AN ANALYSIS ON SINGLE SERVER RETRIAL QUEUE WITH TWO TYPES OF SERVICE, SETUP TIME AND MULTIPLE VACATIONS

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Abstract : In this paper analysis two types of services are provided using single server and vacation by setting up time. Consider two types of arrival for customer to provide service. First using setup time, service is provided to the bulk customer. After completion of this, the normal service is provided to the regular customer on the FCFS basis. If the customer is dissatisfied with the service, both bulk and regular, customer may enter the pool the called Orbit. After completion of bulk and regular customer service is provided to the head the customer in the orbit. If the orbit becomes empty then the server goes to the vacation. If any one of the customer is bulk and regular and in the orbit then the server return to the service station. In the steady state, the probability of generating function, queue length has been obtained. Expected number of customer in the retrial group and expected waiting time of the customer in the orbit are also obtained. I also obtained for the performance measures of the system.

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Key Words: Single server, setup time, Multiple vacations Steady-state solution, Performance measures.

I. INTRODUCTION

There are many situations in daily life when queue is formed. For example people waiting before a counter in bank, departmental store, theater, machines waiting to be repaired etc. If the service required by the customer is not immediately available the queue is formed. Thus, Queueing theory deals with the problem of interest to achieve a balance between the cost associated with long waiting lines (queues) and the cost associated with the prevention of waiting in order to maximize the profit.

There are many ways to increase the quality of service, some of which includes increasing the resource (namely the service provides), increasing the service rates and controlling the arrival. Queues with repeated attempts have been widely used to model many problems in telephone switching systems, Computer and communication systems, Local area network and online shopping to purchase the customer due to the time period.

For detailed survey can Yang and Templeton (1987), Falin (1992) and Choi and Chang (1999) investigated an M/G/1 retrial queue with two types of customers in which the service time distribution for both types of customers are the same. Khalil et al.(1992) investigated the above model at Markovian level; in detail. Falin et al.(1993) investigated a similar model in which they assumed different service time distribution for the two types of customers. Kalyanaraman and srinivasan (2004) studied an M/G/1 retrial queue with geometric loss and with type I batches arrivals and type II single arrivals. In 2011 the authors have analyzed feedback retrial queueing systems with two types of arrivals and type I arrival being in batches of fixed size. In 2009 Pazhanibalamurugan and Kalyanaraman authors have analyzed a vacation queue with additional optional service in batches. In 2014, Rajadurai et al. analyzed on M/G/1 feedback retrial queue with subject to server breakdown and repair. In 1990 H Takagi studied time dependent analysis of M/G/1 vacation models with exhaustive service queueing system.

In 2000 Choudhury, analyzed on $M^{[x]}/G/1$ queueing system with a setup period and vacation period. In 2011 Song fang ja et al. studied discrete queue with vacation and setup time. In 2014 Shymala and Ayyappan, studied on analysis of retrial queueing system with second optional service random breakdown, setup time and Bernoulli vacation. This paper generalized the work of Shymala and Ayyappan, studied on an analysis of retrial queueing system with second optional service random breakdown, setup time and Bernoulli vacation. To the author's best of knowledge, there is no work published in the queueing literature with combination of an analysis on single server retrial Queue with two types of service, Setup time and multiple vacations. Some of the authors put their contribution on a setup time which plays a significant role in the study of queueing systems and which is defined as at every ends of the busy period the server enters into a random time process before actually providing service to a new customer or batch of customer joins the system in the renewed busy period.

The random setup time (during which no proper work is done) in order to set the system into operative mode before actual service begins; with an idea to set up starting time to provide service for bulk customers.

