5 *Let (X,T) be an IFTS, For $\alpha\in I_0,\beta\in I_1$ with $\alpha+\beta\le 1,(X,T)$ is an $IFec_5^{\alpha,\beta}$-connected iff there is no exist IFSs $A,B\in\zeta^X$ such that $B=\overline{A}, B=\overline{ecl_{\alpha,\beta}A}, A=\overline{ecl_{\alpha,\beta}B}, A\neq 0$ and $B\neq 0$.*

Proof. Assume that there exist $A,B\in\zeta^Y$ such that $B=\overline{A}, B=\overline{ecl_{\alpha,\beta}A}, A=\overline{ecl_{\alpha,\beta}B}, A\neq 0_{\sim}$, $B\neq 0_{\sim}$. Then, $A=\overline{cl_{\alpha,\beta}B}=(cl_{\alpha,\beta}B)^c$ is (α,β)-ifeo set and $\overline{A}=B=\overline{cl_{\alpha,\beta}A}=(cl_{\alpha,\beta}A)^c$ is (α,β)-ifeo set. Then ,(X,T) is an $IFec_5^{\alpha,\beta}$-disconnected, a contradiction. Conversely, suppose that (X,T) is an $IFec_5^{\alpha,\beta}$-disconnected. Then there exists an IFS $A\in\zeta^Y$ such that A and A^c are (α,β)-ifeo set, $A\neq 0_{\sim}$ and $A\neq 1_{\sim}$. Let $B=\overline{A}$. Then, $B\neq 1_{\sim}$, $B\neq 0_{\sim}$ and since A^c is (α,β)-ifeo set then, $A=cl_{\alpha,\beta}A$, this implies that $B=\overline{cl_{\alpha,\beta}A}$. Since $B^c=\overline{A}^c=A$ is ifeo set, then $A=\overline{B}=\overline{cl_{\alpha,\beta}B}$, a contradiction.

Definition 4.2 *Let (X,T) be an IFTS. For $\alpha\in I_0,\beta\in I_1$ with $\alpha+\beta\le 1, X$ is called an (α,β)-intuitionistic fuzzy strong e-connected (briefly, $(\alpha,\beta)IF$-strong e-connected) if there exist IFSeS $A,B\in\zeta^X$ with A^c and B^c are (α,β)-ifeo sets, such that $\mu_A+\mu_B\le 1,\gamma_A+\gamma_B\ge 1, A\neq 0_{\sim}$ and $B\neq 0_{\sim}$.*

Remark 4.1*The notions of $IFec_5^{\alpha,\beta}$-connectedness and $(\alpha,\beta)IF$-strong e-connectedness are independent as indiciated by the following examples.*

Example4.1 *Let $X=\{a,b,c\}$ and $G_i\in\zeta^X$ $(i=1,2,3,4)$ defined as follows:*

$G_1=\langle x,(\frac{a}{0.2},\frac{b}{0.3},\frac{c}{0.1}),(\frac{a}{0.3},\frac{b}{0.4},\frac{c}{0.2})\rangle$; $G_2=\langle x,(\frac{a}{0.3},\frac{b}{0.4},\frac{c}{0.2}),(\frac{a}{0.2},\frac{b}{0.3},\frac{c}{0.1})\rangle$;

$G_3=\langle x,(\frac{a}{0.2},\frac{b}{0.3},\frac{c}{0.1}),(\frac{a}{0.3},\frac{b}{0.4},\frac{c}{0.2})\rangle$; $G_4=\langle x,(\frac{a}{0.3},\frac{b}{0.4},\frac{c}{0.2}),(\frac{a}{0.2},\frac{b}{0.3},\frac{c}{0.1})\rangle$;

Let $T:\zeta^X\to I\times I$ defined as follows:

$$T(A)=\begin{cases}\tilde{1}, & \text{if } A\in\{0_{\sim},1_{\sim}\}\\ \langle 0.3,0.4\rangle, & \text{if } A\in\{G_1,G_2\}\\ \langle 0.4,0.4\rangle, & \text{if } A\in\{G_3,G_4\}\\ \tilde{0}, & \text{otherwise.}\end{cases}$$

Let $\alpha=0.2,\beta=0.4$. Then, X is an (α,β)-strong e-connected but not an $IFec_5^{\alpha,\beta}$-connected.

