SMALL ANGLE APPROXIMATION FOR SIMPLE HARMONIC MOTION IN PENDULUM

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ABSTRACT

The Small angle approximation is just a basic simplification of trigonometric functions which holds true when the considered angle is small or tending towards zero. In this examination we utilized picture preparing calculations to decide pendulum ball movement and present a methodology for settling the nonlinear differential condition. A basic estimation equation is inferred here for the reliance of the time of a basic pendulum on plentifulness that just requires a pocket number cruncher and outfits a blunder of under 0.25% as for the definite period. It is demonstrated that this estimation equation is more exact than other basic equations

KEYWORDS*:* Acceleration, Oscillations, Pendulum, Small angle approximation.

INTRODUCTION

The simple pendulum is a standout amongst the most mainstream precedents examined also, it is maybe the most explored oscillatory movement in material science. Numerous nonlinear wonders in reality are administered by pendulum-like differential conditions, which emerge in numerous fields of science and innovation (e.g., examination of acoustic vibrations, motions in little atoms, optically torqued nanorods, electronic channels, gravitational lensing when all is said in done relativity, propelled models in field hypothesis, motions of structures amid seismic tremors, what's more, others. The intermittent movement shown by a basic pendulum is harmonic just for little point motions. Past this farthest point, the condition of movement is nonlinear. In spite of the fact that a necessary equation exists for the time of the nonlinear pendulum, it is typically not examined in early on material science classes since it is preposterous to assess the essential precisely. Hence practically all starting material science course readings what's more, lab manuals talk about just little point motions for which the estimate sin is legitimate. Most looks into including basic harmonic movement and little point approximations, numerous individuals demonstrated diverse hypotheses introductory physics textbooks and lab manuals discuss only small angle oscillations for which the approximation $\sin\theta \approx \theta$ is valid.

In this paper we model the motion of the simple harmonic pendulum from Newton’s second law, then compare this with the small angle approximation model.

LITERATURE SURVEY

Most researches involving simple harmonic motion and small angle approximations, many people proved different theories. . In 2003 Millet proposed a numerical support for the Kidd and Fogg equation by thinking about trigonometric connection and little point approximations for sine and cosine capacities, just as the correlation between the sine capacity and different of its straight approximations [Millet L. E., 2003]. In 2005 Hite three approximations are examined for the recurrence of a simple pendulum. In 2006 Lima and Arun. A determined a simple rough articulation for the reliance of the time of a basic pendulum on the sufficiency. The estimate is more exact than other simple relations.

METHODOLOGY

ASSUMPTIONS:-

:- All models are brimming with suppositions. A portion of these suspicions are exceptionally precise, for example, the pendulum is unaffected by the day of the week. A portion of these suspicions are less precise yet we are as yet going to make them, grating does not impact the framework. Here is a rundown of a portion of the more outstanding assumptions of this model of a pendulum..

- Friction from both air opposition what's more, the framework is unimportant
- The pendulum swings in an ideal plane.
- The arm of the pendulum can't twist or extend/pack.
- The arm is massless.
- Gravity is a steady 9.8 meter/second^2
- Friction from both air resistance and the system is negligible.

VARIABLES:-

m = mass at the swinging end of the pendulum (kilograms)

g = acceleration due to gravity (meter/second^2)

L = length from the swivel point to the center of mass (meters)

θ = angle between the string position to the string position at rest (radians)

t = time (seconds)

T = period of the pendulum (time for one complete cycle) (seconds)