II. MATHEMATICAL DESCRIPTION OF THE MODEL:

The server made some setup time to provide service for the first arrive bulk customers only. The bulk customers arrive according to Poisson process with rate λ_b. If the bulk arrive customer finds the server busy in the setup time it becomes impatient and leaves the system with probability (1 - α) and with probability α, it enters into orbit.
Next, server gives the service to regular customer by FCFS basis. Arrival pattern of the regular customer is FCFS according to Poisson process with rate λ_r. The regular customer finds the server busy then leaves the system with probability (1- β) and with probability β goes to orbit. Service for all the above is in exponential distribution with rate $\mu_i, i=1,2$.
As soon as all the service is completed server may go to the vacation. A after the vacation server checks the system at least one customer in any server return to the system.
Server continues to the vacation with v_i or any one of the customer in the system with probability 1-v_i. This vacation time follows an exponential distribution with rate s. The service time distribution for both type of customers are independently distributed random variables with same distribution. A supplementary variable technique is used for the analysis and the variable considered is the residual service time of a customer in service.

The service time density functions are $b_i(x)$; i=1,2 and $B_k^*(s)=\int_0^\infty e^{-sx}b_k(x)dx, k=1,2$ is the Laplace transformation of the distribution function $b_k(x)$.

The vacation time density function is $v_i(x)$; i=1,2,… and $V_k^*(s)=\int_0^\infty e^{-sx}v_k(x)dx, k=1,2,...$ is the Laplace transformation of the distribution function $v_k(x)$.

The stochastic process related to the model is $X(t)=\{S(t), A_b(t), A_r(t), V(t); t\geq 0\}$

Where $A_b(t)$ =Number of bulk customer in the system at time t.

$A_r(t)$ = Number of regular customer in the system at time t.

V (t) = Server goes vacation at time t.

$$S(t)=\begin{cases}0, \text{ if the server is idle}\\ 1, \text{if the server is busy to the bulk customer due to setup time}\\ 2, \text{if the server is busy to the regular customer}\\ 3, \text{if the server go for vacation}\end{cases}$$

The process $\{N(t), X(t), w(t), t\geq 0\}$ is a continuous time Markov process. We define the probabilities,

$$P_{0,0}(t)=P\{S(t)=1, N(t)=0\},$$
$$P_{1,n}(t)dx=P\{S(t)=1, N(t)=n, x\leq \tau_1(t)<x+dx\}, x\geq 0, n\geq 1,$$
$$P_{2,n}(t)dx=P\{S(t)=2, N(t)=n, x\leq \tau_2(t)<x+dx\}, x\geq 0, n\geq 0,$$
$$P_{3,n}^k(t)dx=P\{S(t)=3, N(t)=n, x\leq \tau_3(t)<x+dx\}, x\geq 0, n\geq 0, k=1,2,...M$$

III. TRANSIENT STATE EQUATIONS:

Let N(t) be the system size of the server at time t. Let $A_b^0(t)$, $A_r^0(t), V_k^0(t)$, denote the elapsed times of retrial units, service time and vacation state at time t.

We introduced the supplementary variable, $w(t)=\begin{cases}A_b^0(t) \text{ if } C(t)=0,1\\ A_r^0(t) \text{ if } C(t)=2\\ V_k^0(t) \text{ if } C(t)=3\end{cases}$

Where $A_b^0(t)$ elapsed service time of the bulk customer arrives at time t. $A_r^0(t)$ elapsed service time of the regular customer in service at time t. $V_k^0(t)$ elapsed vacation time of the server when he is on multiple vacation. The process $\{S(t), N(t), w(t); t\geq 0\}$ is a continuous time Markov process.

To Analysis the mathematical model of queueing system, the transient probabilities are defined as:

(i) $S_n(t)$ = Probability that at time t, the server is in setup time while there are bulk customers in the queue;

(ii) $P_n^{(1)}(x,t)$ = Probability that at time "t" the server is active providing bulk customer's service and there are "n" ($n \geq 1$) customers in the queue excluding the one being served and the elapsed service time for this customer are x.

(iii) $P_n^{(2)}(x,t)$ = Probability that at time "t," the server is active providing regular service and there are "n" ($n \geq 1$) customers in the queue excluding the one being served and the elapsed service time for this customer are x.

