DC-DC Buck Converter using Sliding Mode Control

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***Abstract*— DC-DC converters are power electronic converters that convert one level of DC voltage to other level using switching action. They are widely used in industrial applications such as DC motor drives, computer systems, adapters, communication equipments. One of the main problems of DC-DC converters is the adequate selection of control scheme because these systems are non-linear and show complex behavior patterns. Crucial to the performance of power converter, is the choice of control methods. A linear PID controller designed for regular operation cannot give satisfactory performance when the operating point changes due to line variation and load variation. Sliding Mode Control has gained importance as a design tool for the robust control of linear and non-linear systems. SMC provides inherent order reduction, robustness against system uncertainties, disturbances, and implicit stability proof. The design allows a high-performance control system at low cost. The application of the Sliding Mode Control techniques to DC-DC converter is analyzed with respect to buck converter using MATLAB/SIMULINK™ based simulation.**

Index Terms—Buck Converter, DC-DC Converter, Sliding Mode Control (SMC),
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I. INTRODUCTION

The DC-DC converters are power converters that convert one level of DC voltage to another level of DC voltage using switching action. The DC-DC conversion technology has been developing very quickly, besides they are extensively employed in industrial applications such as dc motor drives, computer systems, adapters and communication equipment. DC-DC converter circuits are utilized in a controllable and lossless DC voltage transformation, offering voltage isolation through the incorporation of a high-frequency electrical device.

One of the main problems in Power Electronics is the adequate selection of the control scheme for switch mode DC-DC converters, as these systems are nonlinear and show complex behavior patterns. The significant part of the performance of power converters is the choice of control methods. Traditional frequency-domain analog methods are mainly used in compensator design. There are a few drawbacks that obstruct the execution of analog controllers such as temperature float of the segments, the necessity for modifying numerous physical parts, and susceptibility to electromagnetic impedance (EMI). To overcome these limitations, four class of non-linear controllers are employed which are Robust, adaptive, Fuzzy and Neural controller. An essential alternative to linear controllers in the field of power electronics are Intelligent (Fuzzy Logic and Neural Network) control techniques[1].

After pioneer study of DC-DC converters, a great deal of effort has been directed in developing the modeling and control methods of various DC-DC converters[7]. The classic linear approach depends on the state averaging methods to get the state-space averaged equations. Specific Perturbations are introduced into the state variables around the operating point from the state-space averaged model, and small-signal state-space equations are therefore derived. Linear transfer functions of the open-loop plant can be obtained based on the equations. A linear controller is easy to design with these necessary transfer functions based on the small-signal state-space equations. The procedure is well known. However, the stability under significant variations of state condition changes cannot be ensured with these methods.

Sliding mode control is a powerful methodology that produces a very robust closed-loop system under plant uncertainties and external disturbances because the sliding mode control can be designed entirely independent of these effects. This technique offers several benefits compared to old control methods, they are Stability, even for broad line and load variations, excellent dynamic response and simple implementation. The SMC has been designed to improve the toughness, and dynamic response in switch mode power supplies as SMC is a control approach that complies with non-linear nature of switch mode power supplies[1]-[4].

II. MODELLING OF BUCK CONVERTER

The power processor usually consists of more than one power conversion stage. Each power conversion refers as a converter. It is a basic module of power electronic systems. It utilizes power semiconductor devices and possible energy storage elements such as inductors and capacitors.

The DC-DC converters converts fixed dc input voltage to a controllable dc output voltage. They are widely used in regulated switch-mode power supplies, in DC motor drive applications, subway cars, trolley trucks, and battery-driven vehicles.

A. Buck Converter

The DC-DC buck converters are widely used in regulated switch-mode power supplies and dc motor drive application. They provide smooth acceleration control, high efficiency, and fast dynamic response. The converter switch can be executed by using power Bipolar Junction Transistor (BJT), power Metal Oxide Semiconductor Field Effect Transistor (MOSFET), Gate Turn Off thyristor (GTO), or Insulated Gate Bipolar Transistor (IGBT). Although a dc converter can be operated either at a fixed or variable frequency, it is usually operated at a fixed frequency with a variable duty cycle.

The buck converter, also known as a step-down converter, is a switching converter that produces an output voltage lower than the dc source voltage. The buck converter has four primary components, namely a power semiconductor switch, a diode, an inductor, and a capacitor[7].

