INTUITIONISTIC FUZZY BI-IDEALS IN TERNARY SEMIRINGS

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Abstract: The purpose of this paper is to study the concept of intuitionistic fuzzy bi-ideals in ternary semirings. We give some characterizations of ternary semiring using intuitionistic fuzzy bi-ideals.

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1. INTRODUCTION

Ternary semirings are one of the generalized structures of semirings. The notion of ternary algebraic system was introduced by Lehmer[8]. He investigated certain ternary albebraic systems called triplexes which turn out to be commutative ternary groups. Dutta and Kar[1] introduced the notion of ternary semiring which is a generalization of the ternary ring introduced by Lister[9]. Good and Hughes[3] introduced the notion of bi-ideal and Steinfeld[11], [12] introduced the notion of quasi-ideal. In 2005, studied quasi-ideals and bi-ideals of ternary semirings.

Ternary semiring arises naturally, for instance, the ring of integers Z is a ternary semiring. The subset Z^+ of all positive integers of Z forms an additive semigroup and which is closed under the ring product. Now, if we consider the subset Z^- of all negative integers of Z, then we see that Z^- is closed under the binary ring product; however, Z^- is not closed under the binary ring product, i.e., Z^- forms a ternary semiring. Thus, we see that in the ring of integers Z, Z^- forms a semiring whereas Z^- forms a ternary semiring. More generally; in an ordered ring, we can see that its positive cone forms a semiring whereas its negative cone forms a ternary semiring. Thus a ternary semiring may be considered as a counterpart of semiring in an ordered ring.

The theory of fuzzy sets was first inspired by Zadeh[14]. Fuzzy set theory has been developed in many directions by many scholars and has evoked great interest among mathematicians working in different fields of mathematics. Rosenfeld[13] introduced fuzzy sets in the realm of group theory. Fuzzy ideals in rings were introdued by Liu[10] and it has been studied by several authors. Jun[4] and Kim and Park[7] have also studied fuzzy ideals in semirings. In 2007, [6] we have introduced the notions of fuzzy ideals and fuzzy quasi-ideals in ternary semirings.

Our main purpose in this paper is to introduce the notions of fuzzy bi-ideal in ternary semirings and study regular ternary semiring in terms of these two subsystems of fuzzy subsemirings. We give some characterizations of fuzzy bi-ideals.

2. PRELIMINARIES

In this section, we review some definitions and some results which will be used in later sections.

Definition 2.1.

A set R together with associative binary operations called addition and multiplication(denoted by + and . respectively) will be called a semiring provided:

- Addition is a commutative operation.
- there exists $0 \in R$ such that $a+0=a$ and $a0=0a=0$ for each $a \in R$,
- multiplication distributes over addition both from the left and the right. i.e.,

$a(b+c)=ab+ac$ and $(a+b)c=ac+bc$

Definition 2.2.

A nonempty set S together with a binary operation, called addition and a ternary multiplication, denoted by juxtaposition, is said to be a ternary semiring if $(S,+)$ is an additive commutative semigroup satisfying the following conditions:

- $(abc)de=a(bcd)e=ab(cde)$
- $(a+b)cd=acd+bcd$
- $a(b+c)d=abd+acd$
- $ab(c+d)=abc+abd,$ for all $a,b,c,d,e \in S.$

Definition 2.3.

- An additive subsemigroup $(Q,+)$ of a ternary semiring S is called a quasi-ideal of S if $QSS \cap (SQS+SSQSS) \cap SSQ \subseteq Q.$
- An additive subsemigroup $(Q,+)$ of a ternary semiring S is called a bi-ideal of S if $QSQSQ \subseteq Q.$

Now, we review the concept of fuzzy sets ([10], [13], [14]). Let X be a non-empty set. A map $\mu : X \to [0,1]$ is called a fuzzy set in X, and the complement of a fuzzy set μ in X, denoted by $\overline{\mu}$, is the fuzzy set in X given by $\overline{\mu}(x)=1-\mu(x)$ for all $x \in X.$