(iv) V(x, t) = probability that at time "t," the server is on vacation with elapsed vacation time x, and there are "n" ($n \geq 0$) customers waiting in the queue for service.

(v) Q(t) = probability that at time "t," there are no customers in the system and the server is idle and available in the system.

IV. GOVERNING EQUATIONS:

The queueing model is then governed by the following set of differential-difference equations:

$$\frac{d}{dt}S_n(t) = -(\lambda+v)S_n(t) + \lambda\sum_{i=1}^{n} c_i S_{n-i}(t) + \lambda c_n Q(t), n \geq 1 \quad \text{.....(1)}$$

$$\frac{\partial}{\partial t}P_n^{(1)}(x,t) + \frac{\partial}{\partial x}P_n^{(1)}(x,t) + (\lambda+\mu_1(x)+\alpha) + P_n^{(1)}(x,t) = \lambda\sum_{i=1}^{n-1} c_i P_{n-1}^{(1)}(x,t), n \geq 1 \quad \text{.....(2)}$$

$$\frac{\partial}{\partial t}P_n^{(2)}(x,t) + \frac{\partial}{\partial x}P_n^{(2)}(x,t) + (\lambda+\mu_2(x)+\alpha) + P_n^{(2)}(x,t) = \lambda\sum_{i=1}^{n-1} c_i P_{n-1}^{(2)}(x,t), n \geq 1 \quad \text{.....(3)}$$

$$\frac{\partial}{\partial t}V_n(x,t) + \frac{\partial}{\partial x}V_n(x,t) + (\lambda+v(x))V_n(x,t) = \lambda\sum_{i=1}^{n-1} c_i V_n(x,t), n \geq 1 \quad \text{.....(4)}$$

$$\frac{\partial}{\partial t}V_0(x,t) + \frac{\partial}{\partial x}V_0(x,t) + (\lambda+v(x))V_0(x,t) = 0 / \quad \text{.....(5)}$$

$$\frac{d}{dt}Q(t) = -\lambda Q(t) + \int_0^\infty V_0(x,t)v(x)dx + \int_0^\infty P_1^2(x,t)\mu_2(x)dx \quad \text{.....(6)}$$

The above set of equation can be solved using the steady state boundary condition at x=0,

$$P_n^{(1)}(0,t) = \int_0^\infty V_n(x,t)v(x)dx + (1-p)\int_0^\infty P_{n+1}^{(2)}(x,t)\mu_2(x)dx + vS_n(t), n \geq 1, \quad \text{.....(7)}$$

$$P_n^{(2)}(0,t) = \int_0^\infty P_n^{(1)}(x,t)\mu_1(x)v(x)dx, n \geq 1, \quad \text{.....(8)}$$

$$V_n(0,t) = v\int_0^\infty P_n^{(2)}(x,t)\mu_2(x)dx, n \geq 0, \quad \text{.....(9)}$$

The normalization condition is given by

$$P_0 + \sum_{n=1}^{\infty}\int_0^\infty P_n(x)dx + \sum_{n=1}^{\infty}\int_0^\infty Q_{n,b}(x)dx + \sum_{n=1}^{\infty}\int_0^\infty Q_{n,v}(x)dx = 1, \quad \text{.....(10)}$$

Let us define the probability generating functions as,

$$P_q^{(i)}(x,z,t) = \sum_{n=0}^{n} P_n^{(i)}(x,t) \;;\; P_q^{(i)}(z,t) = \sum_{n=0}^{\infty} P_n^{(i)}(t); i = 1,2, \; P_q^{(i)}(z,t) = \sum_{n=0}^{\infty} P_n^{(i)}(t); i = 1,2$$

$$V_q(x,z,t) = \sum_{n=0}^{\infty} z^n V_n(x,t), V_q(z,t) = \sum_{n=0}^{\infty} z^n V_n(t); S_q(z,t) = \sum_{n=0}^{\infty} z^n S_n(t)$$

