Dynamic Analysis and Experimental Investigation for Vibration Response of Suspension System of Indian Railway Vehicle

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ABSTRACT: A vibration analysis of suspension of rail vehicle has to be performed which reflects both the varying load and response. A rail vehicle experiences lateral and vertical effect in very high response due to track imperfection, tractive effort, breaking effort, curving and tracking. The differential equation for vibration analysis gives the time dependent response in the superstructure by including damping forces and inertial forces in the equation and each node is subjected to a sinusoidal function of the peak amplitude for that node. This paper represents the vibration response of rail vehicle having primary and secondary suspension with vertical, lateral and longitudinal damping. At first, the complex structure of dynamics of rail vehicle is to simplify in two mass damper suspension models to solve complex differential equation to find its natural frequency and amplitude. The aim is to check dynamic sinusoidal variations for the failure response of primary suspension spring by finding the natural frequency of model and excitation frequency of rail. The vibration response of two mass suspension systems with base excitation is obtained analytically using differential equation which gives peak amplitude by steady state solution. The mathematical model for increasing complexity i.e. for lateral and roll dynamics has also been developed and vibration analyzer is used to find dynamic behavior of rail vehicle in running condition at an average speed of 80-100 km/hr for number of instances during smooth track, curving and tracking.

KEYWORDS: Spring mass damper system, vibration analyzer, rail vehicle, dynamic analysis, modal & harmonic analysis.

I. INTRODUCTION

Mechanical vibration is the study of oscillatory motions of a dynamic system which is a repeated motion with equal interval of time. Free vibrations are oscillations about a system equilibrium position that occurs in the absence of an external excitation force [1]. A rail vehicle is the dynamic multibody system which has the repeated oscillation of its suspension system due to unevenness of track condition or due to fluctuations of load. The dynamic behavior of a railroad vehicle also depends on the load and the mechanical systems, such as springs, dampers, etc., which interact with the wheels, the vehicle body and bogies. A Rail vehicle has primary and secondary suspension system mounted between bogie and wheelbase. The wheelbase has the primary suspension spring with inclined damper at end axle spring and concentric inner and outer spring assembly at middle axle. The body has secondary suspension system mounted between frame and bogie with vertical, lateral and longitudinal damping. Figure 1 shows the primary and secondary suspension system of dynamic multibody railroad vehicle.
The dynamics of the railway vehicle represent a balance between the forces acting between the wheel and the rail, the inertia forces and the forces exerted by the suspension and articulation. In a complete model of the dynamics of a railway vehicle, the vehicle is considered to be assembled from wheel sets, car bodies and intermediate structures which are flexible, and which are connected by components such as springs and dampers. Similarly, the vehicle is considered to run on a track which has a complex structure with elastic and dissipative properties. An objective of the study of railway vehicle dynamics is to develop analytical or numerical models describing the mechanics of various phenomena by the simplest model possible. These can be used to explore suspension and vehicle concepts and to develop a basis for physical understanding and insight [1, 2].

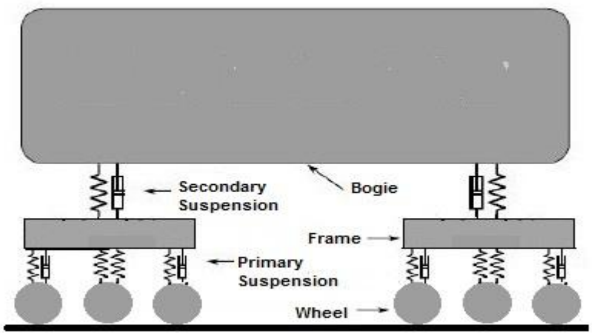


Figure 1: Rail vehicle three wheel engine with primary and secondary suspension system

Primary and secondary suspension is responsible for performance of isolation and absorption of shock loads and vibration and smooth control over movement between the car body and bogie. Springs permit free movement in all directions but lateral buffers and dampers restrict the amount and rate of lateral movement. Vertical dampers and rebound limit chains restrict the amount and rate of the vertical rebound of the locomotive car body. Pitch rate of car body is controlled by yaw (longitudinal) dampers.

The primary suspension, located between the axles and the bogie frame and vertical hydraulic dampers are used to dampen the rebound rate of the springs. This “Flexi coil” arrangement permits lateral movement of the axle. Longitudinal control of the axle, and the transmission of tractive and braking effort to the bogie frame, is provided by guide rods connected between the axle journal boxes and bogie frame. Secondary suspension, located between the bogie frame and locomotive under frame is also provided by coil springs and vertical hydraulic dampers, on each side of the bogie. The weight of the locomotive car body is carried by the secondary suspension springs. The “Flexi-Float” arrangement of the secondary suspension allows the locomotive car body to move both laterally and vertically within certain limits relative to the bogies [1,2].

Equations of motion governing the stability and dynamic response of vehicles will now be derived which encompass the essential features of the wheel-rail geometry, the frictional forces acting between wheel and rail and the elastic and damping forces generated by the suspension. For this analysis the stiffness and damping coefficient of primary and secondary suspension system are provided in table 1.

Table 1: Stiffness and damping coefficient of Primary and secondary suspension system

Spring	Stiffness (N/m) $\times 10^3$	Damper	Damping Coefficient (N-s/m)
Primary Middle Axle outer spring	470	Yaw Damper	30,000
Primary Middle Axle inner spring	144	Horizontal Damper	70,000
Primary End Axle spring	868	Inclined Axle Damper	50,000
Secondary Suspension Spring	612	Vertical Damper	1,10,000

II. MATHEMATICAL MODELING FOR VERTICAL OSCILLATION OF RAIL VEHICLE

Damping control in the primary suspension is applied to the vertical axle-box dampers to suppress the vertical vibrations of the bogies, and hence to reduce the first vertical bending mode of the car body. Furthermore, damping in the secondary suspension is applied to the spring to suppress the rigid vibration modes bounce and pitch [6]. The secondary suspension limits the relative vertical displacements between car body and bogie frame, with the purpose of isolating the car body from excitation transmitted from track irregularities via the wheel sets and bogie frames. The forces on the wheelset arise from creepages between rail and wheel, small relative velocities which arise because of elastic deformation of the steel at the point of contact and which apply in both the longitudinal and the lateral directions [7].

Railway vehicles are dynamically-complex multi-body systems. Each mass within the system has six dynamic degrees of freedom corresponding to three displacements (longitudinal, lateral and vertical) and three rotations (roll, pitch and yaw). The simplified and complex version is specified by design model and simulation for applying control to complex systems. The design model is a simplified version used for synthesis of the control strategy and algorithm, whereas the simulation model is a more complex version used to test fully the system performance [5]. This paper has more influence over the oscillation of suspension system in vertical direction i.e. simplified design model which is responsible for the displacement of rigid frame and vehicle body. In dynamic condition, the displacement of vehicle body occurs by base excitation due the irregularities in rail and relative displacements between the wheels and the rails which are small in the sub-critical range of velocities and, hence, the influence of the contact non-linearities can be neglected and, consequently, a linear model is recommended for studying the vertical vibrations of the vehicle. A rail vehicle has primary and secondary suspension having dashpots in vertical direction with primary spring and vertical, lateral and longitudinal direction with secondary spring. Hence to examine more relative displacement between primary and secondary suspension, a model is formulated for two degrees of freedom system with base excitation.

Designing a rail vehicle suspension system is an interesting and challenging control problem. When the suspension system is designed, a 1/4 model is used to simplify the problem to multiple spring-damper system. This model is for passive suspension system with base excitation of the rail carbody [8]. Figure 2 illustrates 1/4th model of vehicle body shows suspension with primary and secondary spring stiffness and dashpots attached with them and also illustrates the free body diagram of spring force and viscous force. Since the behavior of springs is considered to be linear for solution of harmonic excitation of system, but governing equation has been prepared for linear and nonlinear behavior of spring stiffness i.e. for k_1 and k_2 and displacement and force transmissibility is obtained by linear 2DOF base excited system.