In a DC-DC buck converter, with a given input voltage, the average output voltage is controlled by controlling switch ON and switch OFF time period. Switch S operates at a fixed frequency, period T and with a duty cycle D. The inductor current is assumed to be continuous, circuit components to be lossless and output capacitor ripple voltage is negligible.

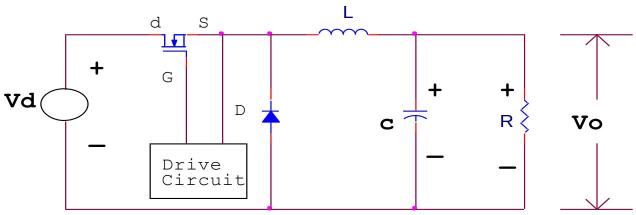

Fig. 1 Ideal DC-DC Buck converter

The average output voltage can be calculated in terms of switch duty ratio as

$$V_o = \frac{1}{T_s}\int_0^{T_s} v_o(t)\,dt$$

$$= \frac{1}{T_s}\left[\int_0^{t_{on}} V_d\,dt + \int_{t_{on}}^{T_s} 0\,dt\right] = \frac{t_{on}}{T_s} \times V_d \quad (1)$$

Relationship between steady state output voltage and duty ratio D is

$V_o = DV_d$

Where $D = t_{on}/T_s$ (2)

t_{on} is the ON time of the switch and T_s is the switching period of the switch.

When switch S is ON, diode D becomes reverse biased, and input supply provides energy to the load and to the inductor L. During the interval when the switch is turned OFF, the inductor current flows through the diode, transmitting some of its energy to the load.

When switch S is ON for time ton, switch S conducts the inductor current and reverse biases the diode D. This results in a positive voltage VL = Vd - Vo across the inductor. This voltage increases the inductor current linearly. When switch S is turned OFF, because of inductor energy storage, inductor current continues to flow. This current now flows through the diode, and now voltage across the inductor is VL = -VO.

In continuous conduction mode, the output voltage varies linearly with the duty ratio of a switch for given input voltage and does not depend on any other circuit parameters. In continuous conduction mode, the buck converter is equivalent to a DC transformer where turns ratio can be continuously controlled in a range of 0-1 by controlling the duty ratio of the switch.

B. State Space averaged Model of dc-dc buck converter

The switching operation of the power electronic converters results in the circuit components connected in periodically changing configurations. A different set of equations is describing each configuration. Since many equations must be solved in sequence the transient analysis and control design for converters is complicated. The technique of averaging helps in solving the problem. A single equation may be used to describe the behavior of converter over several switching cycles by taking the average of the separate equations for each switched configuration of the converter. State space averaging is the most commonly used method and is used to model the dc-dc converters.

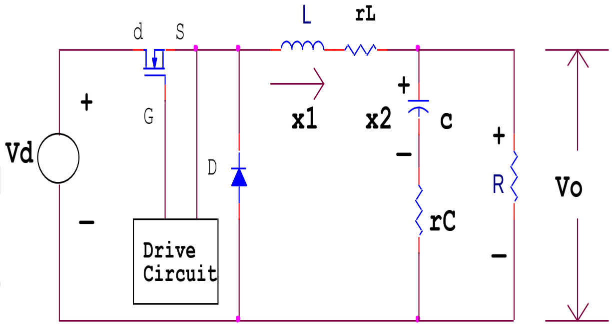

Fig. 3 DC-DC Buck Converter

Consider a practical buck converter shown in fig 3. Let current through the inductor, x1 and voltage across the capacitor, x2 are the state variables.

The steady-state operation of a buck converter in continuous conduction mode consists of two circuit modes.

Mode 1: switch is ON, the current flows from supply to the load

Mode 2: switch is OFF, the load current continues to flow through freewheeling diode D

<u>Mode 1</u>

The buck converter circuit when switch S is ON is shown in Fig. 4.