On multiplying the equations (3) by $\alpha + \sum_{n=1}^{\infty}\alpha^{m+1}$ times equation (1) on multiplying the equations (4) by $\alpha + \sum_{n=1}^{\infty}\alpha^{m+1}$ times equation (2), on multiplying the equations (6) by $\alpha + \sum_{n=1}^{\infty}\alpha^{m+1}$ times equation (5) rearranging the terms we have,

$$\{[\lambda(1-\alpha)+\mu_1]\alpha - \mu_1 q\}P_n^{(1)}(\alpha) = \mu_1 p P_{n-1}^{(1)}(\alpha) - \mu_1 q P_n^{(1)}(\alpha) - \mu_1 p P_{n-1}^{(1)}(\alpha), (0 < n \leq b-1) \quad \text{...(11)}$$

$$\{[\lambda(1-\alpha)+\mu_1]\alpha - \mu_1 q\}P_0^{(1)}(\alpha) = \mu_2 P_0^{(2)}(\alpha) + \theta V_0(\alpha) - (\lambda+\theta)V_0 - \lambda(1-\alpha)P_0^{(1)} \quad \text{.....(12)}$$

$$[\lambda(1-\alpha)+\mu_2]P_0^{(2)}(\alpha)=\mu_1 pP_{b-1}^{(1)}(\alpha)+\mu_1 p\sum_{n=1}^{b-2}P_n^{(1)} \quad \text{.....(13)}$$

Equation (8) + $\sum_{n=1}^{b-1}\alpha^n$ times equation (7), we have

$$[\lambda(1-\alpha)+\theta]V_0(\alpha)=(\lambda+\theta)V(0,0) \quad \text{.....(14)}$$

Performing equation (12) + $\sum_{n=1}^{b-1}\beta^n$, times equation (11) and rearranging the term and simplifying, we have,

$$P^{(1)}(\alpha,\beta)=\frac{\mu_2 P_0^{(2)}(\alpha)+\theta V_0(\alpha)-(\lambda+\theta)V(0,0)-\lambda(1-\alpha)P_0^{(1)}-\mu_1(q+p\beta)P_0^{[1]}(\beta)+\mu_1 qP_0^{(1)}}{[\lambda(1-\alpha)+\mu_1]\alpha-\mu_1(q+p\beta)} \quad \text{.....(15)}$$

Let au assume $\beta=0, P^{(1)}(\alpha,\beta)=P_0^{(1)}(\alpha)$ and $P_0^1(0)=P_{0,0}^{(1)}$ .Thus for $\beta=0,$ equation (15) gives,

$$P_0^{(1)}(\alpha)=\frac{\mu_2 P_0^{(2)}(\alpha)+\theta V_0(\alpha)-(\lambda+\theta)V_{00}-\lambda(1-\alpha)P_{0,0}^{(0)}}{[\lambda(1-\alpha)+\mu_1]\alpha-\mu_1 q} \quad \text{.....(16)}$$

Equation (13) for b=1 gives

$$P_0^{(2)}(\alpha)=\frac{\mu_1 pP_0^{(1)}(\alpha)}{\lambda(1-\alpha)+\mu_2} \quad \text{.....(17)}$$

Substituting equations (14) and (17) in (16) and simplifying we have

$$P_0^{(1)}(\alpha)=\frac{\lambda[\lambda(1-\alpha)+\mu_2]\{(\lambda+\theta)V_{00}+[\lambda(1-\alpha)+\theta]P_{0,0}^{(0)}\}}{[\lambda(1-\alpha)+\theta][\lambda^2\alpha^2-\lambda(\lambda+\mu_1+\mu_2)\alpha+\mu_1(\lambda q+\mu_2)]} \quad \text{.....(18)}$$