Let X and Y be two non-empty sets and $f: X \to Y$ a function, and let μ and ν be any fuzzy sets in X and Y respectively. The image of μ under f, denoted by $f(\mu)$, is a fuzzy set in Y defined by

$$f(\mu)(y)=\begin{cases} sup_{x \in f^{-1}(y)} \mu(x) & \text{if} f^{-1}(y) \neq \emptyset, \\ 0 & otherwise, \end{cases}$$

for each $y \in Y$. The preimage of ν under f, denoted by $f^{-1}(\nu)$, is a fuzzy set in X defined by $(f^{-1}(\nu))(x)=\nu(f(x))$ for each $x \in X.$

Definition 2.4.

A fuzzy ideal of a semiring R is a function $A: R \to [0,1]$ satisfying the following conditions:

- A is a fuzzy subsemigroup of $(R,+)$; i.e., $A(x+y) \geq min\{A(x), A(y)\}$,
- $A(xy) \geq max\{A(x), A(y)\}$, for all $x, y \in R$

Definition 2.5.

Let A and B be any two subsets of S. Then $A\cap B, A\cup B, A+B$ and $A\circ B$ are fuzzy subsets of S defined by

$$(A\cap B) = min\{A(x), B(x)\}$$

$$(A\cup B) = max\{A(x), B(x)\}$$

$$(A+B)(x) = \begin{cases} sup\{min\{A(y), A(z)\}\}, & \text{if} x = y+z, \\ 0 & otherwise, \end{cases}$$

$$(A\circ B)(x) = \begin{cases} sup\{min\{A(y), A(z)\}\}, & \text{if} x = yz, \\ 0 & otherwise, \end{cases}$$

For any $x\in S$ and $t\in(0,1]$, define a fuzzy point x_t as

$$x_t(y) = \begin{cases} t, & \text{if} y = x, \\ 0 & \text{if} y \neq x, \end{cases}$$

If x_t is a fuzzy point and A is any fuzzy subset of S and $x_t \leq A,$ then we write $x_t \in A$. Note that $x_t \in A$ if and only if $x \in A_t$ where A_t is a level subset of A. If x_r and y_s are fuzzy points, than $x_r y_s = (xy)_{min\{r,s\}}$.

Definition 2.6.

The following result is evident. A fuzzy subset A of a fuzzy subsemigroup of S is called a fuzzy ternary subsemigroup of S if:

- $A(x+y) \geq min\{A(x), A(y)\}$, for all $x, y\in S$
- $A(xyz) \geq min\{A(x), A(y), A(z)\}$, for all $x, y, z\in S.$

Definition 2.7.

The following result is evident. A fuzzy subsemigroup A of a ternary semiring S called a fuzzy ideal of S if $A: S\to[0,1]$ satisfying the following conditions:

- $A(x+y) \geq min\{A(x), A(y)\}$, for all $x, y\in S$
- $A(xyz) \geq A(z)$
- $A(xyz) \geq A(x)$ and
- $A(xyz) \geq A(y)$, for all $x, y, z\in S$

A fuzzy subset A with conditions (i) and (ii) is called a fuzzy left ideal of S. If A satisfies (i) and (iii), then it is called a fuzzy right ideal of S. Also if A satisfies (i) and (iv), then it is called a fuzzy lateral ideal of S. A fuzzy ideal is a ternary semiring of S, if A is a fuzzy left, a fuzzy right and a fuzzy lateral ideal of S. It is clear that A is a fuzzy ideal of a ternary semiring S if and only if $A(xyz) \geq max\{A(x), A(y), A(z)\}$ for all $x, y, z\in S$, and that every fuzzy left(right, lateral) ideal of S is a fuzzy ternary subsemiring of S.

Example 2.1

Consider the set of negative inter with zero $S = Z_o^-$, $(Z_0^-, +, .)$ forms a ternary semiring S with zero. Define a fuzzy subset $\mu: Z \to[0,1],$ as follows

$$(A)(x) = \begin{cases} 0.6, & \text{if} x \in \langle -3 \rangle, \\ 0.7 & otherwise, \end{cases}$$

Then μ is an $(\varepsilon, \varepsilon \vee q)$ fuzzy ideal of the ternary subsemiring of S.