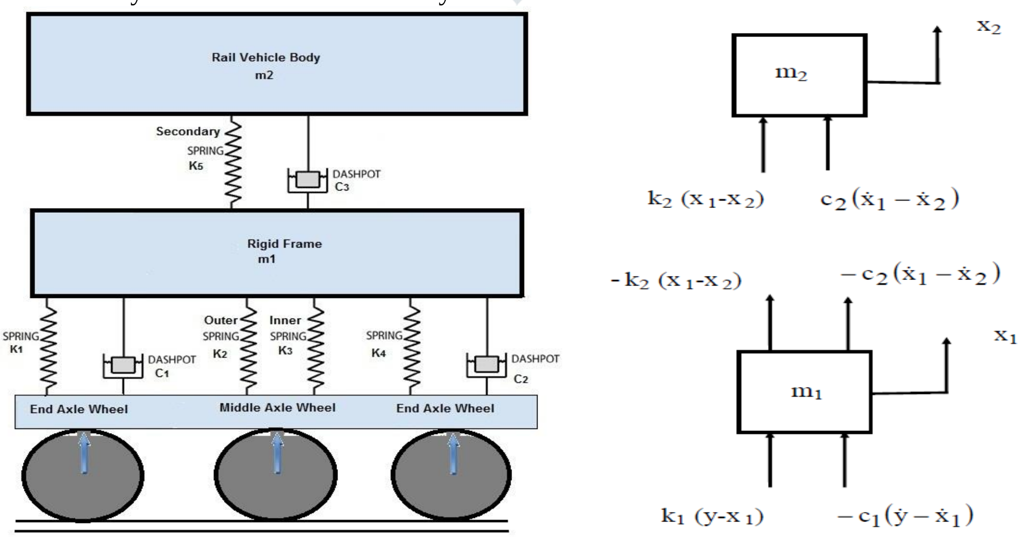


Figure2: 1/4th model of rail vehicle with its free body diagram

a. Equation of motion for linear stiffness

A rail passive vibration isolator 2DOF base excited system has linear stiffness and damper. The equation of motion is define by relative displacement terms as,

$$u_2 = x_2 - y \qquad u_1 = x_1 - y$$

$$x_2 = u_2 + y \qquad x_1 = u_1 + y$$

Note that, $x_1 - x_2 = u_1 - u_2$

Differential equation for primary and secondary suspension system i.e. for two DOF of rail vehicle is obtained by using Newton's second law of motion as,

Determine the equation of motion for mass 1.

$$\sum F = m_1\ddot{x}_1$$

$$m_1\ddot{x}_1 = -c_2(\dot{x}_1 - \dot{x}_2) + c_1(\dot{y} - \dot{x}_1) - k_2(x_1 - x_2) + k_1(y - x_1) \tag{1}$$

Substitute relative displacement terms in equation (1)

$$m_1\ddot{u} + m_1\ddot{y} = -c_2(\dot{u}_1 - \dot{u}_2) - c_1\dot{u}_1 - k_2(u_1 - u_2) - k_1u_1$$

$$m_1\ddot{u} + (c_1 + c_2)\dot{u}_1 - c_2\dot{u}_2 + (k_1 + k_2)u_1 - k_2u_2 = -m_1\ddot{y} \tag{2}$$

Determine the equation of motion for mass 2.

$$\sum F = m_2\ddot{x}_2$$

$$m_2\ddot{x}_2 = c_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2\ ) \tag{3}$$

Substitute relative displacement terms in equation (3)

$$m_2\ddot{u}_2 + m_2\ddot{y} = c_2(\dot{u}_1 - \dot{u}_2) + k_2(u_1 - u_2)$$

$$m_2\ddot{u}_2 + c_2(-\dot{u}_1 + \dot{u}_2) + k_2(-u_1 + u_2) = -m_2\ddot{y}$$

$$m_2\ddot{u}_2 + c_2\dot{u}_2 - c_2\dot{u}_1 + k_2u_2 - k_2u_1 = -m_2\ddot{y} \tag{4}$$

The equation of motion 1 and 2 can be written in matrix form,

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} -m_1\ddot{y}_1 \\ -m_2\ddot{y}_2 \end{Bmatrix} \tag{6}$$

The above matrix represented in symbolic matrix notation,

$$\mathbf{M\ddot{x}+C\dot{x}+Kx=P_{eff}\ (t)}$$

Where, $P_{eff}(t)$ is the effective force vector related to base acceleration by equation.

$$P_{eff}(t) = \begin{Bmatrix} -m_1\ddot{y}_1 \\ -m_2\ddot{y}_2 \end{Bmatrix}$$

To find the natural frequency of system consider undamped condition. Homogeneous form of equation,

$$M\ddot{u} + Ku = F \tag{7}$$

Solution of the form with q vector is the generalized coordinate vector,

Displacement, $u = qe^{(i\omega t)}$

Velocity, $\dot{u} = i\omega qe^{(i\omega t)}$

Acceleration, $\ddot{u} = -\omega^2 qe^{(i\omega t)}$

Substitute these equations in to equation (7)

$$-\omega^2 Mqe^{(i\omega t)} + Kqe^{(i\omega t)} = 0$$

$$(-\omega^2 M + K)qe^{(i\omega t)} = 0$$

The eigenvalues can be found by setting the determinant equal to zero.

$$\det\{K - \omega^2 M\} = 0$$

$$\det\left\{\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} - \omega^2\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\right\} = 0$$

$$\left[(k_1 + k_2) - \omega^2 m_1\right]\left[k_2 - \omega^2 m_2\right] - k_2 = 0$$

$$-\omega^4 m_1 m_2 + \omega^2 \left[-m_2(k_1+k_2) - m_1 k_2 \right] + k_1 k_2 = 0 \tag{8}$$

The Eigen values are the roots of the polynomial.

$$\omega_1^{\,2} = \frac{-b-\sqrt{b^2-4ac}}{2a} \quad \& \quad \omega_2^{\,2} = \frac{-b+\sqrt{b^2-4ac}}{2a} \tag{9}$$

Where,

$$a = m_1 m_2$$

$$b = [-m_2(k_1+k_2) - m_1 k_2]$$

$$c = k_1 k_2$$

The natural frequencies obtained using above equations is,

$$\omega_{n1} = 7.92 rad/\sec = 1.26 Hz$$

$$\omega_{n1} = 24.72 rad/\sec = 3.934 Hz$$

Mode shape for natural frequency ω_{n1},

$$\left(\frac{q_1}{q_2}\right)_1 = \frac{k_2}{(k_1+k_2) - m_1\omega_1^{\,2}} = 0.231$$

Mode shape for natural frequency ω_{n2},

$$\left(\frac{q_1}{q_2}\right)_2 = \frac{k_2 - m_2\omega_2^{\,2}}{k_2} = -6.49$$

Positive value of first mode shape means both masses move simultaneously up and down. Negative value of second mode shape means two motions are out of phase i.e. when one mass moves up, the other moves down or vice-versa.
It can be found out that rail vehicle excitation using steady state solution for base excitation which is sum of particular integral and complementary function.

x(t) = Particular integral + complementary function (10)

Note that for initial conditions, it is needed to find the complimentary solution and weight, the sum of the complimentary and particular solutions such that the initial conditions are satisfied. However, due to the damping in this system, the complimentary solution would die away exponentially and after a period of time only the particular solution (i.e. steady state solution) would remain with amplitude of U, forcing or excitation frequency of ω and phase angle of ϕ.