EQUATIONS AND DERIVAITON:-

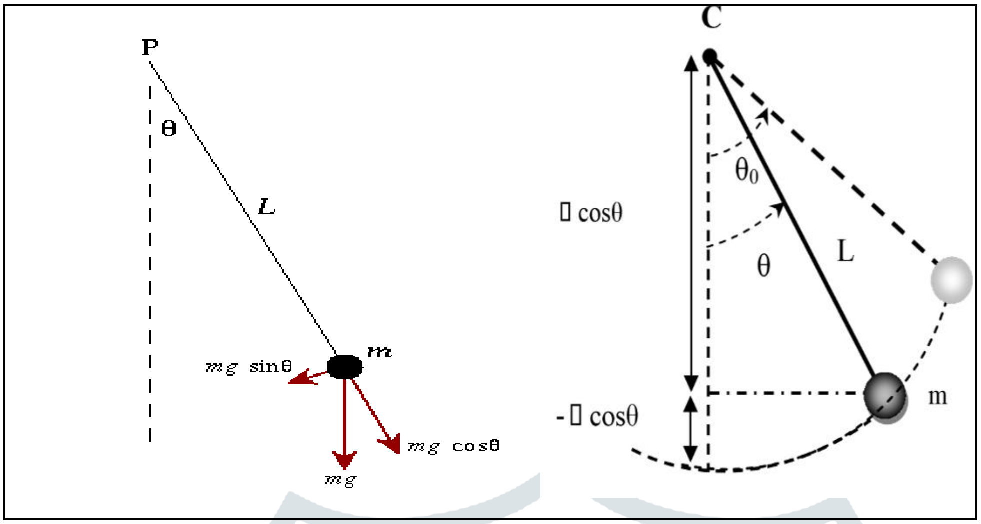

Fig.1. Simple pendulum and it's back and forth motion.

A simple pendulum comprises of a mass m dangling from a string of length L and fixed at a rotate point P. At the point when uprooted to an underlying edge and discharged, the pendulum will swing forward and backward with occasional movement. We will presently determine the simple harmonic movement condition of a pendulum from Newton's second Law

$$F = ma$$

Acceleration due to gravity will be a function of θ. At θ = π 2 −→|a| = g and at θ = 0 −→ a = 0, considering the relation of acceleration and θ we arrive at

$$a = -g \sin\theta$$

Arc length (arcL) of the pendulum can be thought of as the "position" of the system

$$ArcL = L\theta$$

Now, the acceleration of the system will be

$$a = L(d^2\theta/dt^2)$$

$$(\frac{d^2\theta}{dt^2}) + (\frac{g}{L})sin\theta = 0$$

Now we will solve this equation to get T (period) reduce

The second order differential equation to a first order

$$(\frac{d\theta}{dt}) * (\frac{d^2\theta}{dt^2}) + (\frac{d\theta}{dt}) * (\frac{g}{L}) \sin\theta = 0 \longrightarrow \frac{d}{dt}[1/2(d\theta/dt)^2 - (g/L)\cos\theta] = 0$$

Integrating to get the differential equation

$$(\frac{d\theta}{dt})^2 - (2g/L)\cos\theta = C$$

With the initial conditions of $\theta_0 (0) = 0$ and $\theta (0) = \theta_0$we can solve for C

$$C = -(2g/L)\cos\theta_0$$

Putting the value of C into the equation

$$\left(\frac{d\theta}{dt}\right)^2 = (2g/L)(\cos\theta - \cos\theta 0)$$

Now take the square root of both sides while ignoring the negative because we are solving for time and either time will be the same distance from our t0. It is just a matter of forwards or backwards in time. On the left hand of our equation lies the rate of change of the angle with respect to time, but we are going to solve for the period, so we need the time with respect to the angle, because of this we are going to inverse the entire equation and integrate from 0 to θ0. We will now multiply the whole thing by four to get the period. The change in time to get from 0 to θ0 is only one fourth of the entire cycle of the pendulum. This gives us our new equation of

$$T = 4\sqrt{(L/g)} *(1/\sqrt{2}) *\int(0 -> \theta 0)\ (1/\cos\theta - \cos\theta_0)\, d\theta$$