Using equation (18) in (17), we have

$$P_0^{(2)}(\alpha)=\frac{\mu_1 p\dfrac{\lambda[\lambda(1-\alpha)+\mu_2]\{(\lambda+\theta)V_{00}+[\lambda(1-\alpha)+\theta]P_{0,0}^{(0)}\}}{[\lambda(1-\alpha)+\theta][\lambda^2\alpha^2-\lambda(\lambda+\mu_1+\mu_2)\alpha+\mu_1(\lambda q+\mu_2)]}}{\lambda(1-\alpha)+\mu_2} \quad \text{.....(19)}$$

V. THE STEADY STATE ANALYSIS

In this section, we will derive the steady state probability distribution for our queueing model. To define the steady state probabilities, suppress the argument "*t*" where ever it appears in the time dependent analysis. By using well known Tauberian property,

$\lim_{s\to 0} sf(s)=\lim_{t\to\infty} f(t)$ we get $S_q(z)=\dfrac{\lambda QC(z)}{\lambda-\lambda C(z)+v}$

We define $P_q(\alpha)=S_q(z)+P_{0,0}^{(0)}+V_0(\alpha)+P^{(1)}(\alpha,\beta)+P_0^{(1)}(\alpha)+P_0^{(2)}(\alpha)$ .....(20)

To replace all the above value we get,

$$P_q(\alpha)=\frac{A}{B}[P_{00}^0] \quad \text{.....(21)}$$

Where A= $\left\{\dfrac{\lambda}{\theta}(\lambda+\theta)+[\lambda(1-\alpha)+\theta]\right\}\left\{\lambda^2(\alpha^2-2\alpha+1)+\lambda(\mu_1+\mu_2)(1-\alpha)+\mu_1\mu_2\right\}$

B= $[\lambda(1-\alpha)+\theta][\lambda^2\alpha^2-\lambda(\lambda+\mu_1+\mu_2)\alpha+\mu_1(\lambda q+\mu_2)]$, Now, we determine unknown P_{00}^0 by substituting $\alpha=1$ in (21) we have $P_{00}^0=\left[1-\dfrac{\lambda}{\mu_1}\left(\dfrac{p\mu_1}{\mu_2}+1\right)\right]\left[\dfrac{\theta^2}{\lambda(\lambda+\theta)+\theta^2}\right]$ .....(22)

Which implies that the utilization factor is $\rho=\dfrac{\lambda}{\mu_1}\left(\dfrac{p\mu_1}{\mu_2}+1\right)$ and the steady state condition is therefore $\dfrac{\lambda}{\mu_1}\left(\dfrac{p\mu_1}{\mu_2}+1\right)<1$ .....(23)

VI. PERFORMANCE MEASURES:

(i) The Mean Number in the system

Now, we define $P_q(z)$ as the probability generating function of the queue size. Then we have $P_q(z) = S_q(z) + V_q(z) + z(P_1(z) + P_2(z) + P_3(z))$

Let Lq and L denote the steady state average queue size and system size respectively. We have

$$L_q = \lim_{z\to 1} \frac{d}{dz} P_q(z) = P_q^{'}(1) = \lim_{z\to 1} \frac{B(z)A'(z) - B'(z)A(z)}{(B(z))^2} \frac{V_0(0)}{\lambda},$$

We have obtained L_q in closed form. Further, we find the average system size L using little formula.

Thus we have $L = L_q + \rho$.

(ii) The Mean Waiting Time :

Let W_q and W denote the mean waiting time in the queue and the system respectively. Then using little's formula, we obtain

$$W_q = \frac{L_q}{\lambda}$$

***CONCLUSION*:**

This paper clearly analyses the two types of services provided bulk and regular customer, steady state results and the various performance measures of the queueing system with two stages of service with multiple server vacation, server provides essential service in all the stages to the arriving customers. .Further performance measures like average number of customers in the queue and the average waiting time of a customer in the queue are obtained. This model can be utilized in large scale manufacturing industries and communication networks.

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