APPLICATION OF NEWTON'S LAW OF COOLING ESTIMATION OF TIME OF DEATH IN MURDER

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ABSTRACT

It is a major responsibility of a forensic department to find out the time of death. Our research paper shows a how to use Newton's Law of Cooling to calculate the time of death of someone. We try to find out time of death as soon as possible and it is based on four parameters: temperature of the body at time t, temperature of the surrounding area at time t, the weight of the body and the condition of the body. The equations are based on a first-order linear differential equation formulated by Newton. Many calculations can be done in order to find out what is the effect of above four points on the result.

• INTRODUCTION

We cannot find out the time of death with 100% correctness. If we witness death then only the time of death can be found, but a large amount of information is available so we can find out approximate time of death. We can say that if we do post-mortem in less time, then our estimation of time of death has short range. Vice versa, the longer time we take to do postmortem, range of estimation gets broader and chance of error increases dramatically. We cannot estimate the time just by doing one observation. Promising estimation is only made when a number of observations of the dead body are made. The surrounding environment and proper body observations can give us a reliable estimate of when the person died.

We need to find time of death in real world cases as death of a person can tell us many things and help to find the suspect and can be used as a proof for witnesses. One of the most important things in post-mortem is estimation of time of death.

- **History of Newton's Law of Cooling**

  The Newton's law of cooling is named after an English physicist Sir Isaac Newton who, in the late 17th century, conducted the first experiments on the nature of cooling. Specifically, noting that when the difference in temperature between the two bodies is small, approximately less than 10º C, than the rate of heat loss will be proportional to the temperature difference. The inaccuracy of Newton's law becomes considerable at high temperatures. The corrected Newton's law was formulated in 1817 by French Physicist Chemist Pierre Dulong and physicist Alexis Petit.

- **Newton's Law of Cooling**

Newton's law of cooling states that the rate of heat loss of a body is directly proportional to the difference in the temperatures between the body and its surroundings provided the temperature difference is small and the nature of radiating surface remains same.

Newton noted after some mathematical manipulation that the rate of temperature change of a body is proportional to the difference in temperatures between the body and its surroundings.

When stated in terms of temperature differences, Newton's law (with several further simplifying assumptions, such as a low Biot number and temperature-independent heat capacity) results in a simple differential equation for temperature-difference as a function of time. This equation has a solution that specifies a simple negative exponential rate of temperature-difference decrease, over time. This characteristic time function for temperature-difference behaviour, is also associated with Newton's law of cooling.

Equation Methodology

We will solve the first order linear differential equation to find the general solution. Then multiplying it with an integrating factor. We get an ordinary first order differential equation. After solving the equation, we have the value which is the time and after subtracting it with the time the body was found, we can get the time of murder.

Newton's law of cooling is stated mathematically as :

$$\frac{dT}{dt} = -k(T - T_a) \tag{3.01}$$

Where,

T : Temperature of the cooling object at time t

t : time in hours since the first reading

T_a : Temperature of surrounding medium (Ambient temperature)

k : Constant of proportionality

Solution of Newton's Equation

$$\frac{dT}{dt} = -k(T - T_a) \tag{3.02}$$

$$\frac{dT}{dt} + kT = kT_a \tag{3.03}$$

Multiplying both sides by the integrating factor e^{kt}

$$\frac{d}{dt}\left[T(t)e^{kt}\right] = kT_a e^{kt} \tag{3.04}$$

Integrating both sides with respect to t

$$\int (T(t)e^{kt})\,dt = \int kT_a e^{kt} dt \tag{3.05}$$

$$T(t)e^{kt} = T_a e^{kt} + C \tag{3.06}$$

$$T(t) = T_a + Ce^{-kt} \tag{3.07}$$

At $t = 0$, $T(0) = T_o$

$$T_o = T_a + C \quad (3.08)$$

$$C = T_o - T_a \quad (3.09)$$

$$T(t) = T_a + (T_0 - T_a)e^{-kt} \quad (3.10)$$

Table 1 Definitions of Variables used in the Model

Parameters	Definitions
k	Rate constant known as the cooling factor
C	Constant
T	Denotes temperature at any time
T_a	Denotes ambient temperature
M	Weight (Mass) of body
T_o	Denotes temperature at death (t=0)

- **Rate of Post Mortem Cooling**

Linear rate of post mortem cooling is affected by environmental factors and other than the environmental temperature and the body temperature at the time of death. These include:

The size of the body: The greater the surface area of the body relative to its mass, the more rapid will be its cooling. Consequently, the heavier the physique and the greater the obesity of the body, the slower will be the heat loss. The exposed surface area of the body radiating heat to the environment will vary with the body position. **Clothing and coverings**: These insulate the body from the environment and therefore, cooling is slower. Cooling of a naked body is half as fast as when clothed. **Movement and humidity of the air**: Air movement accelerates cooling by promoting convection and even the slightest sustained air movement is significant. Cooling is said to be more rapid in a humid rather than dry atmosphere because moist air is a better conductor of heat. The humidity of the atmosphere will affect cooling by evaporation where the body or its clothing is wet. **Immersion in water**: For a given environmental temperature, cooling in still water is about twice as fast as in air, and in flowing water, about three times as fast. Clearly this will cool more rapidly in cold water than warm water.

The following Assumptions are also made

In addition to the assumptions of the Newton's Model, the following are also made:

No strong radiation (e.g. sun, heater, cooling system), No uncertain severe changes of the cooling condition during the period between the time of death and examination. (The place of death must be the same as where the body was found), The Ambient temperature is maintained at 37.2^0C .

It is observed that the cooling of a body is being influenced initially by a phenomenon whose effect decays with time. They approximated it with an exponential function Ce^{-pt}.

The expression for the rate of cooling of a deceased body is

$$\frac{dT}{dt} = -k(T - T_a) + Ce^{-pt}$$

$$T(0) = T_o \qquad (2.11)$$

The first –order differential equation

$$\frac{dT}{dt} + k(T - T_a) = Ce^{-pt}$$

has an integrating factor $u(t) = e^{kt}$.

Real Life Applications:

EX-1: The coroner arrives on a scene of homicide at 10pm in a 70 degree room. She takes the temperature of the body and gets 92.4 degrees. It takes her 30 minutes to perform some other duties. She then takes the temperature again and finds it is now 91.8 degrees. Estimate the time of death of this homicide.

Solution: Given, $T(t) = 91.8$

$T_a = 70$

$T_0 = 92.4$

$t = 30$mins.

First we use the observed temperatures of the corpse to find the constant *k*. We have,

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

$$91.8 = 70 + (92.4 - 70)e^{-30k}$$

$$91.8 - 70 = 22.4e^{-30k}$$

$$\ln(21.8/22.4) = -30k$$

$$-0.02715/-30 = k$$

$$k = 0.0009$$

Now, assuming at the time of death his body temperature is $T_0 = 98.6$

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

$$92.4 = 70 + (98.6 - 70)e^{-0.0009t}$$

$$92.4 - 70 = (28.6)e^{-0.0009t}$$

$$\ln(22.4/28.6) = -0.0009t$$

$$-0.2443/-0.0009 = t$$

$$t = 271 \text{ mins OR } 4 \text{ hrs } 31 \text{ mins}$$

which means death occurred at 5:29 pm.

EX-2: Estimate the time of death of this homicide. The coroner arrives on a scene of homicide at 6 pm in a 74 degree room. She takes the temperature of the body and gets 93.5 degrees. It takes her 45 minutes to perform some other duties. She then takes the temperature again and finds it is now 92.9 degrees.

Solution: Given, $T(t)= 92.9$

$T_a = 74$

$T_0 = 93.5$

$t = 45$mins.

First we use the observed temperatures of the corpse to find the constant *k*. We have,

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

$$92.9 = 74 + (93.5 - 74)e^{-45k}$$

$$92.9 - 74 = 19.5e^{-45k}$$

$$\ln(18.9/19.5) = -45k$$

$$-0.0312/-45 = k$$

$$k = 0.0006$$

Now, assuming at the time of death his body temperature is $T_0 = 98.6$

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

$$93.5 = 74 + (98.6 - 74)e^{-0.0006t}$$

$$93.5 - 74 = (24.6)e^{-0.0006t}$$

$$\ln(19.5/24.6) = -0.0006t$$

$$-0.2323/-0.0006 = t$$

$$t = 387 \text{ mins OR } 6 \text{ hrs } 45 \text{ mins}$$

which means death occurred at 11.15 am.