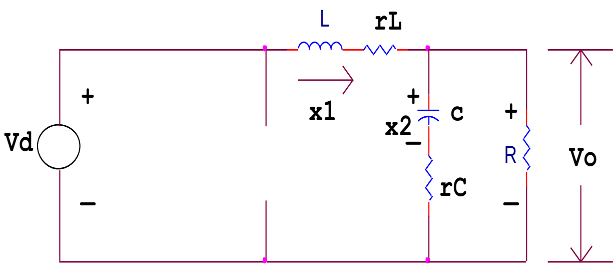

Fig. 4 Circuit of Buck Converter when switch S is ON

To get state space model

$$-V_d + L\dot{x}_1 + r_L x_1 + R(x_1 - C\dot{x}_2) = 0 \quad (3)$$

$$-x_2 - Cr_c\dot{x}_2 + R(x_1 - C\dot{x}_2) = 0$$

(4)

In matrix form the above two equations can be written as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} -\frac{(Rr_C + Rr_L + r_C r_L)}{L(R+r_C)} & -\frac{R}{L(R+r_C)} \\ \frac{R}{C(R+r_C)} & -\frac{1}{C(R+r_C)} \end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}\frac{1}{L}\\ 0\end{bmatrix} V_d$$

(5)

So system matrix and input matrix are:

$$A_1 = \begin{bmatrix} -\dfrac{(Rr_C + Rr_L + r_C r_L)}{L(R+r_C)} & -\dfrac{R}{L(R+r_C)} \\ \dfrac{R}{C(R+r_C)} & -\dfrac{1}{C(R+r_C)} \end{bmatrix} \quad (6)$$

$$B_1 = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} \quad (7)$$

Output equation,

$$v_o = R(x_1 - C\dot{x}_2)$$

$$= \begin{bmatrix} \dfrac{Rr_C}{R+r_C} & \dfrac{R}{R+r_C} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (8)$$

So output matrix and transmission matrix are

$$C_1 = \begin{bmatrix} \dfrac{Rr_C}{R+r_C} & \dfrac{R}{R+r_C} \end{bmatrix} \quad (9)$$

$$D_1 = 0 \quad (10)$$

Mode 2

The buck converter circuit when switch S is OFF is shown in Fig. 5. (Assume ideal diode)

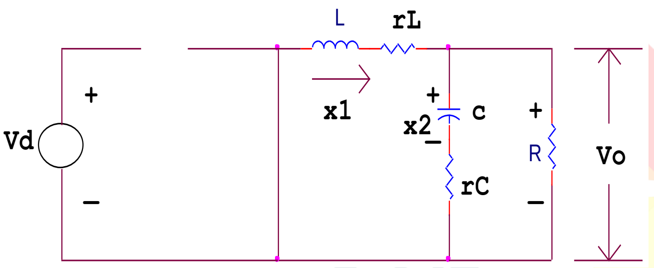

Fig. 5 Circuit of Buck Converter when switch S is OFF

To get state space model,

$$L\dot{x}_1 + r_L x_1 + R(x_1 - C\dot{x}_2) = 0 \quad (11)$$

$$-x_2 - Cr_c\dot{x}_2 + R(x_1 - C\dot{x}_2) = 0 \quad (12)$$

In matrix form the above two equations can be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{(Rr_C + Rr_L + r_C r_L)}{L(R+r_C)} & -\dfrac{R}{L(R+r_C)} \\ \dfrac{R}{C(R+r_C)} & -\dfrac{1}{C(R+r_C)} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (13)$$

So system matrix and input matrix are

$$A_2 = \begin{bmatrix} -\dfrac{(Rr_C + Rr_L + r_C r_L)}{L(R+r_C)} & -\dfrac{R}{L(R+r_C)} \\ \dfrac{R}{C(R+r_C)} & -\dfrac{1}{C(R+r_C)} \end{bmatrix} \quad (14)$$

$$B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (15)$$

To get output equation

$$v_o = R(x_1 - C\dot{x}_2)$$

$$= \begin{bmatrix} \dfrac{Rr_C}{R+r_C} & \dfrac{R}{R+r_C} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (16)$$

So output matrix and transmission matrix are

$$C_2 = \begin{bmatrix} \dfrac{Rr_C}{R+r_C} & \dfrac{R}{R+r_C} \end{bmatrix} \quad (17)$$

$$D_2 = 0 \quad (18)$$

To produce an average description of the circuit over a switching period, the equations corresponding to the foregoing states are time weighted and averaged, resulting in the following equations

$$\dot{x} = [A_1 d + A_2(1-d)]x + [B_1 d + B_2(1-d)]v_d \quad (19)$$

$$v_o = [c_1 d + c_2(1-d)]x \quad (20)$$

Small ac perturbations, represented by ~ are introduced in the dc steady-state quantities (which are represented by the uppercase letters). Therefore

$$x = X + \tilde{x} \quad (21)$$

$$v_o = V_o + \tilde{v}_o \quad (22)$$

$$d = D + \tilde{d} \quad (23)$$

$$v_d = V_d \quad (24)$$

Using (21) through (24) in (19) and recognizing that in steady state $\dot{X} = 0$

[illegible] (25)

The steady state equation can be obtained from (25) by setting all the perturbation terms and their time derivatives to zero.

