Review - Derivation of the Schwarzschild Gravitational Time Dilation

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Abstract

Time dilation is an actual time difference between two events as measured by observers either moving relative to each other or situated differently from gravity masses. In a broader sense, the paper addresses the scientific conceptualization of time dilation. The presented concept focuses on the apparent reason behind the differences in time between different planets. Time dilation also encompasses the age - old mystery of life in outer space and the time constraints they follow which ought to differ with the time standards followed by our universe.

Keywords: Dilation, Gravitational force, Reference frame

Introduction

There has recently been a push within the literature and in textbooks to integrate the ideas of relativity theory into the physics info of college man physics students. Such a push is definitely intelligible considering this state of theoretical physics of that "general theory of relativity (GR) has become Associate in Nursing integral and indispensable half." it's somewhat, unfortunate, therefore, that introductory physics students, each at the college man and high school level area unit seldom if ever, bestowed with a mathematically impelled discussion of relativity theory. Opposition to such a shot is actually applicable. making an attempt to create any progress, during a mathematical sense, against GR, at tier applicable to introductory students is tough, however not possible, if we have a tendency to encourage the discussion by making an attempt to "inform" the reader, instead of describing physical truth. Within the derivation that follows, we are going to commit to extending a general abstract model of relativity into the realm of mathematical formalism. It's vital to worry that what follows is associate degree 'intuitive' approximation of general relativity; nonetheless, what the model lacks in precision; it gains back in friendliness. Before making an attempt this "general relativistic" drawback, it'd profit the scholar vastly to check the derivation of your time dilation in Einstein's special theory of relativity. The derivation, herein given, is implicitly supported the model of special relativistic time dilation. Also, it'd be useful to allow an abstract clarification of the correspondence principle because the organization of this derivation is that the omnipresent fast gedanke starship.

Literature Review

For knowledge of gravitation time dilation we referred several research paper and websites and figured out the higher the gravitational potential the faster time passes that is the further the clock is from the source of gravitation the faster it rotates.

Clocks that are far from massive bodies that is higher gravitational potential run faster as compared to closer clocks o massive bodies which are at lower gravitational potentials

In simple words, time runs slower wherever gravity is strongest, and this is because gravity curves space-time.

Implementation System

Imagine that we tend to area unit floating in associate degree heavenly body starship, distant from any supply of gravity. Also, we've got simply spoken to the captain, associate degreed he assures the North American nation that the ship is in a coordinate system, that's to mention, all the force sensors on board indicate there's no acceleration of any kind influencing the ship. we tend to area unit very happy by this as a result of we wish to conduct associate degree experiment to see what proportion time it takes a beam of sunshine to travel from the ground of the ship to the ceiling, a height Δy.

Being smart theoreticians, we have a tendency to attempt to compute however long it ought to take the beam of sunshine to succeed in the ceiling with no acceleration:

$$\Delta y = \vec{c}\Delta t \qquad (1)^7$$

$$\Delta t_i = \frac{\Delta y}{\vec{c}} \qquad (1a)$$

$\Delta t_i \rightarrow$ time for light to travel a distance in inertial frame

We area unit able to get current to conduct the experiment after we get a message from the captain of the ship. He sends his apologies, however says we have a tendency to simply received a distress call and have to be compelled to begin fast at a continuing

acceleration of $4.5\times10^{15}\frac{m}{s^2}$ (a) (in the +y direction).

Undeterred, we have a tendency to decide merely to redo the calculation for (1a) in an associate fast arrangement.

Thus, we want to seek out an associate equation which will relate what quantity time it'll take the sunshine to succeed in the ceiling, within the presence of a relentless acceleration. this is often a touch bit difficult to conceive conceptually, however, let's modification the model from a space vehicle to standing on the world within the presence of gravitation. Also, rather than shooting lightweight upwards, let's throw a ball upwards and raise, within the presence of gravity, however long will it take the ball to go? we are able to build this 'conceptual' modification due to the Einstein equivalence principle. currently, we are able to conceive terribly simply that the shape of the equation we want is simply one among the quality mechanics equations:

$$\Delta y = \vec{v}\Delta t_a + \frac{1}{2}\vec{a}\Delta t_a{}^2 \qquad (2)$$

Thus, we solve equation (2) for Δt_a using the quadratic equation:

$$\Delta t_a = \frac{-\vec{v} + \sqrt{\vec{v}^2 + 2\vec{a}\Delta y}}{\vec{a}} \qquad (3)$$

We selected this specific answer of the quadratic as a result of it refers to the "first" time the ball reaches Δy on the upward part of its journey. Next, we tend to transition back to the ballistic capsule, wherever the ball we tend to square measure "throwing" corresponds to lightweight, so $v \longrightarrow c$.