Definition 2.8.

The following result is evident. Let A be a fuzzy subset of ternary semiring S. We define

$$SAS + SSASS(z) = \begin{cases} sup\{min\{A(a), A(b)\}\}, & \text{ifz} = x(a + xby)y, \\ 0 & otherwise, \end{cases}$$

for all $x, y, a, b \in S$

3. FUZZY BI-IDEAL OF TERNARY SEMIRING

Definition 3.1.

A fuzzy subsemigroup μ of a ternary semiring S is called a fuzzy quasi-ideal of S [6] if

$$(FQI1) \mu SS \cap S\mu S \cap SS\mu \le \mu$$

$$(FQI2) \mu SS \cap SS\mu SS \cap SS\mu \le \mu$$

i.e., $\mu(x) \ge min\{(\mu SS)(x), (S\mu S + SS\mu SS)(x), (SS\mu)(x)\}$.

To strengthen the above definition, we present the following example.

Example 3.1

Consider the ternary semiring $(Z_5^-, +, .)$.

$Q = \begin{pmatrix} 0 & 0 & 0 \\ a & 0 & 0 \\ 0 & a & 0 \end{pmatrix} : a \in Z_5$ $S = \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & 0 \\ y & z & 0 \end{pmatrix} : x, y, z \in Z_5$ Q-quasi-ideal

Definition 3.2.

A fuzzy ternary subsemiring Q of S is called a fuzzy bi-ideal of S if $QSQSQ \le Q$ i.e., $Q(xs_1ys_2z) \ge min\{Q(x), Q(y), Q(z)\}$ $\forall\ x, y, z, w, v \in S$

Example 3.2

Let $Z^- = S$ be the set of all negative integers. Then Z^- is a ternary semiring under usual addition and ternary multiplication. Let $B = 2S$ Thus $BSBSB = 2SS2SS2S = 6(SSS)SS = 6(SSS) = 6S \subseteq 2S = B.$ Hence B is a bi-ideal of Z^-.

Define $\mu : S \to [0,1]$ by

$$\mu(x) = \begin{cases} 0.6, & \text{if} x \in B = 2S, \\ 0.7, & otherwise. \end{cases}$$

For any $t \in [0,1], \mu_t = \{2S\}$, since $\{2S\}$ is a bi-ideal in Z^-, μ_t is the bi-ideal in Z^- for all t. Hence μ is a fuzzy bi-ideal of Z^-.

Lemma 3.1.

Let μ be a fuzzy subset of S. If μ is a fuzzy left ideal, fuzzy right ideal and lateral ideal of ternary semiring of S, then μ is a fuzzy quasi-ideal of S.

Proof.

Let μ be a fuzzy left ideal, fuzzy right ideal and fuzzy lateral ideal of S. Let $x = as_1s_2 = s_1(b_1 + s_1cs_2)s_2 = s_1s_2d$ where $a,b,c,d,s_1,s_2 \in S$. Consider

$$(\mu SS \cap (S\mu S + SS\mu SS) \cap SS\mu)(x) = min\{(\mu SS)(x), (S\mu S + SS\mu SS)(x), (SS\mu)(x)\}$$

$$= min\left\{sup_{x=as_1s_2}\{\mu(a)\}, sup_{x=s_1(b+s_1cs_2)s_2}\{\mu(b),\mu(c)\}, sup_{x=s_1s_2d}\{\mu(d)\}\right\}$$

$$\leq min\left\{1, sup_{x=s_1(b+s_1cs_2)s_2}\{\mu(s_1(b+s_1cs_2)s_2)\}, 1\right\}$$

(as μ is a fuzzy left, fuzzy right and fuzzy lateral ideal,

$$\mu\{s_1(b+s_1cs_2)s_2\} \geq min\{\mu(b),\mu(c)\}$$

$$= \mu(b)\, if\ \mu(b) < \mu(c), (= \mu(c)\, if\ \mu(b) > \mu(c))).$$

we get

$$(\mu SS \cap (S\mu S + ss\mu SS) \cap SS\mu(x) \leq \mu(x)$$

We remark that if x is not expressed as $x = as_1s_2 = s_1(b_1 + s_1cs_2)s_2 = s_1s_2d$, then

$$(\mu SS \cap (S\mu S + SS\mu SS) \cap SS\mu)(x) = 0 \leq \mu(x).$$

Thus,

$$(\mu SS \cap (S\mu S + SS\mu SS) \cap SS\mu)(x) \leq \mu(x).$$

Hence μ is a fuzzy quasi-ideal of S.