EX-3: A coroner was called to the home of a person who had died during early morning. In order to estimate the time of death the coroner took the person's body temperature twice. At 9am the temperature was 85.7° F and at 9:30am the temperature was 82.8°F. The room temperature stayed constant at 70°F. Find the approximate time of death assuming the body temperature was normal at 98.6°F at the time of death.

Solution: Given, T(t)= 82.8

$T_a = 70$

$T_0 = 85.7$

t =30 mins.

First, we use the observed temperatures of the corpse to find the constant *k*. We have,

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

$$82.8 = 70 + (85.7 - 70)e^{-30k}$$

$$82.8 - 70 = 15.7e^{-30k}$$

$$\ln(12.8/15.7) = -30k$$

$$-0.204/ -30 = k$$

$$k = 0.006$$

Now, assuming at the time of death his body temperature is $T_0 = 98.6$

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

$$85.7 = 70 + (98.6 - 70)e^{-0.006t}$$

$$85.7 - 70 = (28.6)e^{-0.006t}$$

$$\ln(15.7/28.6) = -0.006t$$

$$-0.4490/ -0.006 = t$$

$$t = 74 \text{ mins OR } 1 \text{ hrs } 24 \text{ mins}$$

which means death occurred at 7:36 am.

EX-4: The police officer arrives on a scene of homicide at 1 pm in a 72 degree room. She takes the temperature of the body and gets 91.8 degrees. It takes him exact 35 minutes to perform some other duties. She then takes the temperature again and finds it is now 90.2 degrees. Estimate the time of death of this homicide.

Solution: Given, T(t)= 90.2

$T_a = 72$

$T_0 = 91.8$

t =35 mins.

First we use the observed temperatures of the corpse to find the constant *k*. We have,

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

$$90.2 = 72 + (91.8 - 72)e^{-35k}$$

$$90.2 - 70 = 19.8e^{-35k}$$

$$\ln(18.2/19.8) = -35k$$

$$-0.084/-35 = k$$

$$k = 0.0024$$

Now, assuming at the time of death his body temperature is $T_0 = 98.6$

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

$$91.8 = 72 + (98.6 - 72)e^{-0.0024t}$$

$$91.8 - 70 = (26.6)e^{-0.0024t}$$

$$\ln(21.8/26.6) = -0.0024t$$

$$-0.199/-0.0024 = t$$

$$t = 83 \text{ mins OR } 1 \text{ hrs } 38 \text{ mins}$$

which means death occurred at 11:22 am.

EX-5: An officer was called to the home of a person who had died during afternoon. In order to estimate the time of death the coroner took the person's body temperature twice. At 3:30 pm the temperature was 84.7^o F and after 25 mins the temperature was 81.5^{o}F. The room temperature stayed constant at 71^{o}F. Find the approximate time of death assuming the body temperature was normal at 98.6^{o}F at the time of death.

Solution: Given, $T(t) = 81.5$

$T_a = 71$

$T_0 = 84.7$

$t = 25$ mins.

First, we use the observed temperatures of the corpse to find the constant *k*. We have,

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

$$81.5 = 71 + (84.7 - 71)e^{-25k}$$

$$81.5 - 71 = 13.7e^{-25k}$$

$$\ln(10.5/13.7) = -25k$$

$$-0.266/-25 = k$$

$$k = 0.0106$$

Now, assuming at the time of death his body temperature is $T_0 = 98.6$

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

$$84.7 = 71 + (98.6 - 71)e^{-0.0106t}$$

$$84.7 - 71 = (27.6)e^{-0.0106t}$$

$$\ln(13.7/27.6) = -0.0106t$$

$$-0.7004 / -0.0106 = t$$

$$t = 66 \text{ mins OR } 1 \text{ hrs } 6 \text{ mins}$$

which means death occurred at 2:24 pm.

- **Literature Review**

Textbooks on heat transfer generally refer to Newton's law of cooling but they give no details of Newton's experiment. The purpose of the first part of this paper is to give details of Newton's work. His explanation of why he thought the law was correct, and the experiment that he did to confirm it, are still of interest. It is worth stressing that he did not write his law down in the form of an equation nor did he define or use the heat transfer coefficient. He was however the first to postulate that the rate of loss of temperature of a hot object, with air blowing past, is proportional to the temperature itself. The second part of the paper is an attempt to reconstruct Newton's transient cooling experiment using modern knowledge of heat transfer. It is necessary to allow for varying heat transfer coefficients and specific heats and hence a numerical approach has to be used on a computer. The output of the process is data for temperature versus time for the test section. The next step is to take this simulated cooling time data and analyse it using the same method Newton used, to produce the same type of estimated temperatures that he obtained. By modern standards his estimates of the melting point of various metals were too low. It has been suggested that this was because the metals were impure but a purely heat transfer explanation is shown to be more plausible. A simple extension of his explanation of why the law works is used to derive a result close to accepted modern equations for heat transfer coefficient.