THE SMALL ANGLE APPROXIMATION:-

The small angle approximation states that $\theta \approx \sin(\theta)$ at small angles. Using this we adjust the equation

$$\left(\frac{d^2\theta}{dt^2}\right) = -(g/L)\sin\theta \longrightarrow \left(\frac{d^2\theta}{dt^2}\right) \approx -(g/L)\theta$$

and it is almost the same and far easier to solve.

$$(d^2\theta/\mathrm{d}t^2) \approx -(g/L)\theta$$

$$\Lambda^2 + (g/L) = 0$$

$$\lambda = \pm\sqrt{(g/L)} * i \text{ in the form of } a\pm bi$$

Using the complex roots case for solving second-order equations

$$y(t) = eat[A1 \cos(bt) + A2 \sin(bt)]$$

We can solve for A1 and A2 which are arbitrary constants using the initial conditions θ(0) = θ0 and θ' = 0. We get A1 = θ0 and A2 = 0. Thus we come to the solution

$$\theta(t) \approx \theta 0 \cos(\sqrt{(g/L)}t)$$

Using the above equation we create the period equation by setting θ = 0. We can solve for t, and this gives us one fourth of the total period. Now we multiply by four and get

$$T \approx 2\pi\sqrt{(L/g)}$$

THE EXPERIMENT

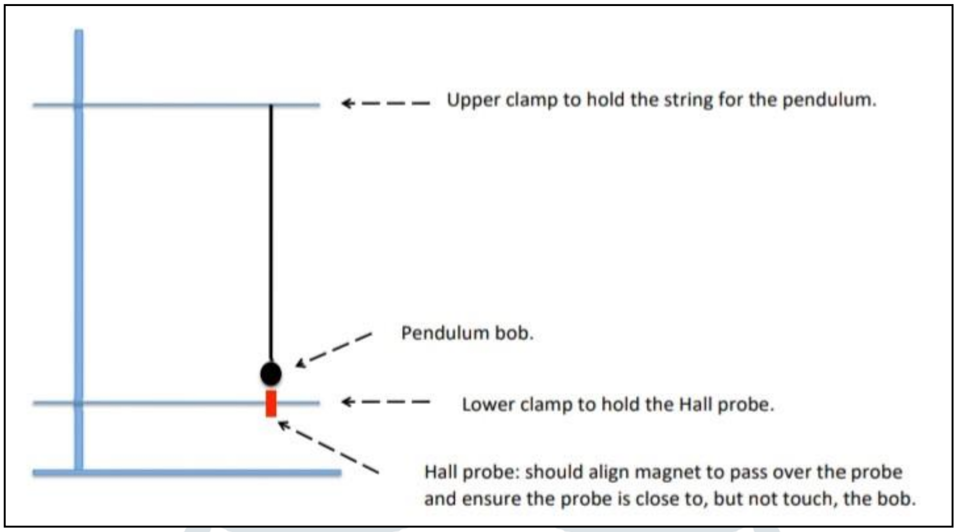

Fig.2. Experimental setup.

We used a Digital Camera for Camera Movement simple pendulum back and forth where we started from the rest of ball and an angle of almost 15°; they arrived to the movement of the pendulum angle of less than 5°. After that, we converted the video clip into still images. Then we used the Segmentation technique to highlight the moving ball only then we easy find the angle at which you make with the vertical axis of the pendulum. We measured the length of a pendulum from the center of the bob to the edge of a pendulum clamp and adjusted it to be as close to 0.20m. Then we have found the angle of pendulum from compute the averages of X-values and Y-values for the indices of object image point. After that using the trigonometric functions to estimate the angle between vertical line and pendulum string in each image. By using Table Curve 2D software, we found at the period time through the relationship between the angular displacements with time and found the wavelength of the oscillating, which is the time period.