$$x_p(t) = x(t) = X\sin(\omega t - \phi) \tag{11}$$

Amplitude or displacement ratio and phase angle is obtained as follows:

$$\frac{X}{Y} = \frac{\sqrt{1+(2\xi\,\omega/\omega_n)^2}}{\sqrt{(1-\omega^2/\omega_n^{\,2})^2 + (2\xi\,\omega/\omega_n)^2}} \quad \& \quad \phi = \tan^{-1}\left(\frac{-2\xi\,\omega^3/\omega_n^{\,3}}{1-(1-4\xi^2)\,\omega^2/\omega_n^{\,2}}\right) \tag{12}$$

b. Equation of motion for nonlinear stiffness

The governing equation has also been prepared if passive vibration isolator is modeled as a parallel combination of a stiffness and damper with cubic nonlinearity for 2DOF system [16,18]. Now consider an isolator is modeled by linear damper c_1 and c_2 and nonlinear spring with stiffness k_1 and k_2.
Consider two degree-of-freedom system whose base is subjected to a known displacement y(t) as shown in Figure 2. The spring is nonlinear with a force-displacement relationship, that is,

$$F(x) = kx + \alpha k x^3 \tag{13}$$

From Fig. 2, the governing equation of the isolator under force excitation can be given as,

$$m_1\ddot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) - c_1(\dot{y} - \dot{x}_1) + k_2(x_1 - \delta_2 - x_2) - k_1(y - \delta_1 - x_1) + \alpha_2 k_2(x_1 - \delta_2 - x_2)^3$$

$$-\alpha_1 k_1 (y - \delta_1 - x_1)^3 = -m_1 g$$

$$m_2\ddot{x}_2 - c_2(\dot{x}_1 - \dot{x}_2) - k_2(x_1 - \delta_2 - x_2) + \alpha_2 k_2 (x_1 - \delta_2 - x_2)^3 = -m_2 g \tag{14}$$

Where x_1 and x_2 is measured from the static equilibrium position of mass m_1 and m_2, y is the absolute displacement of the base, and δ_1 and δ_2 is the static deflection. From the static equilibrium of the system,

$$k\delta + k\alpha\delta^3 = mg$$

Substitute above equation in equation 14,

$$m_1\ddot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) - c_1(\dot{y} - \dot{x}_1) + k_2(x_1 - x_2) - k_1(y - x_1) + \alpha_2 k_2 (x_1 - \delta_2 - x_2)^3$$

$$-\alpha_1 k_1 (y - \delta_1 - x_1)^3 = -k_1 \alpha_1 \delta_1^3$$

$$m_2\ddot{x}_2 - c_2(\dot{x}_1 - \dot{x}_2) - k_2(x_1 - x_2) + \alpha_2 k_2(x_1 - \delta_2 - x_2)^3 = -k_2\alpha_2\delta_2^{\,3} \qquad (15)$$

The isolator model for relative displacement under base excitation can be written as,

$$m_1\ddot{u} + (c_1 + c_2)\dot{u}_1 - c_2\dot{u}_2 + (k_1 + k_2)u_1 - k_2u_2 + (u_1 - \delta_2 - u_2)^3\alpha_2 k_2 - (\delta_1 + u_1)^3\alpha_1 k_1 = -k_1\alpha_1\delta_1^{\,3} - m_1\ddot{y}$$

$$m_2\ddot{u}_2 + c_2\dot{u}_2 - c_2\dot{u}_1 + k_2u_2 - k_2u_1 + k_2\alpha_2(u_1 - \delta_2 - u_2)^3 = -k_2\alpha_2\delta_2^{\,3} - m_2\ddot{y} \qquad (16)$$

Considering, $\tau = \omega_n t$

$$\frac{d^2u_1}{d\tau_1} + 2(\xi_1 + \xi_2)\frac{du_1}{d\tau_1} - 2\xi_2\frac{du_2}{d\tau_1} + u_1 - u_2 + (u_1 - \delta_2 - u_2)^3\alpha_2 - (\delta_1 + u_1)^3\alpha_1 = -\alpha_1\delta_1^{\,3} - \frac{d^2y}{d\tau_1}$$

$$\frac{d^2u_2}{d\tau_2} - 2\xi_2\frac{du_2}{d\tau_2} - 2\xi_1\frac{du_1}{d\tau_1} + u_2 - u_1 + (u_1 - \delta_2 - u_2)^3\alpha_2 = -\alpha_2\delta_2^{\,3} - \frac{d^2y}{d\tau_2} \qquad (17)$$

For the displacement to the base of system, assume a step like disturbance that has variable rise time and a rounded shape given by,

$$y(t) = Y[1 - (1 + \gamma\tau)e^{-\gamma\tau}] \qquad (18)$$

This waveform was selected instead of a unit step function, in part, because its higher order derivatives are continuous. The normalized y(t)/Y is shown in figure 3 for several values of parameter γ.

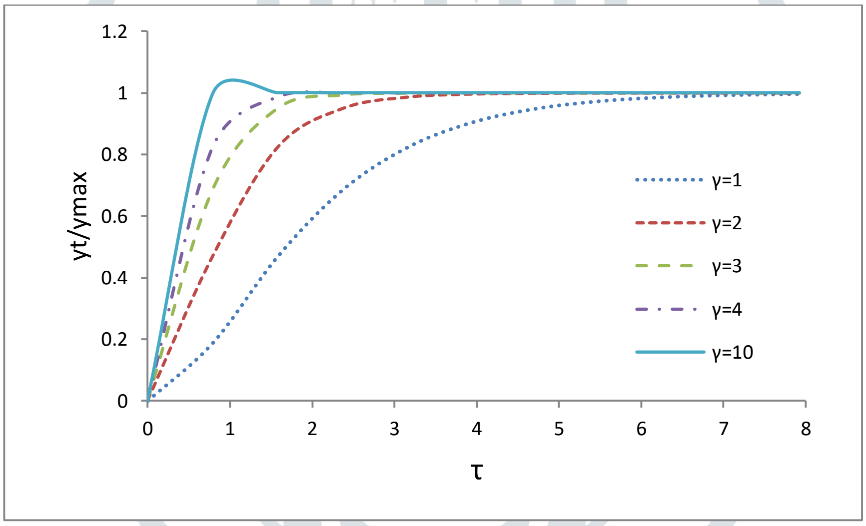


Figure 3: Waveform of base excitation for several values of γ.

Taking the second derivative of Eq. (18) with respect to τ,

$$\ddot{y}(t) = Y\gamma^2(1 - \gamma\tau)e^{-\gamma\tau} \qquad (19)$$

Substitute Eq. (19) into Eq. (17) and introduce the non-dimensional variable $u_n(\tau)=u(\tau)/Y$ to obtain,

$$\frac{d^2u_{n1}}{d\tau_{n1}} + 2(\xi_1 + \xi_2)\frac{du_{n1}}{d\tau_{n1}} - 2\xi_2\frac{du_{n2}}{d\tau_{n1}} + u_{n1} - u_{n2} + (u_{n1} - \delta_{o2} - u_{n2})^3\alpha_{o2} - (\delta_{o1} + u_{n1})^3\alpha_{o1} = -\alpha_{o1}\delta_{o1}^{\,3} - g(\tau_1)$$

$$\frac{d^2u_{n2}}{d\tau_2} - 2\xi_2\frac{du_{n2}}{d\tau_2} - 2\xi_1\frac{du_{n1}}{d\tau_1} + u_{n2} - u_{n1} + (u_{n1} - \delta_{o2} - u_{n2})^3\alpha_{o2} = -\alpha_{o2}\delta_{o2}^{\,3} - g(\tau_2) \qquad (20)$$

Where, $\delta_o=\delta/Y$, $\alpha_o= \alpha Y^2$ and

$$g(\tau) = -\gamma^2(1 - \gamma\tau)e^{-\gamma\tau}$$

The absolute displacement $x(\tau)$ is obtained from relative displacement terms and Eq. (18); that is,

$$x(t) = Yu_n(\tau) + Y[1 - (1 + \gamma\tau)e^{-\gamma\tau}] \qquad (21)$$

c. Solution for Harmonic Excitation

In nonlinear dynamics it is difficult to extract the qualitative essence from simulations alone and also is difficult to solve an algebraic function in nonlinear cubic form to obtain displacement and force transmissibility. A. Carrella[17], explains the response of harmonic function for nonlinear low dynamic stiffness and compared it with linear stiffness system in the form of transmissibility with respect to non dimensional component. Here, it is considered that, a passive suspension system of rail vehicle has linear stiffness and dampers.