- **Graph/Data Survey Chart**

From the chart in Figure 1, it is possible to see that there was very small difference in time between the known time of death and the estimated time of death.

KNOWN TIME OF DEATH	ESTIMATED TIME OF DEATH
11:11:00 AM	11:15:00 AM
5:15:00 PM	5:29:00 PM
7:40:00 AM	7:36:00 AM
11:00:00 AM	11:22:00 AM
2:30:00 PM	2:24:00 PM

Table 2 Summary of estimated time of death and known time of death

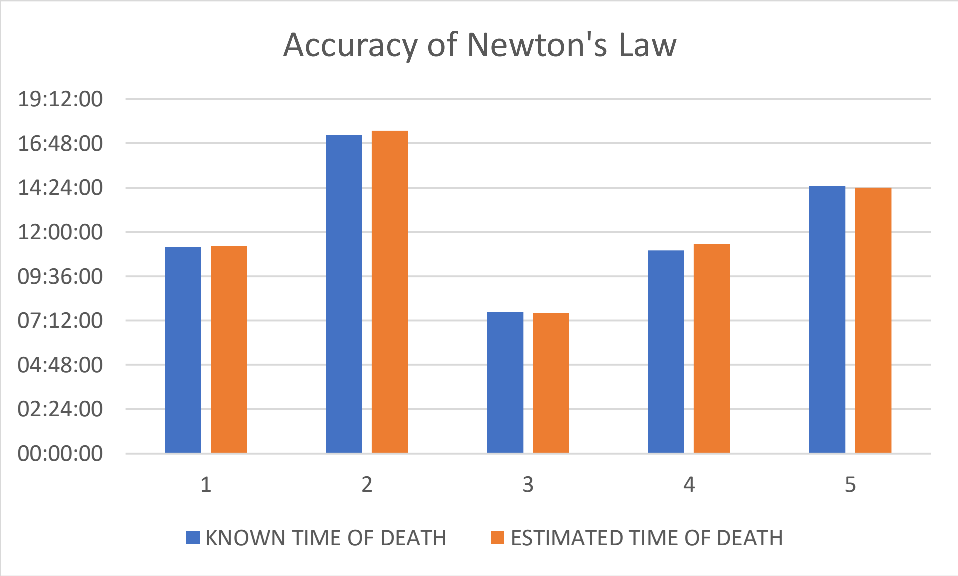

Figure 1 A chart of estimated time and known time of death

The result shows some level of discrepancies from the known time of death, in Chart 5.1. This happened as a result of temperature difference between the environmental (ambient) and body temperatures. When the post mortem period increased above 15hrs, the difference between the ambient temperature and the rectal temperature were very small. The Table gives a very small difference between the known time of death and the estimated time of death because the temperature difference between the ambient and body temperatures were big.

- **PROPOSE ENHANCEMENTS**

Newton's Law of Cooling is a great law. It can help us to find the time of death which will be very useful in case of murder or something related. So, we decided to choose this topic as it has a real-world application which can be helpful for Police Department, CID and FBI.Choosing something that is used by so many departments isn't a hard job, it was a no brainer.We thought that by choosing this topic we can understand the process of finding out the time of death, and we can share it with others.

- **CONCLUSION**

Estimation of time of death by using the model show that accuracy of results largely depends on the length of post-mortem period. From the analysis, it was observed that the shorter the post-mortem period, the more accurate is the estimate of time of death. The inaccuracy becomes considerable at low temperature difference between the rectal and ambient temperatures. Consequently, the longer the post mortem interval, the wider is the range of estimate as to when death probably occurred. We conclude that the Marshall and Hoare double exponential model is the most appropriate model for estimating the time of death in murder.

- **RECOMMENDATIONS**

The estimation of time of death by this method can be helpful in CID, FBI, CBI and various police departments. The determination of time of death could be studied to improve crime investigation to account for the long post mortem period and to accurately estimate time of death. This improvement can be used for scientific crime investigation.

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