Analysis of Imperfect Fault Coverage Reliability of a System

S.Kalaiarasi[1], G.Venkatesan[2], E.Venkatesan[3], N.Nithyapriya[4]

[1,2,3,4] PG & Research Department of Mathematics,
Sacred Heart College (Autonomous),
Tirupattur - 635601, Vellore, Tamil Nadu, India.
Email id: skalaiarasi2@gmail.com [1], venkatesang525@gmail.com [2]

Abstract : In this paper, we deal with the reliability analysis of a system with imperfect fault coverage. A system with two machines, one active with failure rate μ and other a cold standby spare which does not fail is considered. As an active machine fails, it is immediately replaced standby spare with coverage probability ***k***. The life time of a system is assumed to be exponentially distributed. The MTTF of a system and Reliability of a system are derived. Numerical results are also provided with illustrations.

Key words: Imperfect fault coverage, Cold standby, Reliability, MTTF.

1. INTRODUCTION

Reliability prediction is a key concern of the system engineer in any repairable system. Reliability can be defined for a specified period of time as the probability that component will perform its intended function under operating conditions. Complex systems are everywhere among us like as computer and telecommunication networks, manufacturing and production system, transportation, electrical appliances and many more are well-known real life examples. Designing reliable systems and determining their availability are both very important tasks for the system managers and analyst. Many empirical studies were suggested by [5] to improve the reliability of the concerned system.

In many practical scenarios, the production may be interrupted due to the unexpected failure of the machines which brings an undesirable loss of revenue as well as goodwill in the market. Such situation can be controlled up to some extent using the support of standby units and repairs. With the help of standbys, the system remains operative and continues to perform its assigned job in case of failure. The concept of standby support has attracted the attention of several researchers working in the area of machining and reliability theory. Reliability analysis [6] has been done for the machining system with warm standbys. Machine repair problem with standby was considered by [7, 8] in different frameworks. A simple and efficient algorithm [9] was given for computing the reliability and unreliability of systems subject to imperfect fault coverage. In this paper, we discuss the system reliability fault imperfect coverage. The perfect detection and recovery of an active machine is done with probability ***'k'*** which is known as coverage factor and imperfect detection and recovery has been done with complementary probability ***'1 − k'***.

2. BASIC DEFINITIONS

Definition 2.1 Reliability is the probability that an item will Cary out its function satisfactorily for the stated period when used according to the specified conditions.

Reliability is defined as

$$R(t)=\int_t^\infty f(x)dx$$

Definition 2.2 Failure Rate is the frequency with which an engineered system or component fails, expressed in failures per unit of time. It is often denoted by the Greek Letter λ and is highly used in Reliability Engineering.

Definition 2.3 When r sequential phases have identical exponential distributions, then the resulting density is known as ***r-stage (or r-phase) Erlang*** and is given by

$$f(t)=\frac{\lambda^2 t^{r-1}e^{-\lambda t}}{(r-1)!}, t>0, \lambda>0, r=1,2,3,...$$

Two-stage Erlang is

$$f(t)=\lambda^2 te^{-\lambda t}, t>0, \lambda>0$$

Definition 2.4 Mean time to failure is in the case of exponentially distributed times to failures, the sum of the operating time of given items over the total numbers of malfunctions or failures.

$$MTTF=E(T)=\int_0^\infty tf(t)dt$$

Definition 2.5 A cold standby is a redundancy method that involves having one system as a backup for another identical primary system. The cold standby system is called upon only on failure of the primary system.

3. MODEL ASSUMPTIONS

Let the lifetime of a system L with two machines M_1 and M_2 is spare and idle. The failure rate of the machine M_1 is μ and we assume that M_2 does not fail. The indicator random variable of the fault class is considered as G.

G = 0 if the failure is not covered,

G = 1 if the failure is covered.

Then

$p_G(0) = 1-k$ and $p_G(1) = k$.

Using the theorem of total expectation, we obtain the system MTTF

$$E[L] = \frac{1-k}{\mu} + \frac{2k}{\mu} = \frac{1+k}{\mu} \qquad (1)$$

The marginal density of L is computed by summing over the joint density :

$$f_L(t) = f(t,0) + f(t,1) = \mu^2 kte^{-\mu t} + \mu(1-k)e^{-\mu t}$$

Therefore, the system reliability is given by

$$R_L(t) = (1-k)e^{-\mu t} + ke^{-\mu t}(1+\mu t)$$

$$= e^{-\mu t} + k\mu te^{-\mu t}$$

$$= (1+k\mu t)e^{-\mu t} \qquad (2)$$

4. NUMERICAL EXAMPLE

A medical laboratory consists of two equipments, we assume that one is active, the other a cold standby spare and a cold spare does not fail. We calculate the mean life of the medical laboratory based on the failure rate of an active equipment. Also, we calculate Reliability of the medical laboratory