Over a sinusoidal rail irregularity a wheel running with speed v (m/s) and wavelength L (m) will perceive an excitation frequency ω (in Hz). The excitation frequencies vary in the frequency interval is obtained with vehicle speed v=5 to 50m/s and with irregularity wavelengths L= 1m to 20m. A lower frequency is occurred over longer wavelength. The excitation will then induce vibrations and noise in the train and in the track structure and the environment. Resonances in the train-track system will also amplify these vibrations at the resonance frequencies [12]. The wavelength of track can be specified in curvature but it cannot be predicted in straight track as it has smooth surface over very long distance. The overall length of railway engine is 20m running with maximum speed of 100 km/hr. So by considering wavelength of 8m and average train velocity of 80km/hr,
The wheel oscillates vertically with harmonic motion, at frequency ω,

$$y(t) = Y\sin(\omega t)$$

Forcing frequency, $\omega = \frac{2\pi v}{L} = 17.44 rad/\sec = 2.77 Hz$

Damping factor for the dampers provided at primary and secondary suspension as ξ_1 and ξ_2 are,

$$\xi_1 = \frac{c_1}{2\sqrt{k_1 m_1}} = 0.2306 \qquad \& \qquad \xi_2 = \frac{c_2}{2\sqrt{k_2 m_2}} = 0.406$$

To obtain excitation of system, the first natural frequency is considered to find frequency ratio.

Frequency ratio, $\frac{\omega}{\omega_n} = 2.20$

From equation (12), for damping factor ξ_1=0.2306;

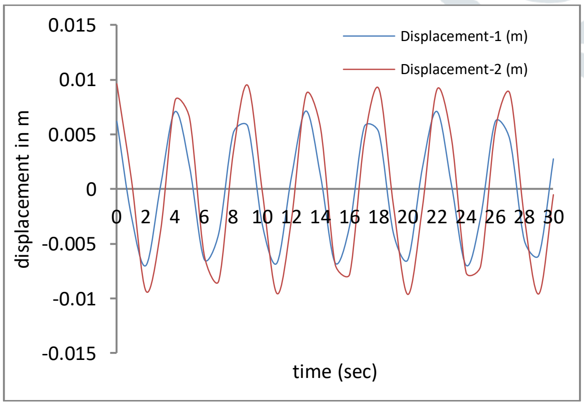
Amplitude ratio, $\frac{X_1}{Y} = 0.357$ and phase angle, $\phi = 1.050 radian$

And for damping factor ξ_2=0.406;

Amplitude ratio, $\frac{X_2}{Y} = 0.483$ and phase angle, $\phi = 1.50 radian$

The peak amplitude for amplitude ratios will be obtained by considering height of track imperfection. If there is track imperfection or mean height of Y= 2cm = 0.02 m, X_1= 0.00715 m and X_2= 0.00966 m

The relative and absolute displacement for harmonic base excitation function is obtained by the equation having particular integral and complementary function. The complementary function has harmonic force subjected to more oscillation during dynamic condition. The derivative of displacement provides velocity and double derivative provides accelerations of vehicle body. For the value of masses i.e. m_1=10,000 kg and m_2=15,000kg, the absolute displacement and acceleration of rail vehicle for base excitation in vertical direction is obtained from equation (11) is shown in figure 4 and its amplitude obtained from equation (12) is shown in figure 5.



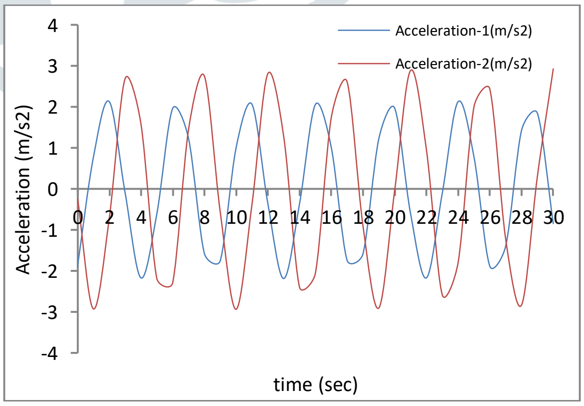
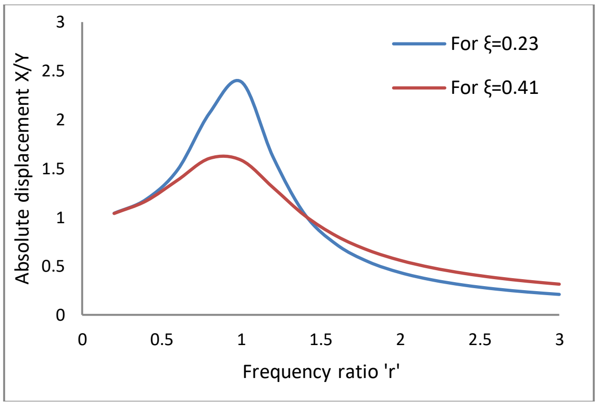
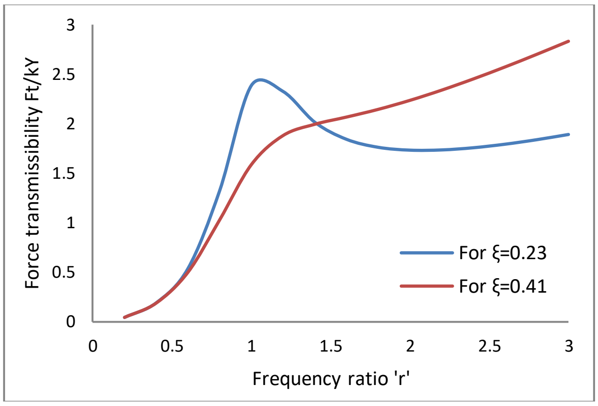


Figure4: Displacement and acceleration of rail vehicle for harmonic base excitation



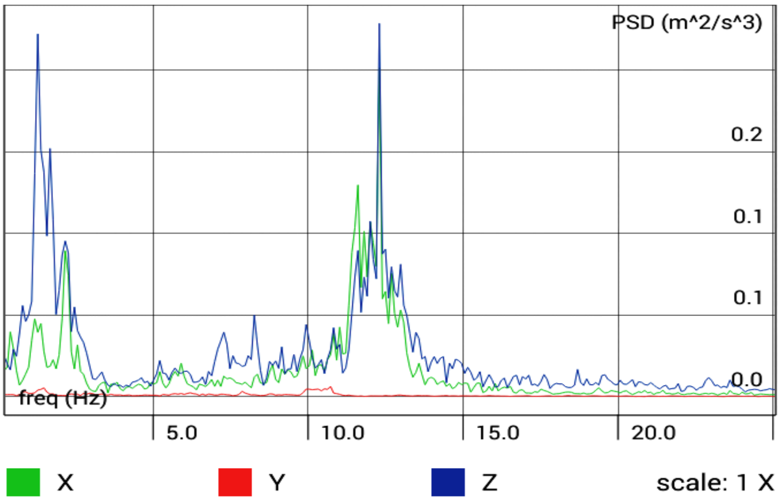
(a)



(b)

Figure 5: frequency response for absolute displacement of base excited 2DOF (a) linear system and (b) nonlinear system

The amplitude of vibration will maximum for frequency ratio is equal to one and it decreases for increasing ratio. The resonance will occurs if excitation frequency is equal to natural frequency of system which results in increase of vibration and may cause failures. The above analysis indicates that the frequency ratio is more than one for rail velocity of 80 km/hr and wavelength of 8m. But for the constant speed of 55km/hr and wavelength of 12m, the frequency ratio is obtained as one i.e. $\omega/\omega_n=1$. The experimental amplitude of power spectral density (PSD) rail vehicle computed from the FFT spectrum is shown in figure 6. The PSD provides a useful way to characterize the amplitude versus frequency content of a random signal. The sinusoidal time domain is converted to frequency response which shows two amplitudes at 1.3Hz and 12Hz respectively.



