# SIMPLE HARMONIC MOTION AND ITS APPLICATION IN CAR SHOCK ABSORBER

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## Abstract

Simple harmonic motion could be a special form of motion or oscillation motion wherever the restoring force is directly proportional to the displacement and acts within the direction opposite to it of displacement. This papers aim is to investigate about optimal damping constant of a quarter car shock absorber that is performed in the frequency domain. The optimum damping constant refers to the worth that minimizes the acceleration of all connected bodies characterizing a 2 degree of freedom system sketching a quarter car. The connected bodies are sprung and unsprung mass severally for quarter of chassis and tire, the last one maintain the contact with the lowest and it's connected with the sprung mass through a damper characterised by spring and fluid damper. Optimal damping constant resolve by imposing analytical conditions on the expression of acceleration of 2 masses. Afterwards, the variation of acceleration and position in function of frequency for the obtained value of damping constant is plotted numerically in two ways using Wolfram Mathematicia and MSC Adams Software.

**KEYWORDS:** Damping constant, Road disturbance, Shock absorber, Sprung mass, Unsprung mass,

## Nomenclature

| Symbol | Description |
|---|---|
| $c_s$ | Damping coefficient of shock absorber |
| $k_t$ | Spring constant of tire |
| $k_s$ | Spring constant of shock absorber |
| $m_s$ | Sprung mass |
| $m_U$ | Unsprung mass |
| $x_S(t)$ | Displacement of Sprung mass |
| $x_U(t)$ | Displacement of Unsprung mass |
| $x1_S(t)$ | Velocity of Sprung mass |
| $x1_U(t)$ | Velocity of Unsprung mass |
| $x2_S(t)$ | Acceleration of Sprung mass |
| $x2_U(t)$ | Acceleration of Unsprung mass |
| y(t) | Road disturbance |
| S | Laplace Variable |
| W(s) | Transfer Function |
| $w_{1s}$ $w_{2s}$ $w_{3s}$ | Frequency response of Sprung mass |
| $w_{1u}$ $w_{2u}$ | Frequency response of Unsprung mass |
| M | Mass Matrix |
| K | Stiffness Matrix |
| C | Damping Matrix |

# INTRODUCTION

Shock absorbers ar hydraulic (oil) pump like devices that facilitate to regulate the impact and rebound movement of your vehicle's springs and suspension. along side smoothening out bumps and vibrations, the key role of the shock is to make sure that the vehicle's tyres stay involved with the road surface the least bit times, that ensures the safest management and braking response from your car. The shock absorber is a component of dynamic system. The suspensions of quarter cars are dynamical systems having as component a shock-absorber. It provides a dissipative effect against exogenous excitation that produces a relative motion between the bodies connected by the shock absorber. The damping constant can be chosen in order to get the minimum acceleration of all masses so that the passenger driving a car, for example, vehicle's tyres in contact with the road surface by dominant the rebound of its suspension springs. As long as your vehicle's tyres remain in contact with the road, steering, road handling and braking response will be optimal, helping to keep you safe. It also used for different purpose for example it is used in a microstructure for the protection of electronic-packaging components in vibration-impact environments shown in Ping (2005) and Ping et al. (2008).
The paper focuses all attention on passive systems as fluid dampers, in order to produce an analytical investigation that shows the values of optimal damping constant.

# Literature Review

A literature review was carried out to know the recent practices and theories in the shock absorber design. It will also help to obtain a superior understanding of internal components and internal flows had been designed and modeled in the past. Reybrouck was the first one to introduce the first brief parametric models of a mono tube damper. Flow restriction forces were found utilizing experimental relationships that enclosed leak restriction, port restriction and spring stiffness correction factors. As individual internal forces were found, another experimental relationship was used to calculate the total damping force. Pressure fall across the precise flow restrictions may even be found. Reybrouck later completed his model to a twin tube damper and included a more physical representation of hysteresis. It was shown that physical phenomenon was caused not solely by oil compressibility, however the compressibility of gas bubbles transferred from the reserve chamber. it absolutely was additionally shown that reserve chamber pressure greatly affects the solubility of nitrogen. Kim also performed a study on a twin tube damper with focus on a vehicle suspension system. Discharge coefficients were through an experiment found and applied to the model.

