

Study on Simple Harmonic Motion And its Application on spring

Abhishek Yernagula^[1], Akshay Thorat^[2], Arbaj Khan^[3], Prof. Parmeshwari Aland^[4]

Professor, Ajeenkya DY Patil University

ABSTRACT

In this paper we have used pendulum and spring-mass to study simple harmonic motion and also perform some of its problems. We have studied about how Bessel function can get approximated by the damped sinusoidal function. The numerical method that is very helpful in small time (independent variable) interval. Results of the case study are also compared with the Language polynomial.

KEYWORDS: - Acceleration, Amplitude, Angular frequency, Harmonic, Oscillations, Velocity.

INTRODUCTION

In mechanics and physics, simple oscillatory motion is a periodic motion or oscillatory motion where the returning force is directly proportional to the displacement and is always directed towards the initial position of the particle.

It can be used as a mathematical model for a many other motions. For ex : oscillation of a spring, motion of pendulum , etc. Motions such as of a simple harmonic motion and simple pendulum can be approximated by studying simple oscillatory motion. In simple oscillatory motion the kinetic energy and potential energy of the particle varies according to the displacement of the particle. When studied using graphical representation we find that the velocity, displacement and time are sine and cosine functions.

Simple harmonic motion shown the motion in both i.e in real space as well as in phase space. The orbit is periodic. (Here the velocity and position axes have been reversed from the standard convention to align the two diagrams) In the diagram, a simple harmonic oscillator, consisting of a weight attached to one end of a spring, is shown. The other end of the spring is connected to a rigid support such as a wall. If the system is left at rest at the equilibrium position then there is no net force acting on the mass. However, if the mass is displaced from the equilibrium position, the spring exerts a restoring elastic force that obeys Hooke's law.

Mathematically, the restoring force \mathbf{F} is given by

$$F = - k x.$$

where \mathbf{F} is the restoring elastic force exerted by the spring (in SI units: N), k is the spring constant ($\text{N}\cdot\text{m}^{-1}$), and x is the displacement from the equilibrium position (m).

LITERATURE REVIEW

1) Describing Vibration:-

Vibration can be defined as “the cyclical change in the position of an object as it moves alternately to one side and the other of some centre. Vibration of rigid bodies can be rectilinear, rotational or a combination of the two.

Mathematically, Harmonic vibration of single degree of freedom system can be given by the expression.

$$u(t) = u_0 \sin \omega t \dots \dots \dots (1)$$

Where,

$u(t)$ = the position of the point with respect to time t .

u_0 = maximum displacement of the point from datum line.

ω = the circular frequency.

t = time

$$T = \frac{2\pi}{\omega} \text{ Sec.}$$

$$f = \frac{\omega}{2\pi} \text{ Cycle present.}$$

Wave form of Simple Harmonic Motion

Describing the velocity and acceleration at the vibration point can be determined by taking the first and second derivative of equation.

Velocity

$$\frac{du}{dt} = u_0 \omega \cos \omega t$$

Acceleration

$$\frac{d^2u}{dt^2} = -u_0 \omega^2 \sin \omega t$$

From studying this equation one can see that the displacement value $u(t)$ and the acceleration value are at a maximum when the velocity is equal to zero.

Reference example:

A particle moves in simple harmonic motion with a frequency of 3 oscillations/s (3.0HZ) and amplitude of 5.0cm.

(a) What total distance does the particle move during one cycle of its motion?

(b) What is its maximum speed? Where does it occur?

(c) Find the maximum acceleration of the particle where does it occur?

Solution: a (amplitude) = 5 cm, f (frequency) = 3Hz

Through a complete cycle the particle moves a distance

$$(a) \text{ Distance } d = 4A = 4 \times 0.5 \\ = 20.0 \text{ cm}$$

$$(b) X = A \cos \omega t$$

$$\text{Also, } f = \frac{\omega}{2\pi} \text{ or } \omega = 2\pi f$$

$$\mu = -A\omega \sin \omega t$$

$$\mu_{\max} = A\omega = A(2\pi f)$$

$$\mu_{\max} = (5) \left(2\pi \times \frac{1}{5} \right)$$

$$\mu_{\max} = 94.2 \text{ oscillation/sec}$$

$$\mu = \mu_{\max} \quad \text{for } x=0$$

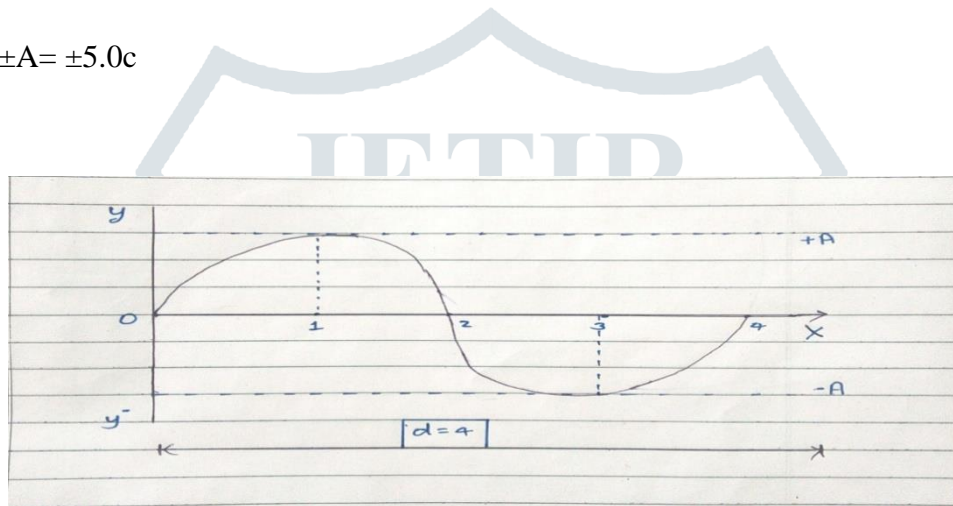
$$a = \frac{d\mu}{dt} = -A\omega^2 \sin\omega t$$

