ABSTRACT

The consequence of the non-newtonian behaviour of micropolar lubricant for a plain circular bearing and two lobe journal bearing was investigated and compared with Newtonian fluid. An empirical fluid flow equation which effectively represents the flow behaviour of lubricant was used to obtain a modified form of Reynolds’ equation. Finite difference numerical solutions were obtained for finite width bearing. The results show a strong influence on the load capacity of journal bearings by using micropolar lubricant for both the bearings. Linearized stiffness coefficients, damping coefficients, critical mass, and threshold speed and whirl frequency ratio for both the bearings for fixed eccentricity were evaluated. It is found from the study that the effect of micropolar parameter is very significant on the performance of journal bearing. The stability of both the bearings is improved while using micropolar fluid compared to Newtonian fluid.

Keywords: Circular bearing, Two-lobe bearing Micropolar fluid, Static and Dynamic Characteristics

1. INTRODUCTION

A hydrodynamic journal bearing system depends on many factors like shape of bearing, lubricant, stability etc. In this paper, the static and dynamic characteristics of a finite plain circular journal bearing and two-lobe journal bearing lubricated with Newtonian and Micropolar fluids are investigated. The earlier studies shows the lubricants are assumed to behave as Newtonian fluids. Micropolar may be noticed in the case of lubricants containing additives or in lubricants with long-chain molecules. Micropolar are particularly significant in lubrication problems, where the film is usually thin, and may have a significant influence on bearing performance.

The Performance characteristics of plain bearing as well as two lobe bearing with Newtonian and Micropolar fluid has been studied by many researchers. Based on the theory of micropolar lubrication the first application of the theory was presented by Eringen [1]. Allen and Kline [2] discussed the theory of lubrication with a micropolar fluid and presented solution for an inclined slider bearing. The analysis shows that for a micropolar fluid, load capacity and frictional force were increased. Zaheruddin and Isa [3], they provided the expressions for different hydrodynamic characteristics such as the load carrying capacity, the volume flow flux and the frictional force for one dimensional journal bearings, both infinitely long and infinitely short, with micropolar lubrication. Huang [4] obtained results for the lowest value of the frictional coefficient for various widths to diameter ratios with the same coupling number occurs at a different value of L. Khonsari and Brewe [5] micropolar fluids do indeed exhibit a beneficial effect in that the load carrying capacity is significantly increased and the friction coefficient is less than that of the Newtonian lubricant. Huang and Weng [6] show that the micropolar fluids exhibited larger normal stiffness coefficient but smaller normal damping coefficient, and comparatively narrower stable region of the journal. Nair and Jaydas [7] presented the effect of deformation of the bearing liner on the static and dynamic performance characteristics of an elliptical journal bearing operating with micropolar lubricant.

2. GOVERNING EQUATION’S

General form of governing equation for a micropolar fluid is obtained as:

Conservation of mass for an incompressible fluid is shown below:
\[ \nabla \cdot \mathbf{V} = 0 \tag{1} \]

Conservation of linear momentum for an incompressible fluid
\[ (\lambda + 2\mu)\nabla(\nabla \cdot \mathbf{V}) - \frac{(2\mu+\lambda)}{2} \nabla \times (\nabla \times \mathbf{V}) + \chi \nabla \times \mathbf{V} + \nabla \cdot \mathbf{F}_B = \frac{\partial \mathbf{V}}{\partial t} \tag{2} \]

