Application of Torsional Pendulum and case study to determine torsional constant

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Abstract

A torsional pendulum, or torsional oscillator, consists of a disk-like mass suspended from a thin rod or wire. When the mass is twisted about the axis of the wire, the wire exerts a torque on the mass, tending to rotate it back to its original position. If twisted and released, the mass will oscillate back and forth, executing simple harmonic motion. This is the angular version of the bouncing mass hanging from a spring. This gives us an idea of moment of inertia.

Keywords: Damping, rotational motion, simple harmonic motion, torsional pendulum, torque.

Introduction

Torsion is a type of stress, which is easier to explain for a uniform wire or a rod when one end of the wire is fixed, and the other end is twisted about the axis of the wire by an external force. The external force causes deformation of the wire and appearance of counterforce in the material. If this end is released, the internal torsion force acts to restore the initial shape and size of the wire. This behavior is similar to the one of the released end of a linear spring with a mass attached.

Attaching a mass to the twisting end of the wire, one can produce a torsion pendulum with circular oscillation of the mass in the plane perpendicular to the axis of the wire.

Methodology

Consider a disk suspended from a torsion wire attached to its centre. See Fig. 1 This setup is known as a torsion pendulum. A torsion wire is essentially inextensible, but is free to *twist* about its axis. Of course, as the wire twists it also causes the disk attached to it to *rotate* in the horizontal plane. Let θ be the angle of rotation of the disk, and let $\theta = 0$ correspond to the case in which the wire is untwisted.

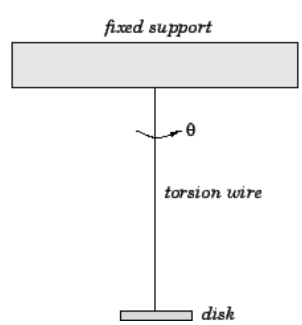


Fig.1 Torsional pendulum

To derive equations of rotational motion of the torsion pendulum, it would be useful to recall a resemblance of quantities in linear and rotational motion. We know that if initially a mass is motionless, its linear motion is caused by force F; correspondingly, if an extended body does not rotate initially, its rotation is caused by torque τ. The measure of inertia in linear motion is mass, m, while the measure of inertia in rotational motion is the moment of inertia about an axis of rotation, I. For linear and angular displacement in a one-dimensional problem, we use either x or θ. Thus, the two equations of motion are:

$F_x = ma_x$ and $\tau = I\alpha$ (1)

where ax and α are the linear and the angular acceleration. If the linear motion is caused by elastic, or spring, force, the Hooke's law gives F = -kx, where k is the spring constant. If the rotation is caused by torsion, the Hooke's law must result in

$\tau = -\kappa\theta$ (2)

where κ is the torsion constant, or torsional stiffness, that depends on properties of the wire. It is essentially a measure of the amount of torque required to rotate the free end of the wire 1 radian.

Where κ > 0 is the *torque constant* of the wire. The above equation is essentially a torsional equivalent to Hooke's law. The rotational equation of motion of the system is written above.

Equation (2) gives the following relationship between the moment of inertia I of an oscillating object and the period of oscillation T as:

$$I = \frac{\mathrm{T}}{2\pi} K$$

This relationship is true for oscillation where damping is negligible and can be ignored. Otherwise the relationship between I and κ is given by

$$I = \frac{K}{{\omega_o}^2}$$

Where, ω_o can be found from $\omega^2 = x = \sqrt{w_0 - \left(\frac{c}{2I}\right)^2}$ (3)

$\omega = \frac{2\pi}{\mathrm{T}} = 2\pi f$; f is the frequency of damped oscillation; and c is the damping coefficient.

The relationship between the torsion constant κ and the diameter of the wire d is given in (3) as

$$K = \frac{\pi G d^4}{32 l} \qquad (4)$$

Where, l is the length of the wire and G is the shear modulus for the material of the wire.