Example 4.2 *Let* $X=\{a,b,c\}$ *and* $G_1,G_2\in\zeta^X$ *defined as follows:*

$G_1=\langle x,(\frac{a}{0.5},\frac{b}{0.5},\frac{c}{0.6}),(\frac{a}{0.3},\frac{b}{0.2},\frac{c}{0.1})\rangle$; $G_2=\langle x,(\frac{a}{0.3},\frac{b}{0.3},\frac{c}{0.2}),(\frac{a}{0.4},\frac{b}{0.5},\frac{c}{0.6})\rangle$.

Let $T:\zeta^X\to I\times I$ defined as follows:

$$T(A)=\begin{cases}\tilde{1}, & \text{if } A\in\{0_{\sim},1_{\sim}\}\\ \langle 0.3,0.2\rangle, & \text{if } A\in G_1\\ \langle 0.5,0.1\rangle, & \text{if } A\in G_2\\ \tilde{0}, & \text{otherwise.}\end{cases}$$

Let $\alpha=0.1,\beta=0.4$. Then, X is an $IFec_5^{\alpha,\beta}$-connected but not an (α,β)-strong e-connected.

Theorem 4.6 *Let* (X,T) *be an IFTS. For* $\alpha\in I_0,\beta\in I_1$ *with* $\alpha+\beta\leq 1,(X,T)$ *is an* $(\alpha,\beta)IF$ *strong* e*-connected iff there is no exists IFSeS* $A,B\in\zeta^Y$ *with* A *and* B *are* (α,β)*-ifeo set such that* $\mu_A+\mu_B\geq 1,\gamma_A+\gamma_B\leq 1, A\neq 1_{\sim}$ *and* $B\neq 1_{\sim}$.

Proof. Let $A,B\in\zeta^X$ with A and B are (α,β)-ifeo set such that $\mu_A+\mu_B\geq 1,\gamma_A+\gamma_B\leq 1, A\neq 1_{\sim}$ and $B\neq 1_{\sim}$. If we take $C=\overline{A}$ and $D=\overline{B}$, then $C^c=\overline{A}^c=A$ is (α,β)-ifeo set, $D^c=\overline{B}^c=B$ is (α,β)-ifeo set $C\neq 0_{\sim}$ and $D\neq 0_{\sim}$. Moreover, $\mu_C+\mu_D=\gamma_A+\gamma_B\leq 1,\gamma_C+\gamma_D=\mu_A+\mu_B\geq 1$, a contradiction.

The converse of the proof is obtained by using a similar technique.

Theorem 4.7 *Let* $f:(X,T_1)\to(Y,T_2)$ *be an intuitionistic fuzzy* e*-continuous and surjective map from an IFTS* (X,T_1) *to another IFTS* (Y,T_2)*. For* $\alpha\in I_0,\beta\in I_1$ *with* $\alpha+\beta\leq 1$*, if* (X,T_1) *is an* $(\alpha,\beta)IF$*-strong* e*-connected then so is* (Y,T_2).