# EQUATION METHODOLOGY

This chapter introduces the mathematical model of vibration system according to Guglielmino et al. (2005), Guiggiani (2007), Guglielmino et al. (2008).The simplest system of two degree of freedom quarter car is produced with minor modifications in Fig .1.

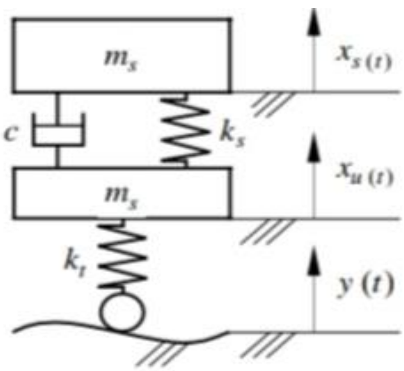


**Fig.1. Simplest system of two degree freedom of the quarter car automotive**

It is described as the system of two ordinary differential matrix equation (1) where the input is the road disturbance y (t) according to the unknown vibration theories of dynamic motions.

$$\begin{bmatrix} m_s & 0 \\ 0 & m_U \end{bmatrix}\begin{bmatrix} \ddot{x}_s(t) \\ \ddot{x}_u(t) \end{bmatrix}+\begin{bmatrix} c & -c \\ -c & c \end{bmatrix}\begin{bmatrix} \dot{x}_s(t) \\ \dot{x}_u(t) \end{bmatrix}+\begin{bmatrix} k_s & -k_s \\ -k_s & k_s+k_T \end{bmatrix}\begin{bmatrix} x_s(t) \\ x_u(t) \end{bmatrix}=\begin{bmatrix} 0 \\ k_T \end{bmatrix}y(t). \qquad (1)$$

The road disturbance y (t) is essential to carry out all information about dynamic behavior of system. The dynamic behavior is evaluated by considering the transfer function that relates the road disturbance to the outputs. In this case the outputs are the acceleration of two masses evaluated in the frequency domain and time domain. It is important to understand that minimization of acceleration can be done in the frequency domain or in time domain, aspect mentioned in Savaresi et al. (2010).

These two different ways to proceed are strictly depended on the particular input, whether the input is harmonic or time depended. In this paper an analytical investigation on damping constant is built up in order to get the optimal value that minimizes the acceleration of the sprung mass so that the entire structure, where the shock absorber is fixed, supports a lower mechanical excitation in order to get the best drive comfort and stability performance. In particular the stability performance is the mean goal of this paper or rather when the landing gear impacts on the ground, the entire structure receives a force directed on upper direction that leads the aircraft to lose the friction force of the wheel rolling on the ground and so a certain delay is introduced for arrest the plane. Moreover the same force leads the aircraft to produce a certain vibration of its structure such as fuselage and wings. This effect is present whatever is the value of damping coefficient but the less force is the less mechanical excitation is on the structure so that life cycle is higher than the others value of damping constant outside the optimal one.

The magneto rheological fluids are more suitable when a low energy is begin in use. The work in such a way the iron particle are subjected to magnetic field so that a long chain of particles produces an increasing of tensile stress, aspect mentioned in Namuduri et al (2003). The magnetic field is produced by the introduction of control current in coil winding the iron. The gap is filled by the magneto rheological fluid defined by a certain fraction of iron particles. The former is characterized by the introduction of a fixed value of control current and injected once a certain value of relative speed between the sprung and unsprung mass is captured by the sensors.

Another way to introduce the control current is by defined a proportional value of magnetic field to the relative speed between the sprung and unsprung mass. In this case a linear study must be introduced in order to delete the complication due to the nonlinearity of magnetic field that is characterized by the saturation problem.

The mathematical model (1) is characterized by a vector containing the kinematic variable with regard on the sprung and unsprung mass that are the outputs of a system shown in this paper. Its dynamic behavior is analyzed according the transfer function that relates the road disturbance to the masses. The transfer function (2) is obtained by Laplace transform according with Giua and Seatzu, (2006) with zero initial conditions:

$$\left(s^2M + sC + K\right)q(s) - sq(0) - \dot{q}(0) = \begin{bmatrix} 0 \\ k_T \end{bmatrix} y(s). \qquad (2)$$

The transfer function(3) is strictly proper because it represent a typical system where the order of derivative input is lower than the kinematic variable mentioned by Giua and Seatzu (2006).

$$W(s) = \left\{ w_{ik} = \left( \frac{\prod_{l=1}^{h} s + z_l(c)}{\prod_{r=1}^{n} s + p_r(c)} \right) / \, w_{ik} = w_{ki} \forall i,k = 1\ldots2 \right\}. \qquad (3)$$

in which: h<n and $w_{ik} = w_{ki} \forall i,k = 1\ldots2$ .