[illegible] (26)

Similarly using (21) through (24) in (20)

[illegible] (27)

The steady state output voltage is given by

[illegible] (28)

Transfer function of the system can be obtained as

$$\frac{\tilde{v}_o(s)}{\tilde{d}(s)} = C_{sys}\left[SI - A_{sys}\right]^{-1} B_{sys} \qquad \text{Since } D_{sys} = 0 \quad (29)$$

$$\frac{\tilde{v}_o(s)}{\tilde{d}(s)} \cong V_d \frac{(1 + SCr_c)}{LC\left\{S^2 + S\left[\dfrac{1}{RC} + \dfrac{(r_c + r_L)}{L}\right] + \dfrac{1}{LC}\right\}} \quad (30)$$

III. Sliding Mode Control

There will be differences between the actual plant and the mathematical model developed for controller design in the formulation of any control problem. This mismatch may be due to the variation in system parameters or the approximation of complex plant behavior by a straightforward model.

It must be ensured that the resulting controller has the ability to produce the required performance levels in practice despite the plant/model mismatches. This aroused intense interest in the development of robust control methods, which seeks to solve this problem. One approach to robust controller design is the SMC methodology[2]-[4].

The SMC is a particular type of the VSCS, which is characterized by a suite of feedback control laws and a decision rule. The decision rule termed the switching function as at its input, some measure of the current system behavior and produces as an output the particular feedback controller, which should be used at that instant of time. A VSS can be considered as a blend of subsystems, with each subsystem having a fixed control structure valid for specified regions of the system behavior.

In the SMC, the VSCS is designed to drive and then constrain the system state to lie within the neighborhood of the switching function. This approach has two advantages:

• The dynamic behavior of the system can be tolerated by the particular reference value chosen for the switching function.

• To a particular class of uncertainty, the closed-loop response becomes totally insensitive.

The latter invariance property makes the methodology an appropriate candidate concept for robust control. Also, the ability to directly specify the performance makes the SMC attractive from the design perspective.

As the field of SMC applications is increasing, researchers are working to prove the efficiency of the SMC in electrical and mechanical systems. Lately, many researchers have researched the implementing of the SMC in electrical and mechanical systems[6].

The sliding mode design approach has two components. The first is the design of a switching function so that the design specifications are satisfied by sliding motion. The second involves the selection of a control law, which will make the switching function attractive to the system state. This control law is not necessarily discontinuous. Considering the following general system with scalar control

$$\dot{x} = f(x, t, u) \qquad (31)$$

Where x is the column vector, f is a function vector with n dimension, and u represents an element that can influence the system motion (control input).

Considering that the function vector f is discontinuous on the surface σ (x, t) = 0, thus

$$f(x,t,u) = \begin{cases} f^{+}(x,t,u^{+}) & \text{for} \quad \sigma \to 0^{+} \\ f^{-}(x,t,u^{-}) & \text{for} \quad \sigma \to 0^{-} \end{cases} \qquad (32)$$

the system is in sliding mode if its representative point (RP) moves on the sliding surface

$$\sigma(x,t) = 0$$

In analyzing the VSS, it is next focused on the behavior of the system operating in a sliding regime. Equation (33) defines a particular class of systems that are linear with the control input, i.e.

$$\dot{x} = f(x,t) + B(x,t)u \qquad (33)$$

The scalar control input u is discontinuous on the sliding surface σ(x, t) = 0, while f and B are continuous function vectors. Under SMC, the system trajectories stay on the sliding surface.

$$\sigma(x,t) = 0 \quad \Rightarrow \dot{\sigma}(x,t) = 0 \qquad (34)$$

$$\dot{\sigma}(x,t) = \frac{d\sigma}{dt} = \sum_{i=1}^{n} \frac{\partial \sigma}{\partial x_i}\frac{dx_i}{dt} = \nabla\sigma\,\dot{x} = G\dot{x} \qquad (35)$$