As any mechanical motion, the torsional oscillation is damped by resistive force originating from excitation of thermal modes of oscillation of atoms inside the crystal lattice of the wire and air resistance to the motion of the oscillating object. We can estimate the torque of the resistive force as being directly proportional to the angular speed of the twisting wire, i.e. the torque $\tau R = -\frac{c d\theta}{dt}$ (recall the drag force on mass on spring in viscose medium as R = -bv). Combining Equation (1), (2) and the expression for τR, we obtain the equation of motion of a torsional pendulum as follows:

$$\frac{Id^2\theta}{dt^2} + \frac{cd\theta}{dt} + K\theta = 0 \qquad (5)$$

The solution of Equation (5) is similar to the solution of the equation for damped oscillation of a mass on spring and is given by

$$\theta = Ae^{-\alpha t}\cos(\omega t + \varphi)$$

Where $\alpha = \frac{c}{2I}$

And $\alpha = \beta^{-1}$ *with* β being the time constant of the damped oscillation; c is the damping coefficient; ω is the angular frequency of torsional oscillation measured in the experiment; and φ can be made zero by releasing the object on the wire at a position of the greatest deviation from equilibrium. Equation (6) can be used to calculate c (damping coefficient) and β (time constant = amount of time to decay e times) with Data Studio interface and software.

Another important formula is $\alpha = \frac{\omega_o}{2Q}$, where Q is the quality factor and $\omega_o{}^2 = \frac{\kappa}{I}$ (seeEq.3'). The ratio

$Ʈ = \frac{\alpha}{\omega_o} = (2Q^{-1})$ is called the damping ratio.

Observations and Results

• The dimensional measurements made:

(1) Radius of the Disc R disc = 6.1cm

(2) Outer Radius of the Annulus Rout = 6.1cm

(3) Inner Radius of the Annulus R in = 5.0cm

(4) Diameter of the Brass Wire d brass = 0.68mm

(5) Diameter of the Steel Wire d steel = 0.44mm

(6) Mass of the Disc M disc = 919gm

(7) Mass of the ring M ring = 327gm

(8) Moment of Inertia of the Disc (theoretically) = I disc = 17097.995gm– cm^2

(9) Moment of Inertia of the Ring (theoretically) = I ring = 10171.335gm– cm^2

(10) Theoretically I ring I disc = 0.595.

• Measurements of Time period for various lengths using a disc hung on a brass wire is listed below

Table 1 : Various length of Dis hung on a brass wire

Sr .No	Length (cm)	Time for 7 oscillation (s)	Time period (s)
1.	43.4	43.40	6.20
2.	42.5	42.92	6.13
3.	36.7	39.31	5.61
4.	33.4	37.81	5.40
5.	28.7	35.28	5.04
6.	27.3	34.35	4.91
7.	14.7	25.75	3.68

Measurements of the Time period for various lengths of a brass wire with the ring with annulus are listed below

Table 2: Various length of Dis hung on a brass wire with the ring with annulus

S.No	Length (cm)	Time for osc.(s)	Time period (s)
1.	46.5	56.5	8.07
2.	40.8	53.49	7.64
3.	34.8	48.82	6.97
4.	30.4	45.81	6.54
5.	25.3	41.72	5.96
6.	19.9	37.28	5.33

• Measurements of Time Period for various lengths of steel wire with the disc hung are listed below

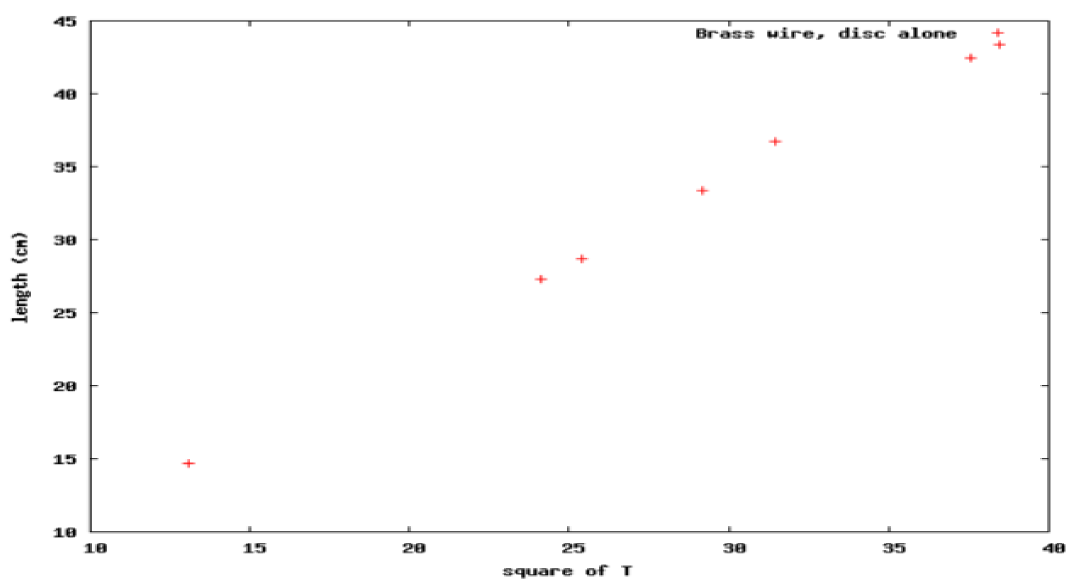
Table 3: Various lengths of steel wire with the disc hung