Conservation of angular momentum for an incompressible fluid
\[ (\alpha + \beta + \gamma)\nabla(\nabla \cdot \mathbf{V}) - \gamma \nabla \times (\nabla \times \mathbf{V}) + \chi \nabla \times \mathbf{V} - 2\chi \mathbf{V} + \mathbf{C}_B = j \frac{D\mathbf{V}}{D\tau} \tag{3} \]
Where, \( \rho \) is the mass density, \( V \) is the velocity vector, \( v \) is the micro-rotational velocity vector, \( \pi \) is the thermodynamic pressure and is to be replaced by the hydrodynamic film pressure, \( p \), since, \( \pi = -\left[ \frac{\partial E}{\partial \rho} \right] = P \). \( E \) is the internal energy and \( P \) is to be determined by the boundary conditions. \( \lambda \) and \( \mu \) are the familiar viscosity coefficients of the classical fluid mechanics, while \( \alpha, \beta \) and \( \gamma \) are the new viscosity coefficients derived as the combinational effects of the gyroviscosities for the micropolar fluid as defined by Eringen [1]. \( \mathcal{X} \) is also a new viscosity coefficient for micropolar fluid, termed as spin viscosity, which establishes the link between velocity vector and the micro rotational velocity vector. \( F_B \) is the body force per unit mass, \( C_n \) is the body couple per unit mass and \( j \) is the microinertia constant. \( D/\text{Dt} \) represents the material derivative. Now consider the flow of thin lubricant layer in journal bearing and assume the components of velocity vector and microrotation vector have the following form:

\[
V = (V_x, V_y, V_z) \quad \text{and} \quad \mathbf{\omega} = (\omega_x, \omega_y, \omega_z)
\]

Using the usual postulations for lubricant theory [13] and ignoring infinitesimal quantities with high order we obtain the governing differential equations for lubricant flow

\[
\begin{align*}
\frac{(2 \mu X)}{2 \gamma} \frac{\partial^2 V_x}{\partial y^2} + \nabla V = \frac{\partial p}{\partial x} = 0 \\
\frac{(2 \mu X)}{2 \gamma} \frac{\partial^2 V_y}{\partial y^2} - \nabla V = \frac{\partial p}{\partial x} = 0 \\
\gamma \frac{\partial^2 V_z}{\partial y^2} + \nabla V = 2 \mathcal{X} = 0 \\
\gamma \frac{\partial^2 V_z}{\partial y^2} - \nabla V = 2 \mathcal{X} = 0
\end{align*}
\]

(4a)-(4d)

For the lubricant layer the ordinary boundary conditions are as follows:

\[
\begin{align*}
\text{At } y = 0: & \quad V_x = U_1, V_y = V_z = 0, \quad \omega_1 = \omega_3 = 0 \\
\text{At } y = h: & \quad V_x = U_2, V_y = V_z = 0, \quad \omega_1 = \omega_3 = 0
\end{align*}
\]

where \( h \) represents the thickness of oil film, \( U_1 \), the tangent velocity of surface 1, \( U_2 \) and \( V_z \), the tangent and normal velocity of surface 2 respectively.

Velocity components can be obtained by solving eqns. (4a-4d). Substituting all these velocity components into the equation (1) and integrating across the film thickness.

The modified Reynolds equation for a micropolar lubricant is obtained as:

\[
\frac{\delta}{\delta x} \left[ \Phi(N, l_m, h) \frac{\partial p}{\partial x} \right] + \frac{\delta}{\delta y} \left[ \Phi(N, l_m, h) \frac{\partial p}{\partial y} \right] = 6U \mu \frac{\partial h}{\partial x} + 12 \mu \frac{\partial h}{\partial x} = 0
\]

where, \( \Phi(N, l_m, h) = h^3 + 12l_m h^2 \pi c \coth \left( \frac{Nh}{2l_m} \right) \) and \( N = \frac{x}{2 \mu + \mathcal{X}} \).

(5)

N and \( l_m \) are two important parameters distinguishing a micropolar lubricant from a Newtonian lubricant. \( N \) is a dimensionless parameter called the coupling number, which couples the linear and angular momentum equations arising from the micro-rotational effects of the suspended particles in the lubricant. \( l_m \) represents the interaction between the micropolar lubricant and the film gap or clearance space of a journal bearing and is termed as the characteristic length of the micropolar lubricant. Non-Dimensional form of modified Reynolds equation is given as:

\[
\frac{\delta}{\delta \theta} \left[ \Phi(N, l_m, \overline{h}) \frac{\partial \overline{p}}{\partial \theta} \right] + \frac{\partial}{\partial \overline{r}} \left[ \Phi(N, l_m, \overline{h}) \frac{\partial \overline{p}}{\partial \overline{r}} \right] = 6 \overline{U} \frac{\partial \overline{h}}{\partial \theta} + 12 \overline{\mu} \frac{\partial \overline{h}}{\partial \theta} = 0
\]