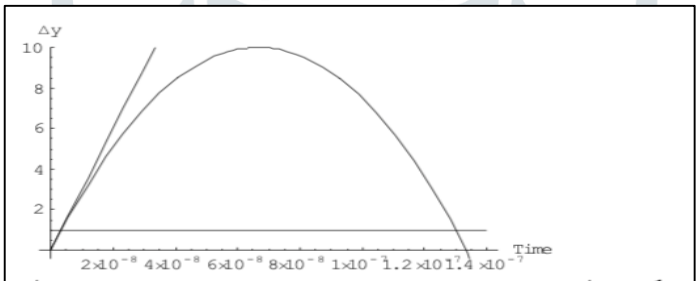
$$\Delta t_a = \frac{-\vec{c} + \sqrt{\vec{c}^{\,2} + 2\vec{a}\Delta y}}{\vec{a}} \qquad (3a)$$

Observation:

We determined to infix numerical values or each equation (1a) and equation (3a) and ascertain this stunning result:

$$\Delta t_i \rightarrow 3.33\times10^{-9}\,\text{s}$$
$$\Delta t_a \rightarrow 3.42\times10^{-9}\,\text{s}$$

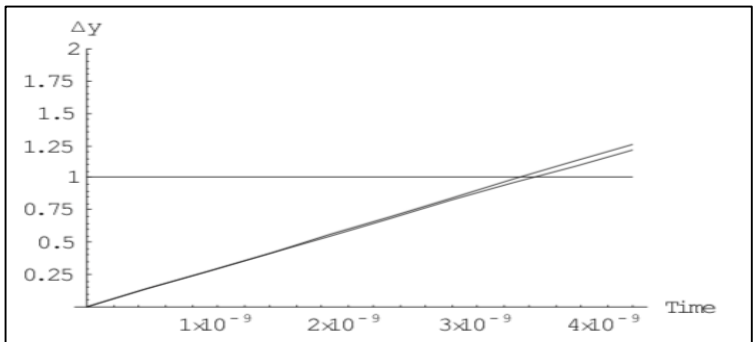
The quantity for the accelerated frame is longer. Surely, this is often a mistake? we will conceive that it is sensible within the case of the spacecraft, as a result of the "ceiling" of the spacecraft moves upward, therefore creating the sunshine travel an extended distance. But, if the Einstein correspondence principle is correct, then this means that within the presence of gravity, time is additionally dilated! unbelievable, we tend to return to the pc and graph equation (1) and equation (2) with the relevant numerical constants



Δy vs t. The horizontal time represents $\Delta y = 1$, the linear line represents the inertial model and the quadratic curve represents the accelerated model.

Graph-1: Time Vs Displacement

It sounds like at each purpose, say zero, the linear curve for the mechanical phenomenon model is larger than the quadratic curve for the accelerated model. To be sure, we have a tendency to squeeze the time ordinal down terribly small:



Δy vs t. The horizontal time represents $\Delta y = 1$. The bottom curve is the accelerated model.

Graph-2: Time Vs Displacement

Incredibly, within the accelerated frame, or by the correspondence principal, the gravitationally affected frame, the interval is longer. Time runs slower within the presence of gravity! Excited, we tend to tell the captain and he asks America to undertake to relate the just the once interval to the opposite.

$$\Delta t_i = \frac{1}{\vec{c}}\left(\vec{c}\Delta t_a + \frac{1}{2}\vec{a}\Delta t_a{}^2\right) \qquad (4)$$

$$\Delta t_i = \Delta t_a\left(1 + \frac{\vec{a}\Delta t_a}{2\vec{c}}\right) \qquad (5)$$

For reasons we'll analyze momentarily, equation (5) is just correct once is incredibly tiny, in fact, when:

$$\Delta t_a \leq \frac{2\Delta y}{c} \qquad (6)$$

Equation (6) substituted into equation (5)

$$\Delta t_i \leq \Delta t_a\left(1 + \frac{\vec{a}\Delta y}{\vec{c}^{\,2}}\right) \qquad (7)$$

Equation (7) represents the attractive force time dilation of our straightforward model.

Discussions

Using simply the precept of a planned experiment and therefore the notion of Einsteinian correspondence, we've determined an easy, however effective, model to clarify attractive force time dilation. Einstein would are proud! Not solely do the results create abstract sense, however within the limit, they become the answer of the precise Schwarzschild reference system. what's additional, we tend to develop all of the ideas with terribly straightforward pure mathematics, at a mathematical level applicable to introductory school students and advanced high school students. If the teacher needed to increase the results herein conferred she may actually take into account things in terms of the relativistic acceleration perform. Such a derivation would definitely demand far more mathematical sophistication from the coed, however, would need way fewer assumptions, and may, in this respect, really be clearer to a mathematically ready student.

Acknowledgement

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