Where G is a 1 by n matrix, the elements of which are the derivatives of the sliding surface with respect to the state variables (gradient vector). Using equations (33) and (35) leads to

$$G\dot{x} = G\, f(x,t) + G\, B(x,t)u_{eq} = 0 \qquad (36)$$

Where the control input u was substituted by an equivalent control ueq that represents an equivalent continuous control input, which maintains the system evolution on the sliding surface. Substituting equation (36) into equation (33) gives

$$\dot{x} = \left[I - B(GB)^{-1}G\right] f(x,t) \qquad (37)$$

Equation (37) describes the system motion under SMC. It is important to note that the matrix $\left[I - B(GB)^{-1}G\right]$ is less than the full rank. This is because, under the sliding regime, the system motion is constrained to be on the sliding surface. As a consequence, the equivalent system described by equation (37) is of the order n-1. This equivalent control description of the VSS in sliding regime is valid also for multiple control inputs. In this case, the system motion is constrained on the hypersurface obtained by the intersection of the individual switching surface Si (x , t) = 0 i.e.

$$\sigma = [s_1 \;\; s_2 \ldots\ldots s_m]^T = 0$$

IV. Design of Sliding Mode Control for DC-DC Converter

Let

$$x^{(n)} = f(X,t) + u + d, \qquad (38)$$

Where

$$X = (x, \dot{x}, \ldots\ldots, x^{(n-1)})^T$$

X is the state vector, d are the disturbances and u is the control variable. Furthermore, let

$$f(X,t) = \hat{f}(X,t) + \Delta f(X,t) \qquad (39)$$

Be a nonlinear function of the state vector X and, explicitly, of time t, where Δf are model uncertainties, and $\hat{f}$ is an estimate of f.

Furthermore, let Δf, d and $x_d^{(n)}$ have upper bounds with known values $\tilde{F}$, D and v:

$$|\Delta f| \le \tilde{F}(X,t); \quad |d| \le D(X,t); \quad |x_d^{(n)}| \le v \qquad (40)$$

The control problem is to obtain the state X for tracking a desired state X_d in the presence of model uncertainties and disturbances. With the tracking error

$$e = X - X_d = (e, \dot{e}, \ldots\ldots, e^{(n-1)})^T, \qquad (41)$$

A sliding surface (switching line for second order systems) is defined as follows:

$$s(X,t) = 0 \qquad (42)$$

$$s(X,t) = (d/dt + \lambda)^{n-1} e; \quad \lambda \ge 0 \qquad (43)$$

For a second order system with $n = 2$ we obtain

$$s(X,t) = \left({}^{d}/_{dt} + \lambda\right)^{n-1} e = \dot{e} + \lambda . e$$

Starting from the initial conditions $e(0) = 0$

(44)

The tracking task $X \to X_d$, which means that X has to follow X_d with the predefined precision, can be considered as solved if the state vector e remains on the sliding surface $s(X,t) = 0$ for all $t \geq 0$. A sufficient condition for this behavior is to choose the control value so that

$$\frac{1}{2}.\frac{d}{dt}\left(s^2(X,t)\right) \leq -\eta\,.|s|; \quad \eta \geq 0 \qquad (45)$$

Considering $s^2(X,t)$ as a Lyapunov function[5], the condition (45) ensures that the system controlled is stable. Looking at the phase plane, we observe that the system is controlled in such a way that the state vector always moves towards the sliding surface. The sign of the control value must change at the intersection of state trajectory $e(t)$ with the sliding surface. In this way, the trajectory is forced to move always towards the sliding surface (refer Fig 6).