Proof. Suppose that (Y,T_2) is not an $(\alpha,\beta)IF$ strong e-connected. Then, there exist $C,D\in\zeta^Y$ with C^c and D^c are (α,β)-ifeo sets such that, $\mu_C+\mu_D\leq 1$, $\gamma_A+\gamma_B\geq 1, C\neq 0_{\sim}, D\neq 0_{\sim}$. By using Theorem 2.4, we have, (α,β)-ifeo set $(f^{-1}(C))^c$ in T_1 is greater than or equal to (α,β)-ifeo set of C^c in T_2 and (α,β)-ifeo set $(f^{-1}(D))^c$ in T_1 is greater than or equal to (α,β)-ifeo set of D^c in T_2 Also, $\mu_{f^{-1}(C)}+\mu_{f^{-1}(D)}=f^{-1}(\mu_C)+f^{-1}(\mu_D)=\mu_C\circ f+\mu_D\circ f\leq 1_{\sim}$ (since, $\mu_C+\mu_D\leq 1_{\sim}$). Similarly, $\gamma_{f^{-1}(C)}+\gamma_{f^{-1}(D)}=f^{-1}(\gamma_C)+\gamma^{-1}(\gamma_D)=\gamma_C\circ f+\gamma_D\circ f\geq 1_{\sim}$ (since, $\gamma_C+\gamma_D\geq 1_{\sim}$). Moreover, $f^{-1}(C)\neq 0_{\sim}$ (For, if $f^{-1}(C)=0_{\sim}$ then, $C=f(f^{-1}(C))=f(0_{\sim})=0_{\sim}$ a contradiction). Similarly, $f^{-1}(C)\neq 0_{\sim}$. This is a contradiction, thus (Y,T_2) is (α,β) IF strong e-connected.

5. (α,β)-intuitionistic fuzzy super e-connectedness

Definition 5.1 *Let* (X,T) *be an IFTS. For* $\alpha \in I_0, \beta \in I_1$ *with* $\alpha+\beta \le 1$,

1. X is called an (α,β)-intuitionistic fuzzy super e-disconnected (briefly, (α,β) IF-super e-disconnected) if there exist an (α,β)-ifeo set A in X such that, $A \ne 0_{\sim}$ and $B \ne 1_{\sim}$,
2. X is called an (α,β)-intuitionistic fuzzy super e-connected if X is not an (α,β) IFSe-disconnected.

Theorem 5.1 *Let* (X,T) *be an IFTS. For* $\alpha \in I_0, \beta \in I_1$ *with* $\alpha+\beta \le 1$, *the following statements are equivalent.*

1. X is an (α,β) IF-super e-connected,
2. For each $A \in \zeta^X, A \ne 0_{\sim}$ such that A is an (α,β)-ifeo set we have $ecl_{\alpha,\beta}A = 1_{\sim}$,
3. For each $A \in \zeta^X, A \ne 1_{\sim}$ such that A^c is an (α,β)-ifeo set we have $int_{\alpha,\beta}A = 0_{\sim}$,
4. There is no exist IFS's, $A,B \in \zeta^X$ with A and B are (α,β)-ifeo sets such that $A \subseteq \overline{B}, A \ne 0_{\sim}$ and $B \ne 0_{\sim}$,
5. There is no exist IFS's, $A,B \in \zeta^X$ with A and B are (α,β)-ifeo sets such that $B = \overline{ecl_{\alpha,\beta}A}, A = \overline{ecl_{\alpha,\beta}B}, A \ne 0_{\sim}$ and $B \ne 0_{\sim}$,
6. There is no exist IFS's, $A,B \in \zeta^X$ with A^c and B^c are (α,β)-ifeo set such that $B = \overline{eint_{\alpha,\beta}A}, A = \overline{eint_{\alpha,\beta}B}, A \ne 0_{\sim}$ and $B \ne 0_{\sim}$.