(6)

where, \( \theta = \frac{x}{R} ; \overline{h} = \frac{h}{c} ; \overline{z} = \frac{z}{R} ; \overline{p} = \frac{pc^2}{\mu \overline{R}} ; \)

\[
\overline{l}_m = \frac{c}{l_m} ; \overline{U} = \omega R ; \overline{\overline{\tau}} = \omega \overline{\tau}
\]

\[
\Phi(N, l_m, \overline{h}) = \overline{h}^3 + 12 \overline{l}_m \overline{h}^2 \pi c \coth \left( \frac{Nh}{2l_m} \right)
\]
3. BOUNDARY CONDITIONS

The following boundary conditions, pertinent to the flow field, are defined in terms of pressure and pressure gradient.

For Plain circular bearing
\[ p = 0 \quad \text{at} \quad \theta = 0, \theta_2 \]
\[ \frac{\partial p}{\partial \theta} = 0 \quad \text{at} \quad \theta = \theta_2 \]
where \( \theta_2 \) is the location of the trailing edge.

For Elliptical bearing
\[ p = 0 \quad \text{at} \quad \theta = \theta_{1m}, \theta_{2m} \]
\[ \frac{\partial p}{\partial \theta} = 0 \quad \text{at} \quad \theta = \theta_{2m} \]
For \( m \)th lobe of the bearing, \( m=1, 2 \)

4. FILM THICKNESS AND ITS GRADIENT

For a plain circular bearing, the film thickness and its gradient is given by
\[ h = 1 - X_J \cos(\theta + \phi) - Y_J \sin(\theta + \phi); \]
\[ \dot{h} = -X_J \dot{\cos}(\theta + \phi) - Y_J \dot{\sin}(\theta + \phi); \]
For an elliptical bearing, the film thickness and its gradient is given by
\[ h = 1 - (X_J - X_L) \cos(\theta + \phi) - (Y_J - Y_L) \sin(\theta + \phi); \]
\[ \dot{h} = - (X_J - X_L) \dot{\cos}(\theta + \phi) - (Y_J - Y_L) \dot{\sin}(\theta + \phi); \]
Where \( X_J \) and \( Y_J \) indicates the centre position of the journal, \( X_L \) and \( Y_L \) indicates the rate of change of position.

5. STATIC CHARACTERISTICS

5.1 Load carrying capacity

Bearing forces are calculated by using:
\[ F_x = \int_0^{\theta_2} \int_0^{\sigma_2} p \cos\theta \, d\theta \, d\sigma \]
\[ F_y = \int_0^{\theta_2} \int_0^{\sigma_2} p \sin\theta \, d\theta \, d\sigma \]
Resultant load is given as:
\[ F = \sqrt{F_x^2 + F_y^2} \]
Where \( F_x \) and \( F_y \) are radial and tangential components of dimensionless hydrodynamic force.

5.2 Attitude Angle

The attitude angle is defined as the angle between the line of action of external force and line of centres.
6. DYNAMIC CHARACTERISTICS

6.1 Stiffness characteristics

The non-dimensional coefficients of stiffness are given as:

\[
\begin{bmatrix}
K_{xx} & K_{xy} \\
K_{yx} & K_{yy}
\end{bmatrix} = \begin{bmatrix}
-\frac{\delta F_x}{\delta x} & -\frac{\delta F_x}{\delta y} \\
-\frac{\delta F_y}{\delta x} & -\frac{\delta F_y}{\delta y}
\end{bmatrix}
\]

The first subscript denotes the direction of force and the second, the direction of displacement.