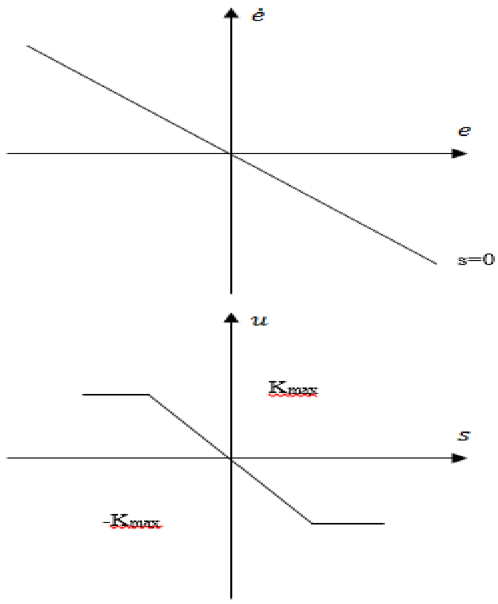

Fig 6: Sliding mode principle with boundary layer

A sliding mode along the sliding surface is thus obtained. By remaining in the sliding mode of (45) the system's behavior is invariant despite model uncertainties, parameter fluctuations, and disturbances. However, the sliding mode causes drastic changes in the control variable, which is an obvious drawback for technical systems[3]-[4]. Returning to equation (45) one obtains the conventional notation for sliding mode:

$s.\dot{s} \leq -\eta\,.|s|$ or alternatively $\dot{s}\,.sgn(s) \leq -\eta$ (46)

In the following, without loss of generality, we focus on second order systems. Hence, from equation (4.6) it follows that

$s = \lambda e + \dot{e}$ and $\dot{s} = \lambda\dot{e} + \ddot{e} = \lambda\dot{e} + \ddot{x} - \ddot{x}_d$ (47)

From the above and (48) it follows

$$s\,.\dot{s} = s\,.(\lambda\dot{e} + f(X,t) + u + d - \ddot{x}_d) \leq -\eta\,.|s| \qquad (48)$$

Rewriting this equation we obtain

$$[f(X,t) + d + \lambda\dot{e} - \ddot{x}_d].sgn(s) + u\,.sgn(s) \leq -\eta \qquad (49)$$

To achieve the sliding mode of equation (4.9) we choose u so that

$u = \left(-\hat{f} - \lambda\dot{e}\right) - K(X,t).sgn(s)$ with $K(X,t) > 0$ (50)

Where $\left(-\hat{f} - \lambda\dot{e}\right)$ is a compensation term and the second term is the controller.

To avoid drastic changes of the control variable mentioned above we substitute the function $sgn(s)$ by $sat(s/\phi)$ in equation (50), where

$$sat(x) = \begin{cases} x, & if|x| < 1 \\ sgn(x), & if|x| \geq 1 \end{cases} \qquad (51)$$

V. Simulation Results

SMC controlled buck converter is created from the MATLAB simulation circuit structure, is an instance circuit for simulation. It is seen in the results that this control can stabilize the power supply and that the output voltage can return to steady state even when it is affected by line and load variation, with a very small overshoot and settling time.

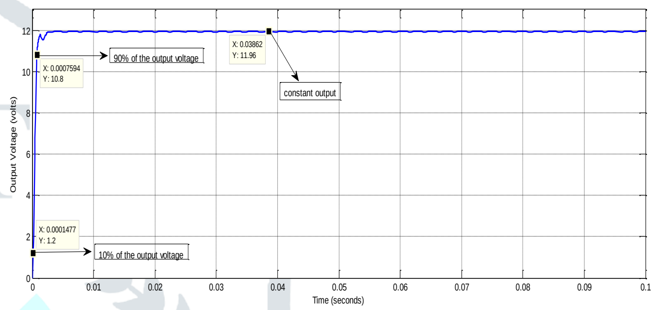

Fig 7: Response of the dc-dc buck converter with sliding mode controller

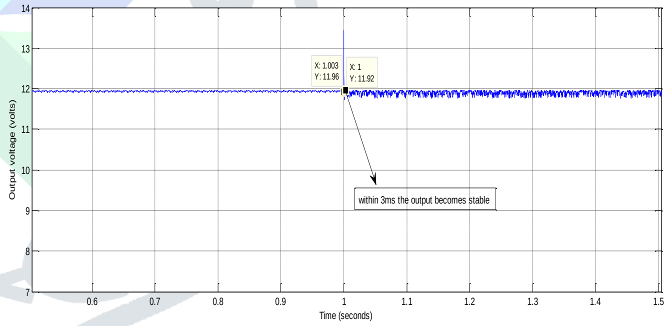

Fig 8: Simulation result when the input voltage changes to 22.5V. The controlling action stabilizes the output within 3ms.