OBSERVATIONS

At that point we utilized the Segmentation strategy to feature the moving ball at exactly that point we simple discover the edge at which you make with the vertical pivot of the pendulum. We estimated the length of a pendulum from the focal point of the bounce to the edge of a pendulum cinch and balanced it to be as near 0.20m. At that point we have discovered the edge of pendulum from register the midpoints of X-qualities and Y-values for the files of object picture point. After that utilizing the trigonometric capacities to evaluate the edge between vertical line what's more, pendulum string in each picture. By utilizing Table Curve 2D programming, we found at the period time through the connection between the precise relocations with time and discovered the wavelength of the swaying, which is the timeframe. Perceptions Comparison among careful and inexact articulation: We look at the exactness of the estimation for the pendulum time frame to that of careful answer for amplitudes not exactly or equivalent to (15°). Figure (3) delineate the proportion between the genuine time of a pendulum and the estimated esteem got for little points, as a capacity of the sufficiency. Shows up from (Fig. 3) the contrast between a power arrangement period time for the power 1 to control 5, additionally we see that the thing that matters is expanding with increment the estimation of edge, it appears to be clear in the edges which more than (20°), while the estimations of the edge that is littler than (15°) - The extent of work in this exploration - the distinction does not show up clear from the estimations of the time, where the most extreme contrast in (θ=15°) achieve (0.037 m/sec), in this way, we will analyze between the power arrangement of intensity 5 with definite period time, which we got from the test. The estimation of the time interim for a few progressive periods is a decent methodology for motions in the little edge routine, where the abundancy does not change fundamentally starting with one swing then onto the next, yet not for expansive point motions, on the grounds that the period diminishes impressively because of air grating

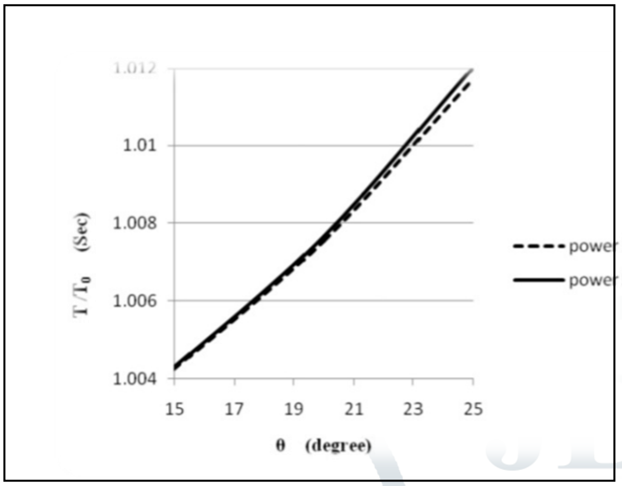

Fig.3. Deviation of period from small-angle approximation

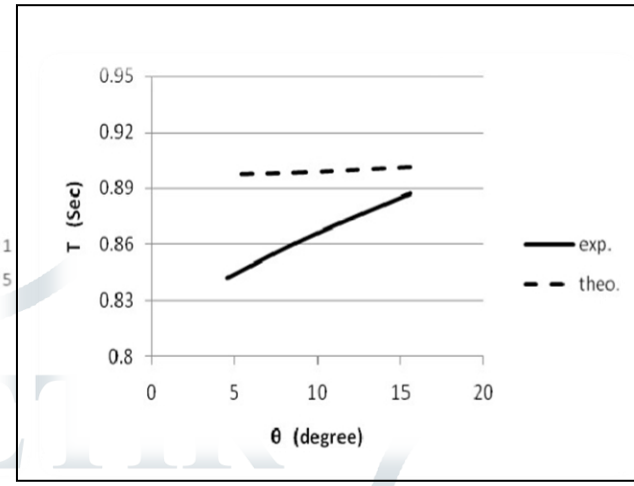

Fig.4. Experimental and Theoretical period time.

At “small angles” less than 15◦ or so, the period of the pendulum is completely dependent on the length of the arm and gravity, because θ ≈ sinθ at “small angles” as demonstrated in the graph below.