$$a_{\max} = A\omega^2 = A \left(4\pi^2 \times 9 \frac{1}{5} \right)$$

$$= 4447 \text{ cm/s}^2$$

$$= 44.4 \text{ m/s}^2$$

a_{\max} occur at $x = \pm A = \pm 5.0 \text{ cm}$



Graph: Implementation of the example

Hence, the diagram shows the implementation of the example in simple harmonic motion.

In that, value of d has been found out, the wave is uniform.

IMPLEMENTATION SYSTEM

Diagram:-

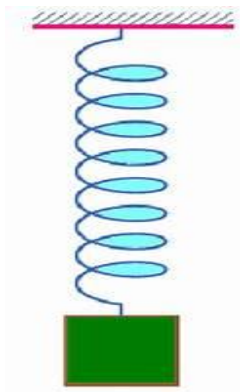


Fig.1:- Simple body a spring
Attached to a fixed wall.

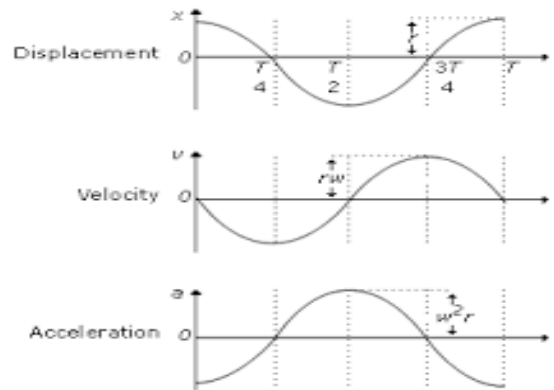


Fig .2:- Different variation in graphs of displacement,
Velocity & acceleration.

The diagram shows a simple body attached with a spring which is attached to a fixed wall. In that, the body is creating tension on the spring which is a cause for motion. The above diagram shows the different variation in graphs with different aspects like displacement, velocity & acceleration.

OBSERVATIONS

Advantages:

- 1) **Displacement** is a vector quantity in which initial and final position are not same. And the standard unit for displacement is meter.
- 2) **Velocity** is a vector which tells about the direction and rate of motion and standard units for velocity are meters per second. Speed and velocity are not the same quantity speed is scalar and velocity is vector quantity. One must use different rules when combining speeds and combining velocities. The average velocity of an object is the total displacement during some extended period of time, divided by that period of time. Instantaneous velocity, on the other hand, describes the motion of a body at one particular moment in time.
- 3) **Acceleration** is a vector which shows the direction and magnitude of changes in velocity. Its standard units are meters per second per second, or meters per second squared. Average acceleration is the total change in velocity (magnitude and direction) over some extended period of time, divided by the duration of that period.

In everyday English, we use the term decelerate to describe the slowing of a body, but physicists use the word accelerate to denote both positive and negative changes in speed.

CONCLUSION

In simple harmonic motion and its applications, solved different applications problems and solved analytically with exact equation of simple harmonic motion. We can calculate the periodic time value of oscillating an object from origin by these methods. Aspiring is an elastic object that stores mechanical energy. Springs are typically made of spring steel with many spring designs. In everyday use, the term often refers to coil springs. When a conventional spring, without stiffness variability features, is compressed or stretched from its resting position, it exerts an opposing force approximately proportional to its change in length (this approximation breaks down for larger deflections). The rate or spring constant of a spring is the change in the force it exerts, divided by the change in deflection of the spring.

REFERENCES

- [1] Werner, A. and Eliezer, J.C. (1969) the Lengthening Pendulum. *Journal of Australian Mathematical Society*, 9, 331- 336.
- [2] Little wood, J.E.(1963) Lorentz's Pendulum Problem. *Annals of Physics*, 21, 233-249.
- [3] Little wood, J.E. (1964) Adiabatic Invariance III. *The Equation & Annals of Physics*, 29, 1-12.
- [4] Little wood, J.E. (1964) Adiabatic Invariance IV: Note on a New Method for Lorentz's Pendulum Problem. *Annals of Physics*, 29, 13-18.
- [5] Little wood, J.E. (1964) Adiabatic Invariance V. Multiple Periods. *Annals of Physics*, 30, 138-153.
- [6] Brearley, M.N. (1966) The Simple Pendulum with Uniformly Changing String Length. *Proceedings of the Edinburgh. Mathematical Society (Series 2)*, 15, 61-66.
- [7] Sánchez-Soto, L.L. and Zoido, J. (2013) Variations on the Adiabatic Invariance: The Lorentz Pendulum. *American Journal of Physics*, 81, 57.
- [8] Boas, M.L. (2006) *Mathematical Methods in the Physical Science*. 3rd Edition, Wiley, 598-599.
- [9] Gil, A., Segura, J. and Temme, N. (2007) *Numerical Methods for Special Functions*. SIAM.
- [10] Garcia, A.L. (2000) *Method for Physics*. 2nd Edition, Prentice-Hall, NJ.