Proof. (i)$\Rightarrow$(ii): Assume that there exist $A \in \zeta^X, A \ne 0_{\sim}$ with A is an (α,β)-ifeo set such that $ecl_{\alpha,\beta}A \ne 1_{\sim}$. Then, $B = eint_{\alpha,\beta}(ecl_{\alpha,\beta}A) \ne 1_{\sim}$ is an (α,β)-ifro set in X and $0_{\sim} \ne A \subseteq eint_{\alpha,\beta}(cl_{\alpha,\beta}A) = B$, which is a contradiction. Then, $ecl_{\alpha,\beta}A = 1_{\sim}$.
(ii)$\Rightarrow$(iii): Let $A \ne 1_{\sim}$ be an IFS in X such that A^c is an (α,β)-ifeo set. Then, $\overline{A} \ne 0_{\sim}$ and $\overline{A} = A^c$ is an (α,β)-ifeo set. By (ii) we have, $ecl_{\alpha,\beta}(\overline{A}) = 1_{\sim}$ implies that $\overline{ecl_{\alpha,\beta}(A)} = 0_{\sim}$ and by Theorem 2.3, we have $eint_{\alpha,\beta}A = 0_{\sim}$.
(iii)$\Rightarrow$(iv): Let $A,B \in \zeta^X$ with, A and B are (α,β)-ifeo sets such that $A \subseteq \overline{B}, A \ne 0_{\sim}$ and $B \ne 0_{\sim}$. Then, $\overline{B} \ne 1_{\sim}$ and B is an (α,β)-ifeo set By (iii) we have $eint_{\alpha,\beta}\overline{B} \ne 0_{\sim}$ and since $A \le \overline{B}$, then $0_{\sim} \ne A = eint_{\alpha,\beta}A \subseteq eint_{\alpha,\beta}\overline{B} = 0_{\sim}$ which is a contradiction.
(iv)$\Rightarrow$(i): Assume for a contradiction that X is an $(\alpha,\beta)IF$ super e-disconnected. Then, there exist an an (α,β)-ifeo set A in X such that $A \ne 0_{\sim}$ and $A \ne 1_{\sim}$. By Theorem 2.5, A is an (α,β)-ifeo set. If we take $B = \overline{ecl_{\alpha,\beta}A}$, then B is an (α,β)-ifeo set and $B \ne 0$ (For, if $B = 0_{\sim} \Rightarrow \overline{ecl_{\alpha,\beta}A} = 0_{\sim} \Rightarrow ecl_{\alpha,\beta}A = 1_{\sim} \Rightarrow A = eint_{\alpha,\beta}(ecl_{\alpha,\beta}A) = 1_{\sim}$ which is a contradiction with the fact $A \ne 0_{\sim}$). We also, have $A \le \overline{B}$ and this is a contradiction too.
(i)$\Rightarrow$(v): Suppose that there exist IFSs $A,B \in \zeta^X$ with, A and B are (α,β)-*ifeo* sets such that $B = \overline{ecl_{\alpha,\beta}A}, A = \overline{ecl_{\alpha,\beta}B}, A \ne 0_{\sim}$ and $B \ne 0_{\sim}$. Then $eint_{\alpha,\beta}(ecl_{\alpha,\beta}A) = eint_{\alpha,\beta}\overline{B} = \overline{ecl_{\alpha,\beta}B} = A$ and $A \ne 0_{\sim}, A \ne 1_{\sim}$ (For, if $A = 1_{\sim}$, then $1_{\sim} = \overline{ecl_{\alpha,\beta}B}$ implies

$0_{\sim} = ecl_{\alpha,\beta}B$ implies $B = 0_{\sim}$). A contradiction with X is an $(\alpha,\beta)IF$ -super e -connected.

(v)$\Rightarrow$(i): Suppose that X is an $(\alpha,\beta)IF$ super e-disconnected. Then, there is an (α,β) -ifro set A in X such that, $A \neq 0_{\sim}, A \neq 1_{\sim}$. Now, take $B = \overline{ecl_{\alpha,\beta}A}$. Then, B is an (α,β) -ifeo set $B \neq 0_{\sim}$ and $\overline{ecl_{\alpha,\beta}B} = \overline{ecl_{\alpha,\beta}(\overline{ecl_{\alpha,\beta}A})} = \overline{\overline{eint_{\alpha,\beta}(ecl_{\alpha,\beta}A)}} = eint_{\alpha,\beta}(ecl_{\alpha,\beta}A) = A$ which is a contradiction.

(v) $\Rightarrow$ (vi): Let A, B IFSs in X with A^c and B^c are (α,β) -ifeo sets such that $B = \overline{eint_{\alpha,\beta}A}, A = \overline{eint_{\alpha,\beta}B}, A \neq 1_{\sim}$ and $B \neq 1_{\sim}$. Take $C = \overline{A}$ and $D = \overline{B}$. Then $C \neq 0_{\sim}, D \neq 0_{\sim}, C = \overline{A} = A^c$ is (α,β) -ifeo set $D = \overline{B} = B^c$ is (α,β) -ifeo-set and $\overline{ecl_{\alpha,\beta}C} = \overline{ecl_{\alpha,\beta}\overline{A}} = \overline{\overline{eint_{\alpha,\beta}A}} = eint_{\alpha,\beta}A = \overline{B} = D$. Similarly, $\overline{ecl_{\alpha,\beta}D} = C$. This is a contradiction.