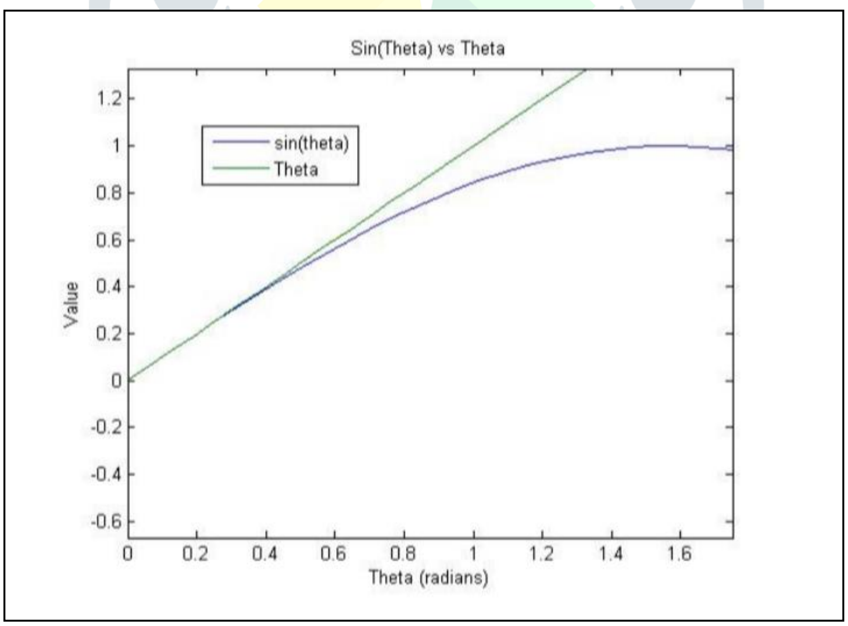

Fig.5. Graphical representation of sin$\theta$and θ in radians

Now, breaking $(\frac{d^2\theta}{dt^2})$+(g/L) sinθ = 0 into a system of two first order equations

$$x_{01} = x_{02}$$

$$x_{02} = -(g/L)\sin(x_1)$$

And breaking $(\frac{d^2\theta}{dt^2}) + (\frac{g}{L})\theta = 0$ into

$$x_{01} = x_{02}$$

$$x_{02} = -(g/L)x_1$$

we get the two graphs below.