In particular for the sprung and unsprung mass the transfer function are respectively (4) and (5):

$$W(s) = \frac{k_T(k_s + c\cdot s)}{k_s\left(k_T + (m_s + m_U)s^2\right) + s\left(m_s s\left(k_t + m_U s^2\right) + c\cdot s\left(k_t + (m_s + m_U)s^2\right)\right)}. \qquad (4)$$

$$W(s) = \frac{k_T(k_s + s(c + m_s s))}{k_s\left(k_T + (m_s + m_U)s^2\right) + s\left(m_s s\left(k_t + m_U s^2\right) + c\cdot s\left(k_t + (m_s + m_U)s^2\right)\right)}. \qquad (5)$$

The optimal damping constant is carried out from a study of the acceleration in the frequency domain and it is based on the research of that point of acceleration point that are maximum in a derivative sense. It is constituted by the substitution of Laplace variable with and the calculation of second derivative for each transfer function as in (6) and (7), respectively for the sprung and unsprung mass.

$$W(j\omega)_{acc\_s} = \omega^2 \frac{\sqrt{(k_s k_T)^2 + (ck_T\omega)^2}}{\sqrt{\left(k_s k_T + m_s m_U \omega^4 - (k_T m_s + k_s(m_s + m_U))\omega^2\right)^2 + \left(-c(m_s + m_U)\omega^3 + ck_T\omega\right)^2}}. \qquad (6)$$

$$W(j\omega)_{acc\_s} = \omega^2 \frac{\sqrt{\left(k_s k_T - k_T m_s \omega^2\right)^2 + (ck_T\omega)^2}}{\sqrt{\left(k_s k_T + m_s m_U \omega^4 - (k_T m_s + k_s(m_s + m_U))\omega^2\right)^2 + \left(-c(m_s + m_U)\omega^3 + ck_T\omega\right)^2}}. \qquad (7)$$

The investigation starts by considering the intersection points of (6) and (7) with the transfer function (40 and (5) (Guiggiani, 2007).

$$W(j\omega)=\omega^2\frac{\sqrt{k_T^2\omega^2}}{\sqrt{\omega^2\left(k_T-(m_S+m_U)\omega^2\right)^2}} \quad (8$$

The intersection points are very important because the calculation of maximum point made in order to reach the minimum of acceleration. It is the obtained from the intersection between (8) and (9):

$$\left.\frac{d\left(\omega^2 W(c,\omega)\right)}{d\omega}\right|_{\omega=\Omega}=0 \quad (9)$$

## Optimal Damping Constant :-

In order to calculate the optimal damping constant with regards to the dynamic system a set of data are used and showed in the table I under the assumption of passive system;

Table 1. The values taking in account in the analysis

| **Symbols** | **Description** | **Values** |
|---|---|---|
| $m_s$ | Sprung mass | 400 Kg |
| $m_U$ | Unsprung mass | 12Kg |
| $k_s$ | Stiffness of shock absorber | 50000N/m |
| $k_T$ | Stiffness of tire | 300000N/m |