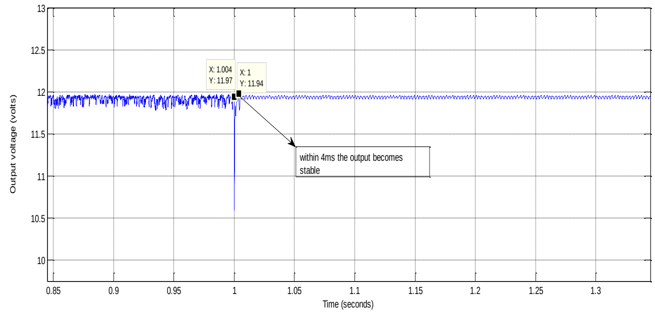

Fig 9: Simulation result when the input voltage changes to 22.5V. The controlling action stabilizes the output within 4ms.

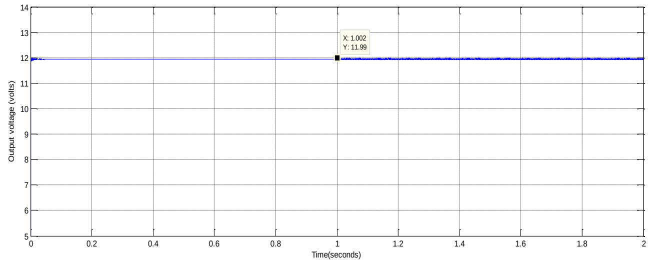

Fig10 : Simulation result for load variation when R= 7Ω

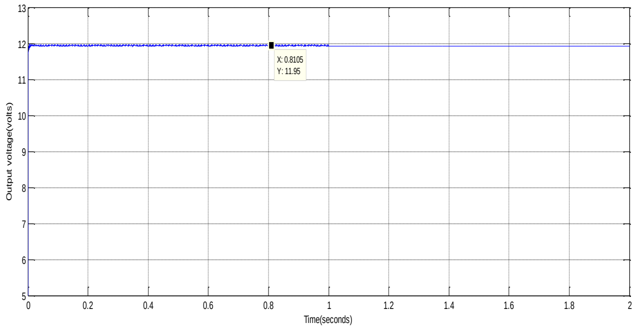

Fig 11: Simulation result for load variation when R= 20Ω

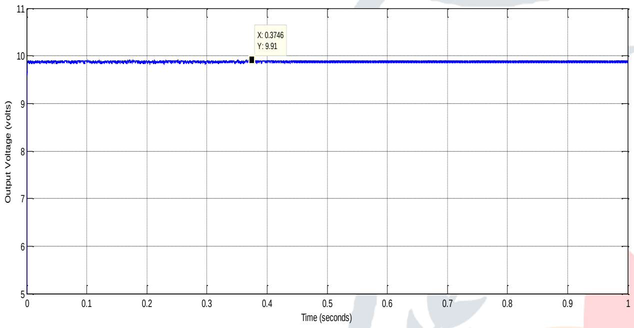

Fig 10: Simulation result when the step input voltage is changed to 10V

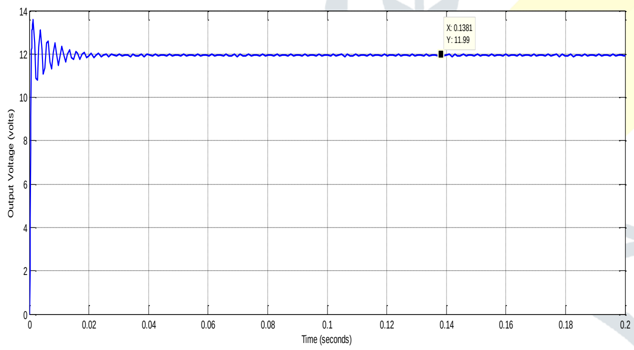

Fig 11: Simulation result with pulse generator of 50 % duty cycle

CONCLUSION

A general-purpose SMC for DC-DC converter is presented. The application of the Sliding Mode Control techniques to DC-DC converter is analyzed in detail with respect to Buck converter. A simulation model in Matlab/Simulink™ for the system is built. The simulation model showed that this control can stabilize the power supply and that the output voltage can return to steady state even when it is affected by line and load variation, with a very small overshoot and settling time. Sliding Mode Control is gaining increasing importance as a design tool for the robust control of linear and non-linear systems. Its strength results from the ease and flexibility of the methodology for its design and implementation. Sliding Mode Control provides inherent order reduction, robustness against system uncertainties disturbances, and an implicit stability proof, so it could be said that the design allows high-performance control system.

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