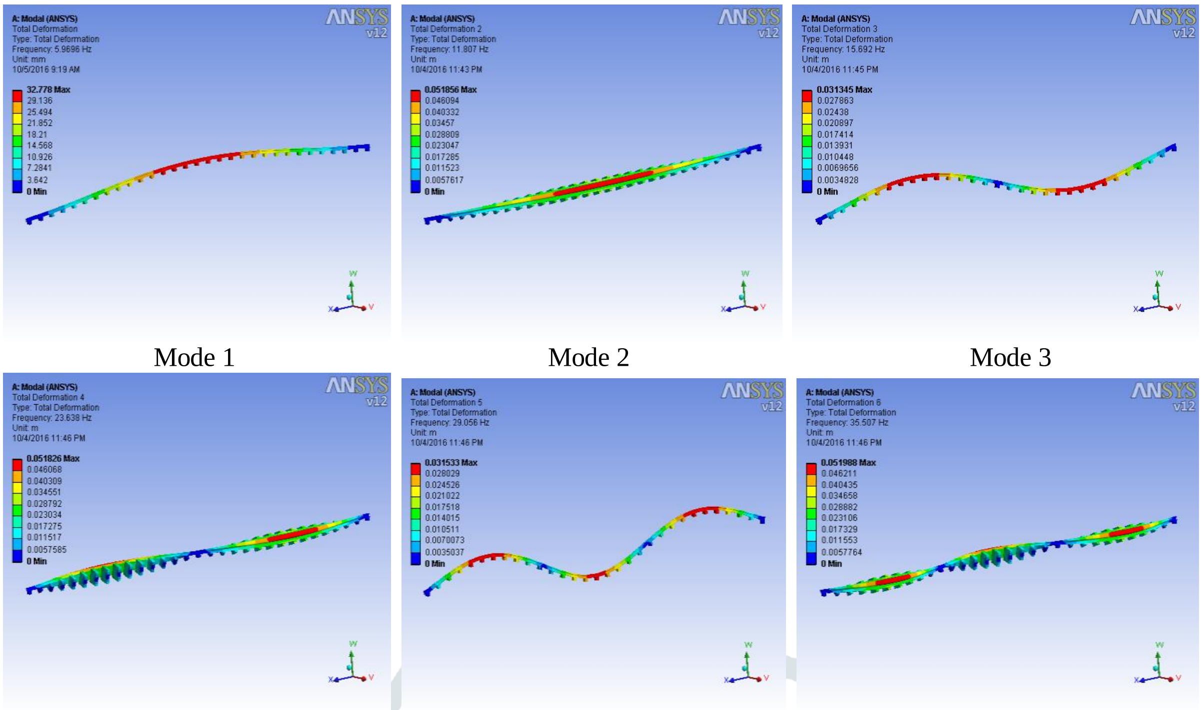
(X- Lateral, Y- Longitudinal, Z- Vertical)

Figure 6: Power spectral density (PSD) for rail vehicle computed from the FFT spectrum

III. VIBRATION USING FINITE ELEMENT ANALYSIS

The dynamics of rail vehicle has the frequent failure of primary inner suspension spring which calls for the investigation. In the above section, the natural frequencies of 2DOF system are calculated analytically and correlated with experimental results. The vehicle-track interaction excites vibration from base to vehicle body may cause failure of inner suspension if its natural frequency match with excitation frequency during running condition. The excitation frequency is based on rail vehicle velocity and track wavelength which will vary with change in velocity and wavelength. Due to the physical nature of track structures, a chain of their structural vibration modes exists depending on the resonant frequencies on the vertical, lateral and longitudinal direction. The lowest possible vertical resonant frequency of the track structure is the full track resonance, which includes both in-phase and out-of-phase vibrations.

Wheel and rail are coupled together at two contact points, where track irregularities are introduced as a dynamic excitation. Vibration transmitted up to the carbody can be suppressed by focusing on reduction of bogie vibrations. A higher primary damping coefficient reduces the vibrations of the bogie so that a smaller amount of vibrations is transmitted to the carbody [5]. For modal analysis of broad gauge rail, the length of rail taken into account in the model is equal to the length of railway engine i.e. 20m to find its natural frequencies and mode shapes. Modal analysis defines the behavior of rail without applying an external force but providing proper constraints [9,14]. A single broad gauge railway track is analyzed for modal analysis in finite element analysis tool, ANSYS where the geometry is mesh with solid 187 10-node tetrahedral element with fine meshing. For analysis purpose, continuous block are modeled below track as sleeper in the distance of 510mm to provide proper constraining. In boundary condition, end two sleepers are fixed in all degree of freedom (i.e. x,y,z) and all middle sleepers are allowing for vertical deformation only (i.e. fixed at x,z). The excited model of single railway track using finite element analysis is shown in figure 7 and their natural frequency with period of system is shown in table 2.



Mode 1 Mode 2 Mode 3

Mode 4 Mode 5 Mode 6

Figure 7: Modal analysis of rail at 6 different mode shapes using finite element analysis

Table 2: Natural Frequency of railway track for 6 different modes using finite element analysis

Mode	Frequency [Hz]	Period (sec)
1.	5.9696	0.1675
2.	11.807	0.0846
3.	15.692	0.0637
4.	23.638	0.0423
5.	29.056	0.0344
6.	35.507	0.0281

Also a rail vehicle is considered for modal analysis to check its natural frequency for vertical, lateral and longitudinal vibration. A rail vehicle model is prepared in ANSYS with contact elements as spring and damper as per the stiffness and damping coefficient respectively as shown in table 1. A mesh model with boundary condition is shown in figure 8 which shows primary and secondary suspension system with vertical, lateral and longitudinal damper. The natural frequencies of suspension system in two modes is found out using modal analysis for all direction i.e. vertical vibration, lateral vibration (roll), longitudinal vibration (Pitch)is shown in figure 9, figure 10 and figure 11 respectively and results are shown in table 3.