$$E = \frac{1+k}{\mu}$$

K \ μ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	10	5	3.3333	2.5000	2.0000	1.6667	1.4286	1.2500	1.1111	1.0000
0.2	12	6	4.0000	3.0000	2.4000	2.0000	1.7143	1.5000	1.3333	1.2000
0.4	14	7	4.6667	3.5000	2.8000	2.3333	2.0000	1.7500	1.5556	1.4000
0.6	16	8	5.3333	4.0000	3.2000	2.6667	2.2857	2.0000	1.7778	1.6000
0.8	18	9	6.0000	4.5000	3.6000	3.0000	2.5714	2.2500	2.0000	1.8000
1.0	20	10	6.6667	5.0000	4.0000	3.3333	2.8571	2.5000	2.2222	2.0000

$$R(t) = (1 + k\mu t)e^{-\mu t}, t = 1$$

K \ μ	0	0.2	0.4	0.6	0.8	1.0
0	1	0.8187	0.6703	0.5488	0.4493	0.3679
0.2	1	0.8515	0.7239	0.6147	0.5212	0.4415
0.5	1	0.9006	0.8044	0.7135	0.6291	0.5518
0.7	1	0.9334	0.8580	0.7793	0.7010	0.6254
1	1	0.9825	0.9384	0.8781	0.8088	0.7358

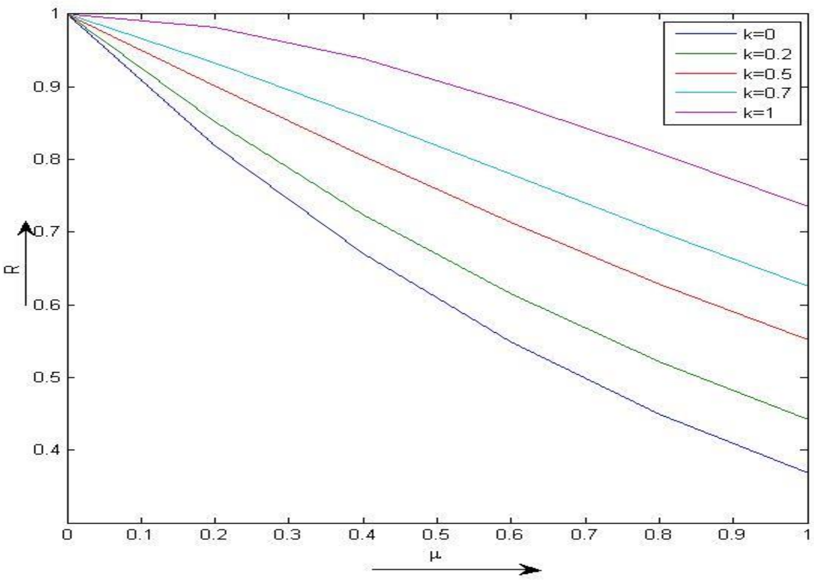

Figure 1. Reliability of system when t = 1

$$R(t) = (1 + k\mu t)e^{-\mu t}, t = 2$$

K \ μ	0	0.2	0.4	0.6	0.8	1.0
0	1	0.6703	0.4493	0.3012	0.2019	0.1353
0.2	1	0.7239	0.5212	0.3735	0.2665	0.1895
0.5	1	0.8044	0.6291	0.4819	0.3634	0.2707
0.7	1	0.8580	0.7010	0.5542	0.4280	0.3248
1.0	1	0.9384	0.8088	0.6626	0.5249	0.4060

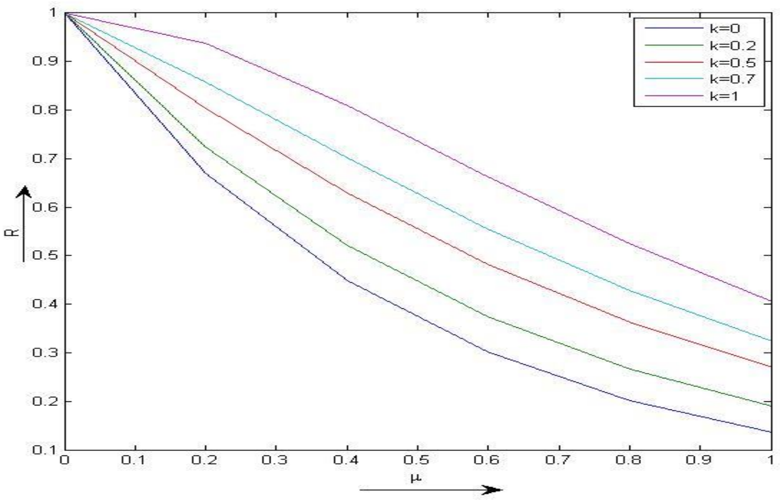

Figure 2. Reliability of system when t = 2

5. CONCLUSION

In this paper, we analyze the Reliability of the system based on the imperfect fault coverage of the unit. We determine the Reliability, Mean time to failure of the system, which would enable us to improve the effectiveness of the health care system and to care of patients safety. We observe that μ increases R(t) is decreases, MTTF rate is also decreases.

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