S.No	Length (cm)	Time for 7 osc.(s)	Time period (s)
1.	50.1	67.28	9.61
2.	47.5	66.83	9.55
3.	42.8	62.55	8.94
4.	38.3	61.07	8.72
5.	32.5	56.13	8.02
6.	28.9	51.84	7.41
7.	19.8	43.54	6.22

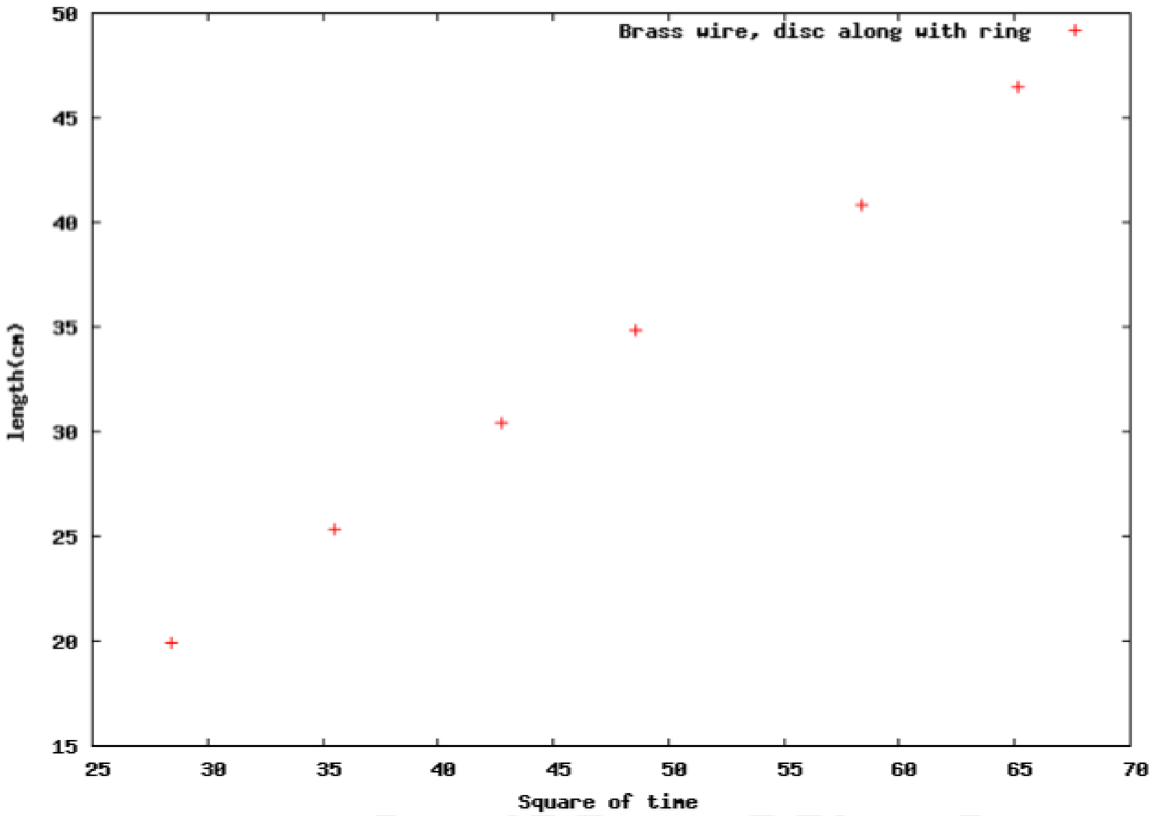
• Measurements of Time period for various lengths of steel wire with the disc and the annulus hung are listed below