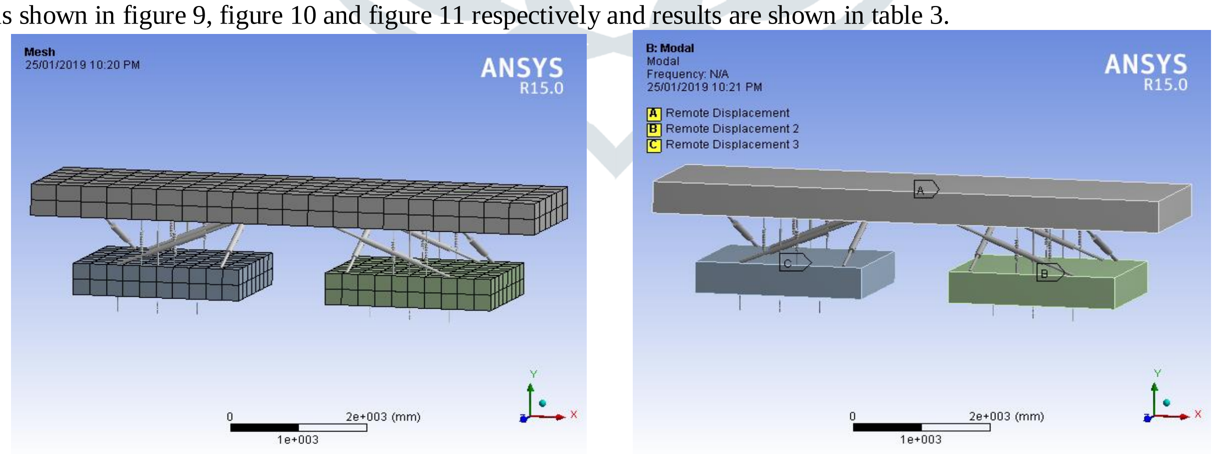


Figure 8: Mesh model of rail vehicle suspension system with boundary condition

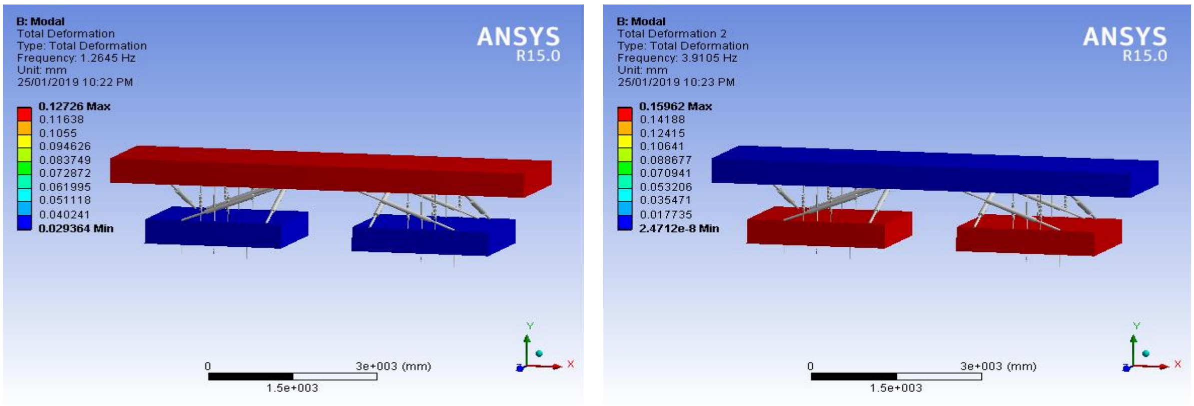


Figure 9: Natural frequencies of rail vehicle suspension system in vertical direction

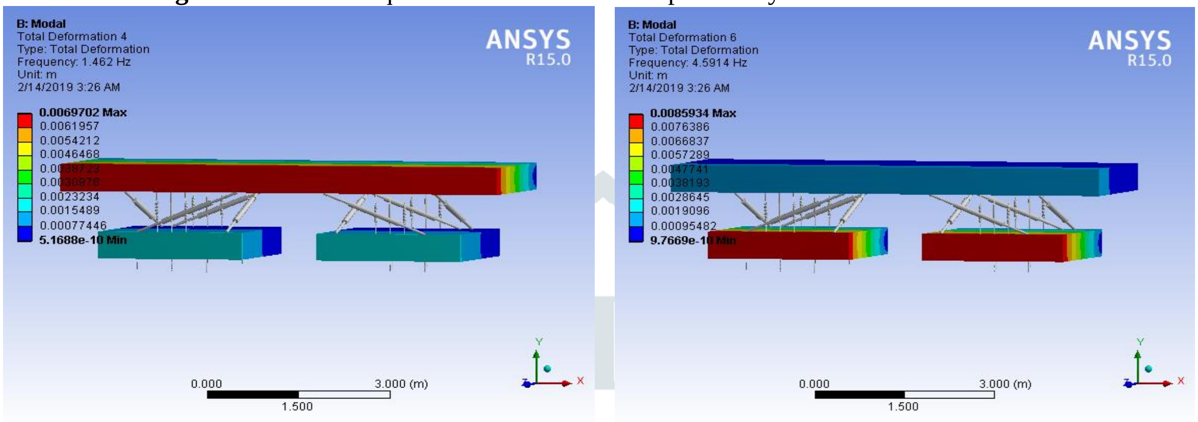


Figure 10: Natural frequencies of rail vehicle suspension system in Lateral vibration (Roll)

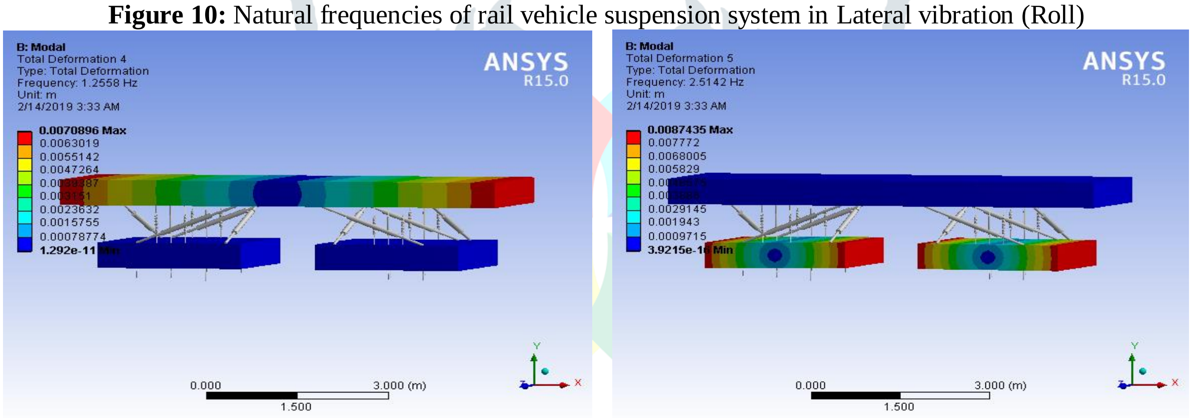


Figure 11: Natural frequencies of rail vehicle suspension system in direction of Longitudinal vibration (Pitch)

Table 3: Natural Frequency of rail vehicle suspension system for 2 different modes using finite element analysis

Mode	**Natural Frequency [Hz]**		
	Vertical Vibration	Lateral vibration (Roll)	Longitudinal vibration (Pitch)
1.	1.26	1.46	1.25
2.	3.91	4.59	2.51

The excitations from track forwarded towards rail body through suspension system i.e. the suspensions possesses harmonic excitations from wheel set. A rail vehicle has total weight of 123 tonne acting over track excite back force over each wheel contains primary suspension. Middle axle in frontal and rear side has composite assemble of inner and outer spring without damping experiences more excitation. The total weight of rail body has been resolved and finds the forces exerted on each suspension spring. This harmonic force has been considered to act over individual rail vehicle suspension system to obtain maximum amplitude of vibration with respect to excitation frequency using finite element analysis. The amplitudes of displacement and acceleration obtained having maximum amplitude at 1.5Hz and 4Hz is shown in figure 12.