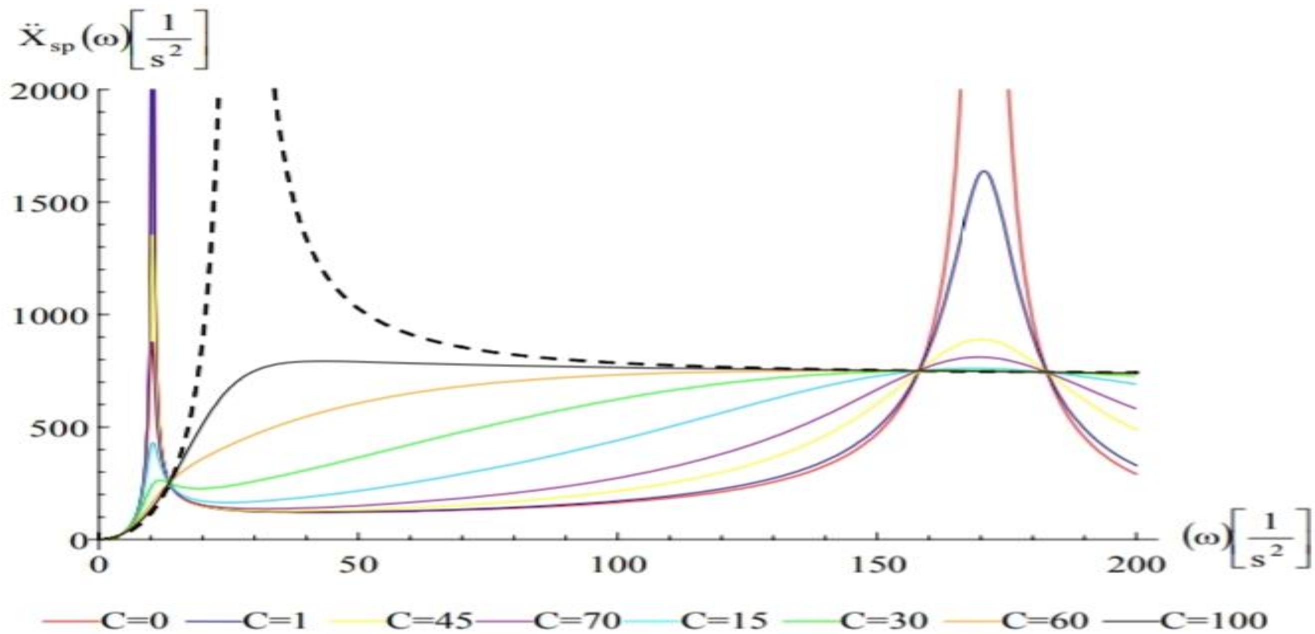


**Fig. 2.** Acceleration of sprung mass
$c_s \geq 0$ for continuous line; c $c_s \rightarrow \infty$ for dashed line.

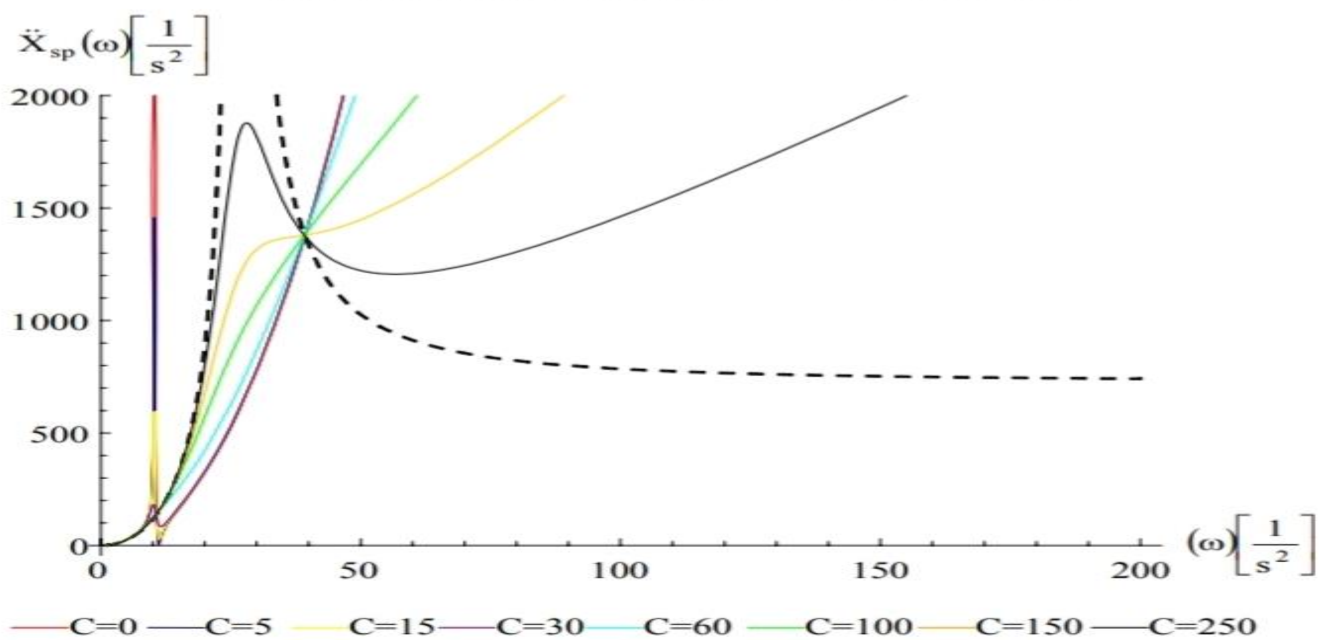


**Fig. 3.** Acceleration of unsprung mass
$c_s \geq 0$ - for continuous line; $c_s \rightarrow \infty$ - for dashed line.

In Fig. 2 and Fig. 3 the intersection points in frequency domain are significantly for purpose of this study; in particular these intersection points lead to a set of data one sprung mass and, respectively, for the unsprung mass. For the sprung and unsprung mass we refer the intersection points corresponding to the frequencies shown in the vectors (10) and (11).

Since that minimum acceleration is verified for the intersection point, the first derivative of acceleration function in the frequency domain must be calculated in order to get the damping constant in those points (Guiggiani,2007)

$$\omega_{1s} = \sqrt{\frac{k_t}{m_u}} . =$$

$$\omega_{2s} = \frac{1}{\sqrt{2}} \sqrt{\frac{2k_s m_s + k_t m_s + 2k_s m_u - \sqrt{-8k_s k_t m_s m_u + (k_t m_s + 2k_s(m_s + m_u))^2}}{m_s m_u}} = \tag{10}$$

$$=$$

$$\omega_{3s} = \frac{1}{\sqrt{2}} \sqrt{\frac{2k_s m_s + k_t m_s + 2k_s m_u + \sqrt{-8k_s k_t m_s m_u + (k_t m_s + 2k_s(m_s + m_u))^2}}{m_s m_u}} =$$

$$=$$

$$\omega_{1u} = \sqrt{\frac{k_s m_s + k_t m_s + 2k_s m_u}{m_s{}^2 + 2m_s m_u} -}$$