Lemma 3.2.

For any non-empty subsets A, B and C of S,

- $f_A f_B f_C = f_{ABC}$
- $f_A \cap f_B \cap f_C = f_{A\cap B\cap C}$
- $f_A + f_B = f_{A+B}$

Proof.

Proof is straight forward.

Lemma 3.3.

Let Q be an additive subsemigroup of S.

- Q is a quasi-ideal of S if and only if f_Q is a fuzzy quasi-ideal of S.
- Q is a bi-ideal of S if and only if f_Q is a fuzzy bi-ideal of S.

Proof.

Proof of (1) can seen in [8]. Proof of (2) Assume that Q is a bi-ideal of S. Then f_Q is a fuzzy ternary subsemiring of S.

$$f_Q f_S f_Q f_S f_Q \leq f_{QSQSQ} \leq f_Q$$

This means that f_Q is a fuzzy bi-ideal of S. Conversely, let us assume that f_Q is a fuzzy bi-ideal of S. Let x be any element of $QSQSQ$. Then, we have

$$f_Q(x) \geq f_Q f_S f_Q f_S f_Q(x) = f_{QSQSQ}(x) = 1.$$

Thus $x \in Q$ and $QSQSQ \subseteq Q$. Hence Q is a bi-ideal of S.

Lemma 3.4.

Any fuzzy quasi-ideal of S is a fuzzy bi-ideal of S.

Proof.

Let μ be any fuzzy quasi-ideal of S. Then, we have

$$\mu S\mu S\mu \subseteq \mu(SSS)S \subseteq \mu SS,$$

$$\mu S\mu S\mu \subseteq S(SSS)\mu \subseteq SS\mu,$$

$\mu S\mu S\mu \subseteq SS\mu SS$ and taking $\{0\} \subseteq S\mu S$

so,

$$\mu S\mu S\mu \subseteq S\mu S + SS\mu SS$$

we have,

$$\mu S\mu S\mu \subseteq \mu SS \cap (S\mu S + SS\mu SS) \cap SS\mu \subseteq \mu$$

Hence, μ is a fuzzy bi-ideal of S.

Remark 3.1.

The converse of Lemma 3.4. does not hold, in general, that is, a fuzzy bi-ideal of a ternary semiring S may not be a fuzzy quasi-ideal of S.

Theorem 3.1.

Let μ be a fuzzy subset of S. If μ is a fuzzy left, fuzzy right and lateral ideal of ternary semiring of S, then μ is a fuzzy bi-ideal of S.

Proof.

As μ is fuzzy left, right, lateral ideal of S and Lemma 3.1 , μ is a fuzzy quasi-ideal of S. Hence by Lemma 3.4, μ is a fuzzy bi-ideal of S.

Theorem 3.2.

The following result is evident. Let μ be a fuzzy subset of S. Then μ is a fuzzy quasi-ideal of S, if and only if μ_t is a quasi-ideal of S, for all $t \in Im(\mu)$.

Theorem 3.3.

Let μ be a fuzzy subset of S. Then μ is a fuzzy bi-ideal of S, if and only if μ_t is a bi-ideal of S, for all $t \in Im(\mu)$.

Proof.