(vi)$\Rightarrow$(v): It is similarly to that (v)$\Rightarrow$(vi).

Theorem 5.2 *Let (X,T) be an IFTS. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, if X is an $(\alpha,\beta)IF$ super e-connected then, X is an $IFec_5^{\alpha,\beta}$ -connected.*

Proof. It is clear.

The converse of Theorem 5.2 is not true in general as shows in the following example.

Example 5.1 *Let $X = \{a,b,c,d\}$ and $G_i \in \zeta^X (i = 1,2,3,4)$ defined as follows:*

$$G_1 = \langle x, (\frac{a}{1.0}, \frac{b}{0.0}, \frac{c}{0.0}, \frac{d}{0.0}), (\frac{a}{0.0}, \frac{b}{0.0}, \frac{c}{1.0}, \frac{d}{1.0}) \rangle ;$$

$$G_2 = \langle x, (\frac{a}{0.0}, \frac{b}{0.0}, \frac{c}{0.0}, \frac{d}{1.0}), (\frac{a}{1.0}, \frac{b}{0.0}, \frac{c}{1.0}, \frac{d}{0.0}) \rangle ;$$

$$G_3 = \langle x, (\frac{a}{1.0}, \frac{b}{0.0}, \frac{c}{0.0}, \frac{d}{1.0}), (\frac{a}{0.0}, \frac{b}{0.0}, \frac{c}{1.0}, \frac{d}{0.0}) \rangle ;$$

$$G_4 = \langle x, (\frac{a}{0.0}, \frac{b}{0.0}, \frac{c}{0.0}, \frac{d}{0.0}), (\frac{a}{1.0}, \frac{b}{0.0}, \frac{c}{1.0}, \frac{d}{1.0}) \rangle$$

Let $T : \zeta^X \to I \times I$ defined as follows:

$$T(A) = \begin{cases} \tilde{1}, & \text{if } A \in \{0_{\sim}, 1_{\sim}\} \\ \langle 0.4, 0.3 \rangle, & \text{if } A \in \{G_1, G_2\} \\ \langle 0.6, 0.2 \rangle, & \text{if } A \in \{G_3, G_4\} \\ \tilde{0} & \text{otherwise.} \end{cases}$$

Let $\alpha = 0.3, \beta = 0.5$. Then, X is an $IFec_5^{\alpha,\beta}$ -connected but not $(\alpha,\beta)IF$ -super e -connected.

Theorem5.3 *Let $(X,T_1), (Y,T_2)$ be two IFTSs and $f : (X,T_1) \to (Y,T_2)$ be a surjective intuitionistic fuzzy continuous map. Then, for $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, if X is an $(\alpha,\beta)IF$ super e-connected, then so is Y.*

Proof. Assume that Y is an $(\alpha,\beta)IF$ -disconnected. By Theorem 5.1(iv), there exist IFSs $C, D \in \zeta^Y$ with C is (α,β)-ifeo set in T_2 such that $C \subseteq \overline{D}, C \neq 0_{\sim}$ and $D \neq 0_{\sim}$. Since f is intuitionistic fuzzy continuous, $T_1(f^{-1}(C)) \geq T_2(C) \geq \langle \alpha, \beta \rangle$ and $T_1(f^{-1}(D)) \geq T_2(D) \geq \langle \alpha, \beta \rangle, C \subseteq \overline{D}$ implies that $f^{-1}(C) \subseteq f^{-1}(\overline{D}) = \overline{f^{-1}(D)}$. Also, $f^{-1}(C) \neq 0_{\sim}$ and $f^{-1}(D) \neq 0_{\sim}$. By Theorem 5.1(i), X is an $(\alpha,\beta)IF$ super e -disconnected, a contradiction.

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