$$- \frac{\sqrt{(-2k_s m_s - 2k_t m_s - 2k_s m_u)^2 - 8k_s k_t (m_s{}^2 + 2m_s m_u)}}{2(m_s{}^2 + 2m_s m_u)} = \tag{11}$$

$$\omega_{2u} = \sqrt{\frac{k_s m_s + k_t m_s + 2k_s m_u}{m_s{}^2 + 2m_s m_u} +}$$

$$+ \frac{\sqrt{(-2k_s m_s - 2k_t m_s - 2k_s m_u)^2 - 8k_s k_t (m_s{}^2 + 2m_s m_u)}}{2(m_s{}^2 + 2m_s m_u)} =$$

The figure (3) shows the comparison of optimal damping constant between sprung and unsprung mass. The first derivative of both functions is referred to the expression (12) and (13) where $\omega$f is the value of relative to the best compromise between maximum and minimum acceleration taken with positive sign. The solutions of (12) and (13) are shown in a vector form as (14) and (15).

$$\left. \frac{\partial\left(\omega^2 W(c,\omega)_{sprung}\right)}{\partial\omega} \right|_{\omega=\omega_f} = 0 \tag{12}$$

$$\left. \frac{\partial\left(\omega^2 W(c,\omega)_{unsprung}\right)}{\partial\omega} \right|_{\omega=\omega_f} = 0 \tag{13}$$

The approximation formula of damping constant produced by Guiggiani, (2007), is obtained by simplifying the expression used here that is carried out by Wolfram Mathematica but since that is too complicated and very long we report only the value of it. The symbolic values of the frequencies where the intersections point exist as follows:

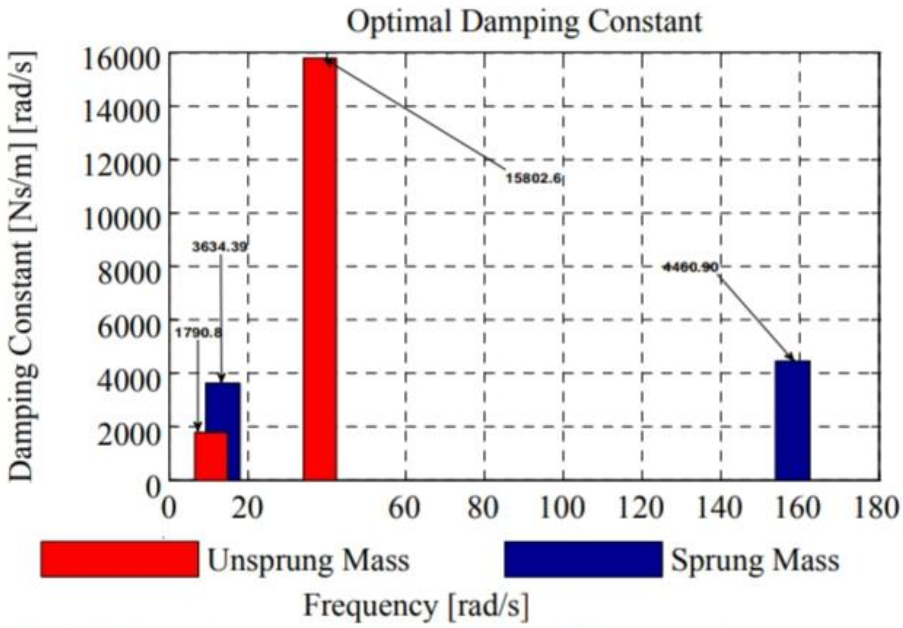


**Fig. 4.** Optimal damping constant value of sprung and unsprung mass

$$c_s = \begin{bmatrix} 3634.39 \\ 4460.94 \end{bmatrix} \left[ \frac{Ns}{m} \right] \tag{14}$$

$$c_u = \begin{bmatrix} 1790.8 \\ 15802.6 \end{bmatrix} \left[ \frac{Ns}{m} \right] \tag{15}$$

The Fig. 4 shows an important result, or rather one that can minimize the acceleration of sprung mass by using the minimum value relative to the blue bar or one that minimize the acceleration of unsprung mass by using the minimum value relative to the red one.

## GRAPHS

The readings are noted down whereas testing the standard & variable damper for various weights and by utilizing these readings, graphs are planned with sure & rebound damping co-efficient in coordinate axis.