Let μ be a fuzzy bi-ideal of S. Let $t \in Im(\mu)$. Suppose $x, y, z \in S$ such that $x, y, z \in \mu_t$. Then

$$\mu(x) \geq t, \mu(y) \geq t, \mu(z) \geq t$$

and

$$min\{\mu(x), \mu(y), \mu(z)\} \geq t.$$

As μ is a fuzzy bi-ideal, $\mu(x-y) \geq t$ and thus $x - y \in \mu_t$. Let $u \in S$. Suppose $u \in \mu_t S \mu_t S \mu_t$. Then there exist $x, y, z \in \mu_t$ and $s_1, s_2 \in S$ such that $u = xs_1ys_2z$. Then,

$$\begin{aligned}(\mu S\mu S\mu)(u) &= \mu(xs_1ys_2z)\\ &\geq min\{\mu(x),\mu(y),\mu(z)\}\geq min\{t,t,t\}=t.\end{aligned}$$

Therefore, $(\mu S\mu S\mu)(u)\geq t.$ As μ is a bi-ideal of S, $\mu(u)\geq t$ implies $u\in\mu_t$. Hence μ_t is a bi-ideal of S. Conversely, let us assume that μ_A is a bi-ideal of S, $t\in Im(\mu)$. Let $p\in S$. Consider

$$(\mu S\mu S\mu)(p)=sup_{p=xs_1ys_2z}\{min\{\mu(x),\mu(y),\mu(z)\}\}$$

Let $\mu(x)=t_1<\mu(y)=t_2<\mu(z)=t_3$. Then, $\mu_{t_1}\supseteq\mu_{t_2}\supseteq\mu_{t_3}$. Thus $x,y,z\in\mu_{t_1}$ and $p=x_{t_1}s_1ys_2z\in\mu_{t_1}S\mu_{t_1}S\mu_{t_1}\subseteq\mu_{t_1}$. This implies $\mu(p)\geq t_1$ and hence $\mu S\mu S\mu\leq\mu$. Therefore, μ is a fuzzy bi-ideal of S.

Definition 3.3.

Let S and T be two ternary semirings. Let f be a mapping which maps from S into T. Then f is called a homomorphism of S into T if

- $f(a+b)=f(a)+f(b)$ and
- $f(abc)=f(a)f(b)f(c)$ for all $a,b,c\in S$

Theorem 3.4.

If λ is a fuzzy bi-ideal of a ternary semiring S and μ is a fuzzy ternary subsemiring of S, then $(\lambda\cap\mu)$ is a fuzzy bi-ideal of S.

Proof.

Let λ be a fuzzy bi-ideal and μ be a fuzzy ternary subsemiring of S. Clearly $(\lambda\cap\mu)$ is a fuzzy ternary subsemiring of S. Next we prove that $(\lambda\cap\mu)$ is a fuzzy bi-ideal of ternary semiring S. Let $t\in S$ and $s_1,s_2,x,y,z\in S$ such that $t=xs_1ys_2z.$ Consider

$$\begin{aligned}((\lambda\cap\mu)S(\lambda\cap\mu)S(\lambda\cap\mu))(t) &= sup_{t=xs_1ys_2z}\{min\{(\lambda\cap\mu)(x),S(s_1),(\lambda\cap\mu)(y),S(s_2),(\lambda\cap\mu)(z)\}\}\\ &= sup_{t=xs_1ys_2z}\{min\{(\lambda\cap\mu)(x),(\lambda\cap\mu)(y),(\lambda\cap\mu)(z)\}\}\end{aligned}$$

Let $min\{(\lambda\cap\mu)(x),(\lambda\cap\mu)(y),(\lambda\cap\mu)(z)\}=t$. This implies that $(\lambda\cap\mu)(x)\geq t,(\lambda\cap\mu)(y)\geq t$ and $(\lambda\cap\mu)(z)\geq t.$ Then $x,y,z\in(\lambda_t\cap\mu_t).$ As λ is the fuzzy bi-ideal and μ is the fuzzy ternary subsemiring, $(\lambda_t\cap\mu_t)$ is a bi-ideal of S. Hence, $xs_1ys_2z\in(\lambda_t\cap\mu_t).$ This implies

$$\begin{aligned}(\lambda\cap\mu)(xs_1ys_2z) &\geq t\\ &= min\{(\lambda\cap\mu)(x),(\lambda\cap\mu)(y),(\lambda\cap\mu)(z)\}.\end{aligned}$$