6.2 Damping Characteristics

The non-dimensional coefficients of damping are given as:

\[
\begin{bmatrix}
C_{xx} & C_{xy} \\
C_{yx} & C_{yy}
\end{bmatrix} = \begin{bmatrix}
-\frac{\delta F_x}{\delta x} \cdot \dot{x} & -\frac{\delta F_x}{\delta y} \cdot \dot{y} \\
-\frac{\delta F_y}{\delta x} \cdot \dot{x} & -\frac{\delta F_y}{\delta y} \cdot \dot{y}
\end{bmatrix}
\]

The first subscript denotes the direction of force and the second, the direction of velocity.

7. STABILITY ANALYSIS

Journal bearing experiences forces when disturbed from its equilibrium position resulting in the journal rotating around its static equilibrium position. The motion trajectories can be obtained by the linear and nonlinear equations of motion of the journal. The linearized equations of motion of a journal centre are:

\[
\begin{bmatrix}
M_j & 0 \\
0 & M_j
\end{bmatrix} \begin{bmatrix}
\ddot{x} \\
\ddot{y}
\end{bmatrix} + \begin{bmatrix}
C_{xx} & C_{xy} \\
C_{yx} & C_{yy}
\end{bmatrix} \begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} + \begin{bmatrix}
K_{xx} & K_{xy} \\
K_{yx} & K_{yy}
\end{bmatrix} \begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

The assumption of complementary solution to eqn. (11) of the form \( x = X e^{\lambda t} \) and \( y = Y e^{\lambda t} \) leads to following polynomial

\[
A_0 \lambda^4 + A_1 \lambda^3 + A_2 \lambda^2 + A_3 \lambda + A_4 = 0
\]

Where \( A_0 = M_j^2 \)

\( A_1 = M_j (C_{xx} + C_{yy}) \)

\( A_2 = M_j (K_{xx} + C_{xx} * C_{yy} + C_{xy} * C_{xy}) - K_{yx} * C_{xy} \)

\( A_3 = K_{xx} * C_{yy} + C_{xx} * K_{yy} - K_{yx} * C_{xy} - K_{xy} * C_{yx} \)

\( A_4 = K_{xx} * K_{yy} - K_{yx} * K_{xy} \)

Making the use of the polynomial equation into Routh-Hurwitz stability criteria, the stability periphery of the journal bearing system is obtained.

7.1 Critical mass

The critical mass is evaluated by using the stiffness and damping coefficients incorporated with Routh-Hurwitz stability criteria in following equation:

\[
\overline{M}_c = \frac{a_0}{b_0 - c_0}
\]

where, \( a_0 = C_{xx} * C_{yy} - C_{yx} * C_{xy} \)

\( b_0 = \frac{(C_{xx} + C_{yy}) * (K_{xx} * K_{yy} - K_{yx} * K_{xy})}{K_{xx} * C_{yy} + C_{xx} * K_{yy} - K_{yx} * C_{xy} - K_{xy} * C_{yx}} \)

\( c_0 = \frac{K_{xx} * C_{xx} + K_{yy} * C_{yy} + K_{yx} * C_{xy} + K_{xy} * C_{yx}}{C_{xx} + C_{yy}} \)

A journal bearing system is stable when its mass \( M_j \) is less than the critical mass \( \overline{M}_c \). For any negative value of \( \overline{M}_c \), the journal bearing system will always be stable.
7.2 Whirl frequency ratio

It is the ratio of the rotor whirl or precessional frequency to the rotor onset speed at instability.

\[
\omega = \sqrt{\frac{(K_{xx} - K_{eq})(K_{yy} - K_{eq}) - K_{xy}K_{yx}}{c_{xx}c_{yy} - c_{yx}c_{xy}}}
\]

where, \( K_{eq} = \frac{K_{xx}c_{yy} + c_{xx}K_{yy} - K_{xy}c_{yx} - K_{yx}c_{xy}}{c_{xx} + c_{yy}} \)

\( K_{eq} \leq 0 \) implies absence of whirl.