Table 4 : Various lengths of steel wire with the disc and the annulus

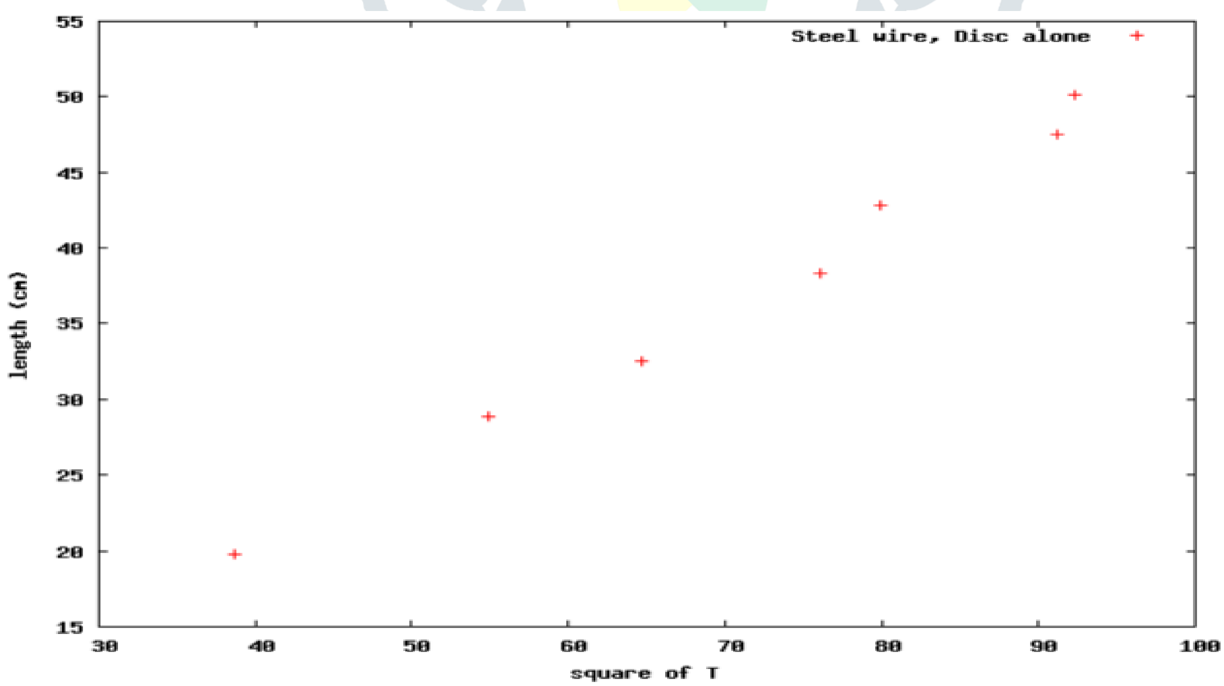
S.No	Length (cm)	Time for 7 osc.(s)	Time period (s)
1.	50.5	87.50	12.5
2.	41.1	80.56	11.51
3.	34.5	72.81	10.40
4.	21.7	57.96	8.28



Graph 1 : Based on observations of Table 1



Graph 2: Based on observations of Table 2



Graph 3 : Based on observations of Table 3

Experiment to find torsional constant

Throughout our experiment the pendulum is formed by attaching a wire to the centre of a meter stick with mass 2.0 kg. If the resulting period is 3.0 minute, then we have just find out torsional constant

$T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$ for our familiar mass on a spring simple pendulum

The similar period equation for a torsion pendulum is

$$T = 2\pi\sqrt{\left(\frac{i}{k}\right)}$$

K = torsion constant

$\tau = -\kappa\theta$

For a rod rotated about its centre,

$$I = \frac{1}{12} ML^2$$

$$I = \frac{1}{12} (2.0\ kg)(1.0m)^2 = 0.167kg.m^2$$

$$T = \frac{3\ \text{min}\ 60s}{\text{min}} = 180s$$

$$T = 180s = 2\pi = \frac{\sqrt{0.167kg.m^2}}{K}$$

$$(180\ m)^2 = 4\pi^2 0.167\ kg.m^2 K$$

$$32400s^2 = \frac{6.59\ kg.m}{32400}$$

$$K = 2.03 \times 10^{-4}\ N\text{-}m$$

$$K = 2.03 \times 10^{-4}\ N.m/rad$$

Conclusion

We conclude that when a torsion pendulum is perturbed from its equilibrium state (i.e. ϴ = 0) it executes torsional oscillations about this state at a fixed frequency ω, which depends only on the torque constant of the wire and the moment of inertia of the disk. Note, in particular, that the frequency is independent of the amplitude of the oscillation [provided ϴ remains small enough that Eq. (2) still applies]. Torsion pendulums are often used for time – keeping purposes. For instance, the balance wheel in a mechanical wristwatch is a torsion pendulum in which the resorting torque is provided by a coiled spring. And then we conclude here that after experimenting we used a massless disc the pendulum is formed by attaching a wire to the centre of a meter stick and found out torsional constant by the above given formulae.

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