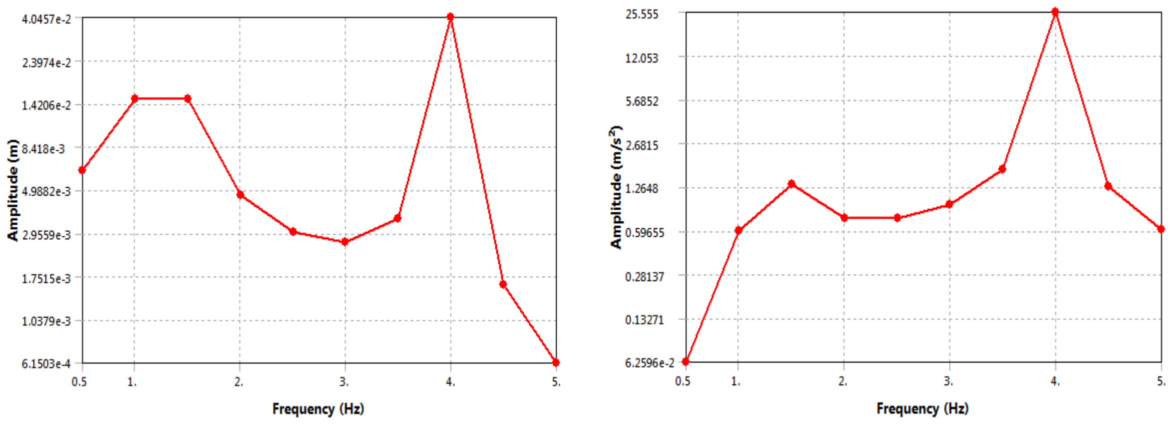


Figure 12: Amplitude of displacement and acceleration

IV. MATHEMATICAL MODELING FOR RAIL VEHICLE ON CURVED TRACK

The modeling of the baseline railway vehicle is based upon a linear end view model version. It includes the lateral and roll dynamics of the body and bogie plus a state from the suspension spring dynamics. The mechanical model for the study of the lateral vibrations in a railway vehicle, excited by the tangent track geometric deviations, is presented in figure 13. The curving acceleration rises at higher speeds, and because of the duration of the transition will reduce, the transition will also be more severe, this is due to change in track. This is where curvature arises; to bring the acceleration back to the level. However, it is not only important to study possibility in the steady curve, but also the dynamic response during the transition [5].

In Indian railway, every rail engine having six axles i.e. three on rear and three on frontal side with couple of wheels on each axle. It is important to understand motion of wheelsets at curvature which directly affects on the suspension system. Helical suspension springs are responsible for axial displacements only but middle axle spring acquires lateral load also while rail moving over curved track. When rail moving at curved track the middle axle slide in lateral direction as the dashpot is not provided near the suspension to avoid its motion but the linkage can restrict its motion upto 16 mm. When middle axle slides, the flanges of remaining two wheelset strikes to its outer rail but due to availability of dashpot, it prevents lateral deformation of end axle springs. The combination of the profiled wheels and the creep forces is to create an oscillatory system, a combined lateral and yaw motion known as "hunting". Hence on minimum curves radius the wheel flange will be in contact with the side of the rail, causing wear of the wheels, wear of the rails and often significant amounts of noise.

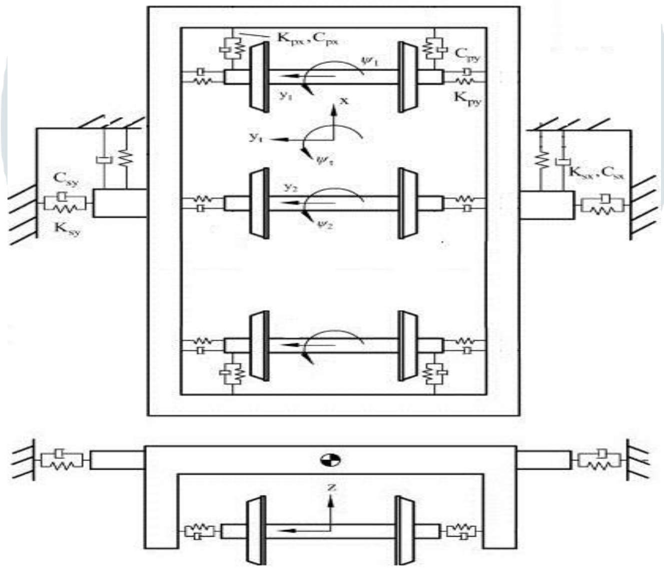


Figure 13: Mechanical model for lateral vibrations rail vehicle

The mathematical models of increasing complexity, via Newton's laws, were studied to encapsulate the lateral and roll dynamics of the tilting vehicle system. The equations of motion are given below with all variables [5].

For the vehicle body:

$$m_v\ddot{y}_v = -2K_{sy}(y_v - h_1\theta_v - y_b - h_2\theta_b) - 2C_{sy}(\dot{y}_v - h_1\dot{\theta}_v - \dot{y}_b - h_2\dot{\theta}_b) - \frac{m_v v^2}{R} + m_v g\theta_o - h_{g1}m_v\ddot{\theta}_o$$

$$i_{vr}\ddot{\theta}_v = -k_{vr}(\theta_v - \theta_b - \delta_a) + 2h_1(k_{sy}(y_v - h_1\theta_v - y_b - h_2\theta_b) + C_{sy}(\dot{y}_v - h_1\dot{\theta}_v - \dot{y}_b - h_2\dot{\theta}_b)$$
$$+m_v g(y_v - y_b) + 2d_1(-k_{az}(d_1\theta_v - d_1\theta_b) - k_{sz}(d_1\theta_v - d_1\theta_r)) - i_{vr}\ddot{\theta}_o$$

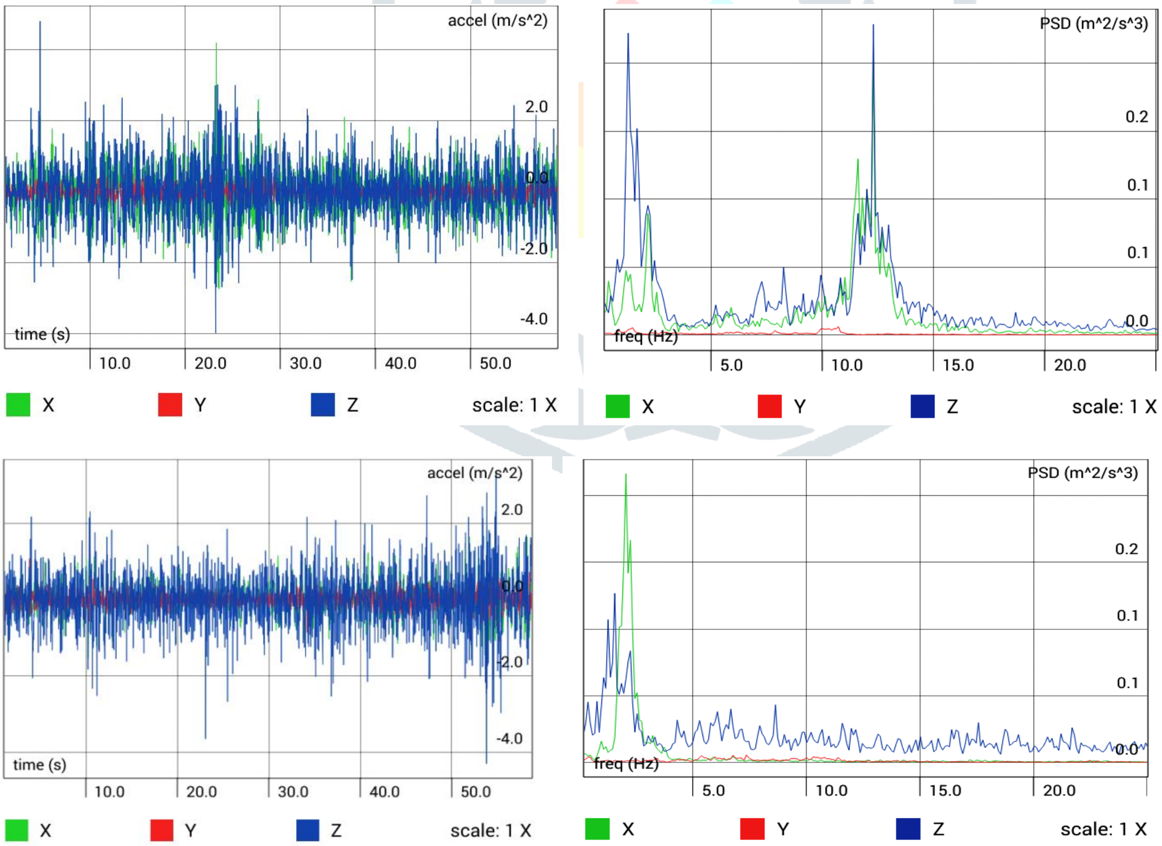
For the vehicle frame:

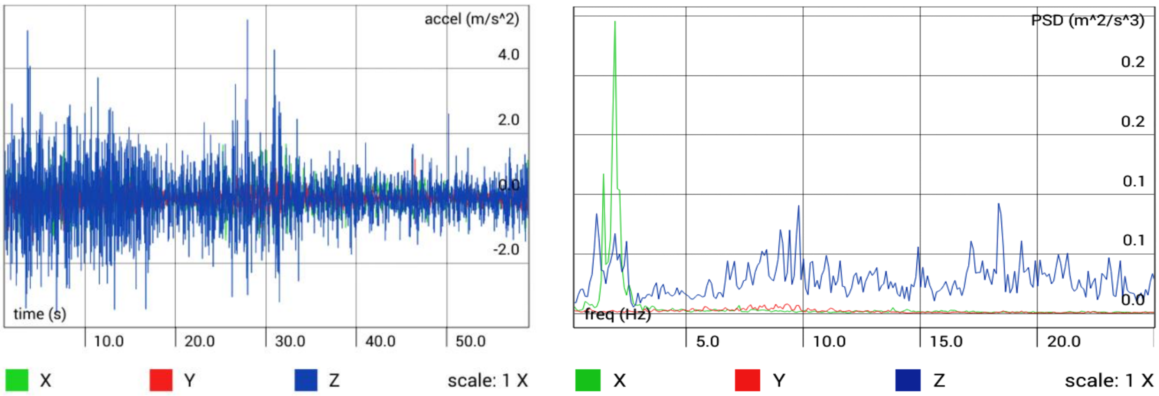
$$m_b\ddot{y}_b = -2K_{sy}(y_v - h_1\theta_v - y_b - h_2\theta_b) + 2C_{sy}(\dot{y}_v - h_1\dot{\theta}_v - \dot{y}_b - h_2\dot{\theta}_b) - 2k_{py}(y_b - h_3\theta_b - y_o)$$
$$-2c_{py}(\dot{y}_b - h_3\dot{\theta}_b - \dot{y}_o) - \frac{m_b v^2}{R} + m_b g\theta_o - h_{g2}m_b\ddot{\theta}_o$$
$$i_{br}\ddot{\theta}_b = -k_{vr}(\theta_v - \theta_b - \delta_a) + 2h_2(k_{sy}(y_v - h_1\theta_v - y_b - h_2\theta_b) + C_{sy}(\dot{y}_v - h_1\dot{\theta}_v - \dot{y}_b - h_2\dot{\theta}_b)$$
$$-2d_1(-k_{az}(d_1\theta_v - d_1\theta_b) - k_{az}(d_1\theta_v - d_1\theta_r)) + 2d_2(-d_2k_{pz}\theta_b - d_2c_{pz}\dot{\theta}_b)$$
$$+2h_3(k_{py}(y_b - h_3\theta_b - y_o) + c_{py}(\dot{y}_b - h_3\dot{\theta}_b - \dot{y}_o)) - i_{br}\ddot{\theta}_o$$

y_v, y_b, y_o is Lateral displacement of vehicle body, bogie frame, track. θ_v, θ_b, θ_r is Roll displacement of vehicle body, bogie frame, spring reservoir, θ_o, R is Track cant, curve radius. v is Vehicle forward speed. m_v is Half body mass, 15,000kg. i_{vr} is Half body roll inertia, 25,000kgm^2. m_b is Bogie mass, 10000kg. i_{br} is Bogie roll inertia, 1500kgm^2. g is gravitational acceleration, 9.81m/s^2.

Vibration analyzer is used to determine vibration response in all direction in the form of acceleration with respect to time. The analyzer is used to find peak value with respect to calibrated value which determines the dynamic behavior of rail vehicle on running track. The readings have been collected on Indian track condition to analyze fluctuation in vertical, lateral and in longitudinal direction. The raw acceleration data high-pass filtered to emphasize vibration. The roll-off frequency is 1Hz in high frequency mode and 0.1Hz in low frequency mode. The power spectrum is calculated from 0Hz to the Nyquist frequency. To integrate over the power spectrum, sum all data and then multiply by the frequency step size [13, 15].

The vibration readings are acquired at an average speed of 80 to 100km/hr. The vibration analyzer has been calibrated with gravity of 9.81 m/s^2 in vertical direction and 0 m/s^2 in lateral and longitudinal direction placed on the platform of frame of suspension system. During acquisition a peak value of acceleration, RMS vibration and PSD resonance in all directions has been recorded at different tracks which are observed to be maximum in vertical and lateral direction during running condition at an average speed of 100km/hr. The responses have been recorded on straight track, curve track and while tracking. The peak responses are observed to be maximum while tracking. An excitation of rail vehicle suspension in the form of acceleration with respect to time and frequency has been recorded and its vibration response and power spectrum density at different tracks captured are shown in figure 14. The maximum peak values of acceleration among three condition is 14.73 m/s^2 in vertical and 4.12 m/s^2 in lateral direction and power spectrum density of 0.23 m^2/s^3 at 1.3 Hz and 0.2 m^2/s^3 at 12 Hz in lateral direction has been recorded during experimentation.





X – Lateral, Y- Longitudinal, Z - Vertical

Figure 14: Vibration and power spectrum density (PSD) resonance response of rail vehicle suspension in dynamic condition with average speed of 100km/hr.

V. CONCLUSION

Dynamic analysis of mechanical structure is of great importance in vibration response where the responses are varying with respect to time. A dynamic of rail vehicle involves the vibrations response of vehicle carbody, bogie frame, and secondary suspension with three mutual damping and primary suspension with inclined damping. Secondary suspension system supports a carbody from vertical as well as lateral and longitudinal oscillation due to its high damping performance to balance the system. As the excitation and natural frequency ratio are more than unity, the resonance will not occur for the speed and wavelength consider but the resonance will occur for speed of 55 km/hr and 12m wavelength. A primary spring sustains the fluctuation of load due to unevenness of track, during curvature and during track change which causes abrupt change of oscillation in vertical and lateral direction. Finite element analysis of track and primary spring reveals that the natural frequency of track varies from 5Hz to 35Hz for different 6 modes which will be input for harmonic analysis of rail vehicle suspension system gives maximum amplitude at 4Hz.

The experimental results reveal that the oscillation are more in vertical and lateral direction on the Indian track and it is experienced to be varying and instantly changes while curving and tracking. Also it is rapidly changes for the increasing speed results in change of excitation frequency which may match with natural frequency of primary spring affects the behavior of suspension system. The analytical, numerical and experimental analysis of primary and secondary suspension system of rail vehicle reveal that the primary spring may cause more fluctuations and peak value obtained at 1.2Hz to 13Hz indicate that track is not common in every location and uneven surface causes more fluctuation and causes failure of spring.

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