7.3 Threshold speed

Threshold speed is the utmost value of speed at which the bearing remains stable. It is obtained using the relation given below

\[
\Omega_t = \sqrt{\frac{\bar{F} + \bar{K}_{eq}}{M_c}}
\]

8. SOLUTION PROCEDURE

The analysis is carried out by the linear study of journal bearing system. The studies are carried out for two types of bearing geometries. Hence to the simulation of journal centers motion two programs are developed. The Operating conditions of journal bearing system are varied by the combination of a characteristic length of the micropolar lubricant \( (\bar{l}_m) \), Coupling number \( (N) \), and eccentricity ratio. Hence with the help of these programs, one can obtain results over a wide range. Hence it becomes necessary to execute a program at each operating condition separately. In this paper, all the solutions obtained for eccentricity ratio=0.3 only.

9. RESULTS AND DISCUSSION

The linear stability of a journal depends on the stiffness and damping characteristics. Table 1 shows the comparison of performance characteristics for a Newtonian and micropolar fluid. The results tabularized for \( L/R=2, \ c=0.3, \ T_m=10, \ 40 \) and \( N^2=0.1, \ 0.3, \ 0.5 \). Figure 1-8 shows that as the coupling number \( (N) \) tends to zero or the characteristic length \( (\bar{l}_m) \) of micropolar fluid goes infinitely; the fluid is converted into Newtonian fluid. It is also observed that critical mass \( (M_c) \) and threshold speed \( (\Omega_t) \) increases with increase in coupling number. It shows that for the same eccentricity the micropolar fluid exhibits better stability than a Newtonian fluid. The results evaluated in Table 1 are compared for Newtonian fluid [12] and the results agree very well and satisfactory. Transient responses of both bearings were evaluated for eccentricity = 0.3 and trajectories plotted and compared for linear and nonlinear analysis are presented in Figure 9-10. For Newtonian fluid at \( M_j=M_c \), motion of a journal for linear analysis exhibit limit cycle. However for the same mass, nonlinear analysis shows instability of journal. When \( M_j<M_c \), journal motion achieve limit cycle for nonlinear analysis while journal is stable for linear analysis. It has observed that if the mass of journal is taken below to this limit then the journal shows stability for both the analysis. There are several cases presented for micropolar fluid for, \( \bar{l}_m=10 \) and \( N^2=0.1 \).

10. CONCLUSION

An important conclusion can be made about the use of micropolar fluid in plain circular bearing and two lobe bearing.

a) It has been observed that load carrying capacity of a journal bearing with micropolar lubricant at a particular eccentricity ratio increases when compared with that of bearing with a Newtonian fluid.

b) The micropolar fluid approaches the Newtonian fluid as characteristic length of the micropolar fluid grows indefinitely or coupling number tends to zero.

c) The critical mass and threshold speed for a circular bearing under micropolar fluid is increased for high coupling number. Hence, the stability of a bearing increases for high coupling number.

d) The critical mass and threshold speed for a circular bearing under micropolar fluid decreases when characteristic length decreases.

e) The results show that nonlinear analysis gives better results than linear analysis. Linear analysis is necessary to complete for predicting the value of in nonlinear analysis. It seems that the nonlinear analysis predicts a lower value of critical mass than linear analysis.

f) It is difficult to get the perfectly circular shape of the bearing, so it is better to make an elliptical bearing.
REFERENCES

Table 1 Comparison of performance characteristics at \( \epsilon = 0.3 \) and \( L/R = 2 \)