Thus,

$$min\{(\lambda\cap\mu)(x),(\lambda\cap\mu)(y),(\lambda\cap\mu)(z)\}\leq(\lambda\cap\mu)(xs_1ys_2z)$$

This shows that

$$sup_{t=xs_1ys_2z}min\{(\lambda\cap\mu)(x),(\lambda\cap\mu)(y),(\lambda\cap\mu)(z)\}\leq(\lambda\cap\mu)(xs_1ys_2z)$$

Thus, we have

$$((\lambda\cap\mu)S(\lambda\cap\mu)S(\lambda\cap\mu)(t)\leq(\lambda\cap\mu)(t)$$

Hence,

$$((\lambda\cap\mu)S(\lambda\cap\mu)S(\lambda\cap\mu)\leq(\lambda\cap\mu)$$

and $(\lambda\cap\mu)$ is a fuzzy ideal of S.

4. REGULAR TERNARY SEMIRING

A ternary semiring S is called regular if for every $a \in S$, there exists an x in S such that $axa = a$. A ternary semiring S is regular if and only if

$$\mu * \gamma * \lambda = \mu \cap \gamma \cap \lambda$$

for every fuzzy right ideal μ, fuzzy left ideal λ and fuzzy lateral ideal γ of S.

Proof. Proof is Straight forward.

Theorem 4.1.

For a ternary semiring S, the following conditions are equivalent:

- S is regular
- $\mu = \mu * S * \mu * S * \mu$, for every fuzzy bi-ideal μ of S.
- $\mu = \mu * S * \mu * S * \mu$, for every fuzzy quasi-ideal μ of S.

Proof.

$(1) \Rightarrow (2)$ First assume that (1) holds. Let μ be any fuzzy bi-ideal of S, and a any element of S. Then since S is regular, there exists an element x in S such that $a = axa (= axaxa)$. Then we have

$$\begin{aligned}
(\mu * S * \mu * S * \mu)(a) &= sup_{a=\sum_{finite} x_i y_i z_i} min\{\mu(x_i), (S * \mu * S)(y_i), (\mu)(z_i)\} \\
&\geq min\{\mu(a), (S * \mu * S)(xax), (\mu)(a)\} \\
&= min\left\{\mu(a), sup_{xax=\sum_{finite} p_i q_i r_i}[min\{S(p_i), \mu(q_i), S(r_i)\}], \mu(a)\right\} \\
&\geq min\{\mu(a), min\{S(x), \mu(a), S(x)\}, \mu(a)\} \\
&= min\{\mu(a), min\{1, \mu(a), 1\}, \mu(a)\} = \mu(a),
\end{aligned}$$

and so $\mu * S * \mu * S * \mu \subseteq \mu$. Since μ is a fuzzy bi-ideal of S, the converse inclusion holds. Thus we have $\mu * S * \mu * S * \mu = \mu$. $(2) \Rightarrow (3)$ Since any fuzzy quasi-ideal of S is a fuzzy bi-ideal of S by Lemma 3.4. $(3) \Rightarrow (1)$ Assume (3) holds. Let Q be any quasi-ideal of S, and a any element of Q. Then it follows from Lemma 3.4. that the characteristic function f_Q is a quasi-ideal of S. Then we have

$$f_{QSQSQ}(a) = (f_Q * f_S * f_Q * f_S * f_Q)(a) = f_Q(a) = 1$$

and so, $a \in QSQSQ$. Thus $Q \subseteq QSQSQ$. On the other hand, Q is a quasi-ideal of S.

$$QSQSQ \subseteq (QSS \cap SQS \cap SSQ)$$

$$QSQSQ \subseteq (QSS \cap SSQSS \cap SSQ)$$

then,

$$QSQSQ \subseteq (QSS \cap (SQS + SSQSS) \cap SSQ) \subseteq Q$$

and so we have $QSQSQ = Q$ and hence, S is a regular ternary semiring.

5. CONCLUSION:

In this article intuitionistic fuzzy bi-ideals in ternary semirings are studied and some properties of these ideals are analyzed.

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