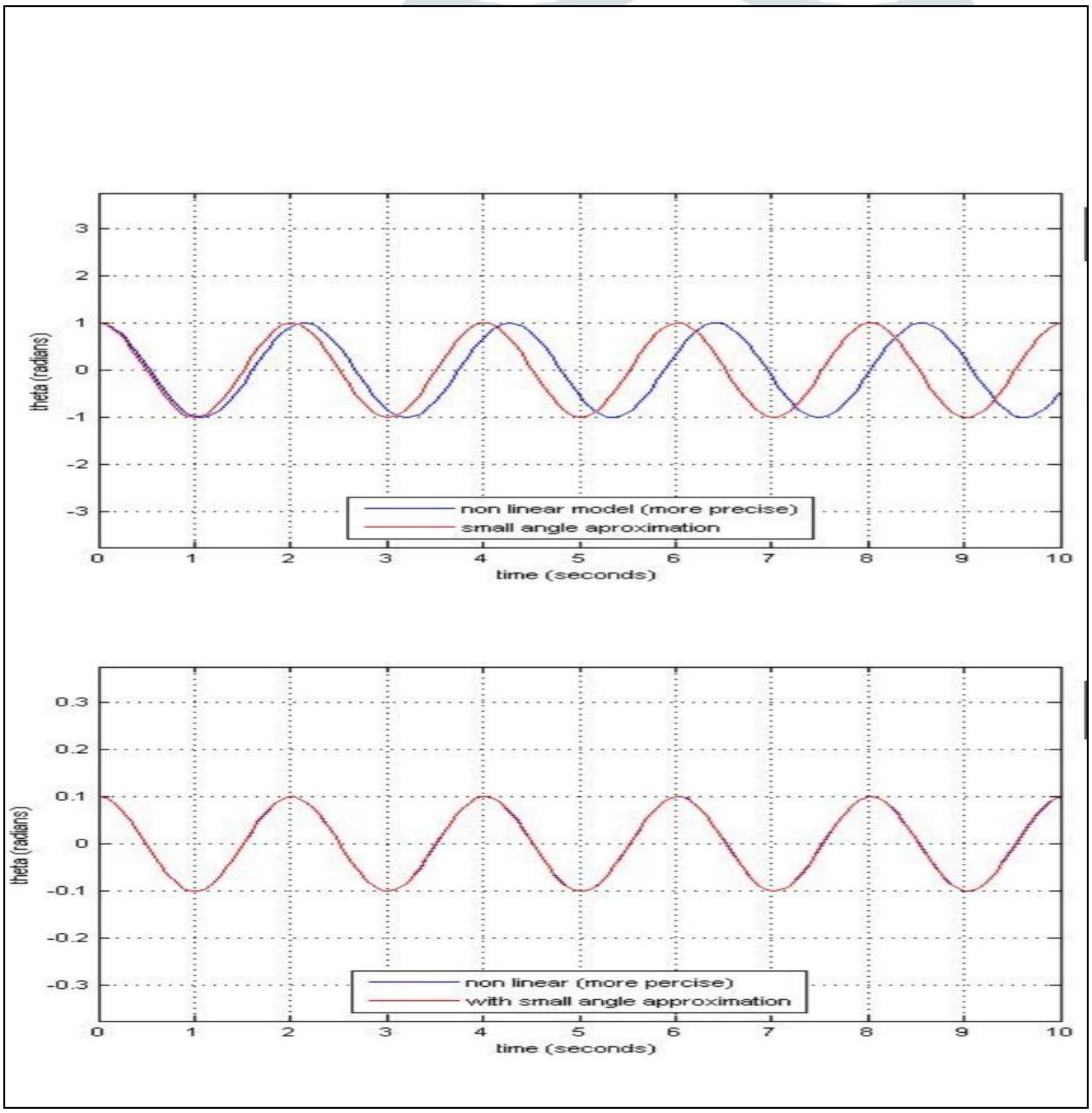

Fig.6. The difference between the simple harmonic model and the small angle approximation model.

APPLICATIONS

Pendulums have numerous applications and were used regularly before the computerized age. They are utilized in tickers and metronomes because of the normality of their period, in destroying balls and play area swings, because of their simple method for structure up what's more, keeping vitality. They are even found in different scientific instruments, from seismographs to early torpedo direction frameworks, because of their affectability to unsettling influence. An ancestor to the seismograph depended on an altered pendulum

CONCLUSION

For small angle oscillations, the approximation $\sin\theta \approx \theta$ is valid and the equation $a = -g \sin \theta$ becomes a linear differential equation analogous to the one for the simple harmonic oscillator. In this regime the pendulum oscillates with a period $T_0 = 2\pi\sqrt{}\ (L/g)$.This relation underestimates the exact period for any amplitude, but the difference is almost imperceptible for small angles. For larger angles T_0 becomes more and more inaccurate for describing the exact period.

In this routine the pendulum wavers with a period $T_0 = 2\pi\sqrt{L/g}$. This connection thinks little of the definite period for any adequacy, however the thing that matters is nearly vague for little points. For bigger points T0 turns out to be increasingly more wrong for depicting the precise period. It appears that the revealed blunders in approximating the accurate time of the estimation formulae for vast amplitudes of a basic pendulum in the range 0° -15° are not exact for material science understudies who play out the simple pendulum test. A basic estimated articulation is determined for the reliance of the time of a simple pendulum on the adequacy. The estimate is more precise than other simple relations.

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