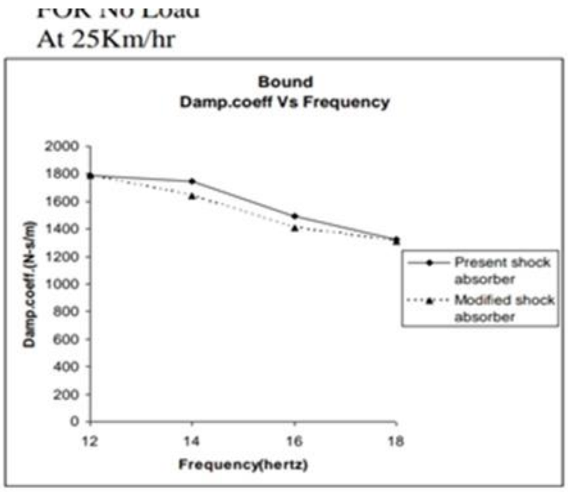


Fig5

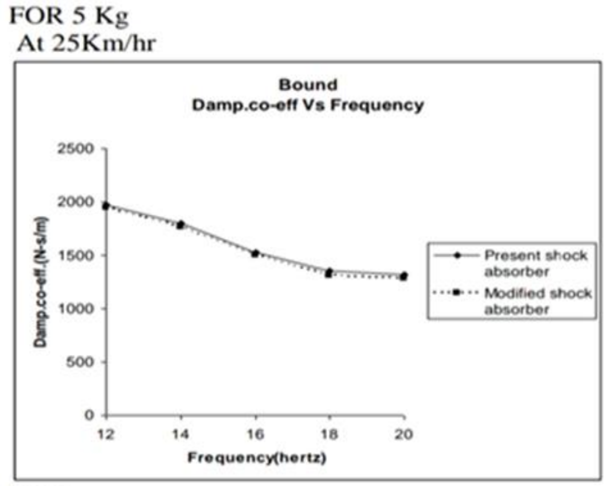


Fig7

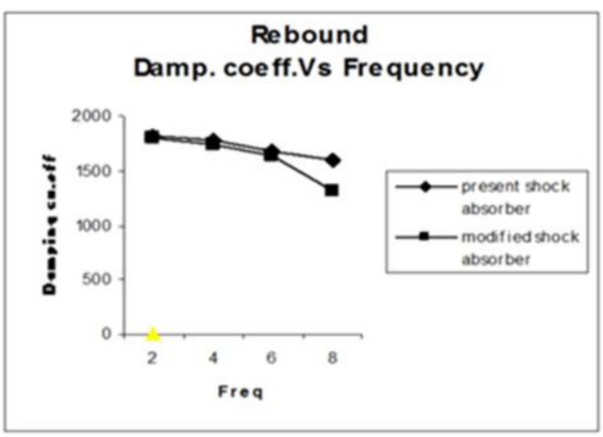


Fig 6

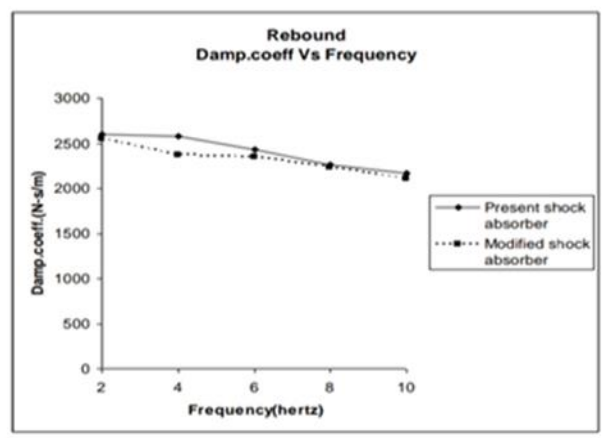


Fig8

**Fig.-5 and Fig-7** shows the variation of Bound Damping co-efficient with respect to frequency for 0 and 5 Kg in order for both present & modified damper. Bound Damping co-efficient for a modified damper found to be less when compared to the present one.

**Fig.-6 and Fig-8** shows the variation of Rebound Damping co-efficient with respect to frequency for 0 and 5Kg in order for both present & modified damper. ReBound Damping co-efficient for a modified damper found to be less when compared to the present one.

## PROPOSED ENCHANCEMENTS

### Advanced DAS

The DAS system used currently has a sampling rate of about 20 readings per second. In order to increase the number of readings obtained per stroke and also to test the system with a higher sampling rate can be used.

### Validation of Test Rig

By testing a New Shock absorber on the test rig, we can evaluate the authenticity of the results obtained from our test rig.

### Use of LVDT and Velocity Transducer

Accuracy of the displacement and velocity measurement can be increased by the use of LVDT and Velocity Transducer.

## CONCLUSION

This paper analyzes a quarter car sketched by a 2DOF system in order to determine the optimal damping constant minimizing the acceleration of connected body;

• The dynamic behavior was analyzed considering a transfer function obtained by Laplace transform that relates the road disturbance to the masses. The input of analytical model was represented in this paper by the road disturbance.

• In order to determinate the value of optimal damping constant it was used the first derivative of both transfer function, for sprung and unsprung masses, relative to the frequency which offer the best compromise between acceleration of connected body.

• For the damping constant determinate previous, a comparative place of displacement and acceleration of sprung an unsprung masses was obtained using Wolfram Mathematicia and MSC Adams.

• The result gained by both methods is practically same.

• The method can be applied in order to provide the optimal damping constant of this system for different value of input data or can be used for analyze another systems configuration.

## ACKNOWLEDGEMENT

It gives us a great pleasure and immense enthusiasm to present this topic of mini project on “ SIMPLE HARMONIC MOTION AND IT’S APPLICATION”, which is the result of unwavering support, expert guidance and focused directions of our guide Prof. Parmeshwari Aland, to whom we express our deep sense of gratitude and humbly thank her for her valuable guidance throughout the presentation work. We sincerely thank all others who directly or indirectly helped us and made numerous suggestions which have surely improved our work and helped us make this project.

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