<table>
<thead>
<tr>
<th>Attribute’s</th>
<th>Plain Circular Bearing</th>
<th>Two Lobe Bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Newtonian Fluid (( \bar{I}_m = 10 ))</td>
<td>Micropolar Fluid</td>
</tr>
<tr>
<td>Coupling Number (( N^2 ))</td>
<td>( \phi )</td>
<td></td>
</tr>
<tr>
<td>Attitude angle (( \phi ))</td>
<td>68.359</td>
<td>68.281</td>
</tr>
<tr>
<td>Load (( \bar{F} ))</td>
<td>1.6357</td>
<td>1.7914</td>
</tr>
<tr>
<td>( K_{xx} )</td>
<td>3.2237</td>
<td>3.5662</td>
</tr>
<tr>
<td>( K_{xy} )</td>
<td>3.8067</td>
<td>4.2355</td>
</tr>
<tr>
<td>( K_{yy} )</td>
<td>2.2251</td>
<td>2.417</td>
</tr>
<tr>
<td>( C_{xx} )</td>
<td>8.7093</td>
<td>9.6921</td>
</tr>
<tr>
<td>( C_{xy} \approx C_{yx} )</td>
<td>-3.4455</td>
<td>-3.847</td>
</tr>
<tr>
<td>( C_{yy} )</td>
<td>-3.5238</td>
<td>-3.9698</td>
</tr>
<tr>
<td>Critical mass (( \bar{M}_c ))</td>
<td>13.13</td>
<td>14.438</td>
</tr>
<tr>
<td>Threshold speed (( \bar{\Omega}_c ))</td>
<td>9.6927</td>
<td>10.695</td>
</tr>
<tr>
<td>Whirl frequency ratio (( \bar{\Omega} ))</td>
<td>3.1133</td>
<td>3.2703</td>
</tr>
</tbody>
</table>
Non-dimensional characteristic length ($l_m$)

Bearing force ($F$)

- $N^2 = 0.1$
- $N^2 = 0.3$
- $N^2 = 0.5$
- $N^2 = 0.7$
- $N^2 = 0.9$
- Newtonian

---

**Figure 1** Variation of Bearing force ($F$) as a function of non-dimensional Characteristic length ($l_m$) for Plain circular Bearing.

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**Figure 2** Variation of Bearing force ($F$) as a function of non-dimensional Characteristic length ($l_m$) for Two lobe Bearing.

---

Critical Mass ($M_c$)

- $N^2 = 0.1$
- $N^2 = 0.3$
- $N^2 = 0.5$
- $N^2 = 0.7$
- $N^2 = 0.9$
- Newtonian

---

**Figure 3** Variation of Critical Mass ($M_c$) as a function of non-dimensional Characteristic length ($l_m$) for Plain circular Bearing.

---

**Figure 4** Variation of Critical Mass ($M_c$) as a function of non-dimensional Characteristic length ($l_m$) for Two lobe Bearing.

---

Whirl frequency ratio ($\omega$)

- $N^2 = 0.1$
- $N^2 = 0.3$
- $N^2 = 0.5$
- $N^2 = 0.7$
- $N^2 = 0.9$
- Newtonian

---

**Figure 5** Variation of Whirl frequency ratio ($\omega$) as a function of non-dimensional Characteristic length ($l_m$) for Plain circular Bearing.

---

**Figure 6** Variation of Whirl frequency ratio ($\omega$) as a function of non-dimensional Characteristic ($l_m$) for Two lobe Bearing.
Figure 7 Variation of Threshold Speed (Ω) as a function of non-dimensional Characteristic length (l_m) for Plain circular Bearing

Figure 8 Variation of Threshold Speed (Ω) as a function of non-dimensional Characteristic length (l_m) for Two lobe Bearing
Figure 9 Transient analysis of Plain circular bearing

Figure 9 Linear and nonlinear trajectories for a plain circular bearing with Newtonian fluid for eccentricity ratio=0.3.

Figure 9 Linear and nonlinear trajectories for a plain circular bearing with micropolar fluid for eccentricity ratio=0.3, $I_m=10$ and $N^2=0.1$. 
Figure 10 Transient analysis of Two lobe bearing

a) $\bar{M}_f = 63.018$

b) $\bar{M}_f = 57.662$

c) $\bar{M}_f = 50.742$

Figure 10 Linear and nonlinear trajectories for an elliptical bearing with Newtonian fluid for eccentricity ratio=0.3 and ellipticity=0.5.

Figure 10 Linear and nonlinear trajectories for an elliptical bearing with micropolar fluid for eccentricity ratio=0.3, ellipticity=0.5, $\bar{I}_m = 10$ and $N^2 = 0.1$. 