A Numerical Approach for the Telegraph Equations using Haar Wavelets

A. Padmanabha Reddy[a], Nagaveni K[b]

Department of Studies in Mathematics, V. S. K. University, Ballari.

Abstract:
In this paper, we present a numerical scheme to solve the telegraph equations based on Haar wavelets. Haar wavelets with the aid of collocation method have become very useful in providing highly accurate solution to the telegraph equations. Some illustrative examples are included to demonstrate the validity and applicability of the present technique. Based on the obtained results, we concluded that the proposed method has good accuracy and efficiency compared to Sinc-collocation method (SCM), Adomian decomposition method (ADM) and Modified Adomian decomposition method (MADM).

Keywords: Telegraph equations; Haar wavelets; Collocation method.

AMS Subject Classification: 35L20; 65T60; 65L60.

1. Introduction

Naturally questions was arises in our mind is that from where this telegraph equation originated and why we need?. The telegraph equation initially found in a paper by Kirchhoff in 1857[9] and later one by Oliver Heaviside [16], who created the transmission line model and are based on Maxwell's equations. In telecommunication system an information can be send over a long or short distance by using radio signals or coded electrical signals sent along a transmission medium [coaxial cable, optical fiber,… etc] are connected to a transmitting and a receiving instruments. During this process there is some loss of information. Telegraph equation is invented to discover these losses and ultimately ensure a maximum output and minimum error.

Wavelets theory is a moderately new and a developing area in mathematical research. It has been applied in a wide range of mathematical problems related to scientific and engineering fields. In wavelet analysis we can use almost accurate function that is contained nicely in finite domain. This makes wavelet analysis more attractive and beneficial. Wavelet study is probably the new solution to succeed the limitations of Fourier transform. Let us consider an example of a piece of music. Fourier transform is a strong tool for studying the pitches of the music on other hand less useful in studying the melody of the music. Wavelet transform allows the analysis of both the pitches and melody of the music. It shows that wavelet transform has capable of yielding the time and frequency data concurrently. In numerical analysis, wavelet derived algorithms have become a vital tool because of features of localization, compactness, fast convergence and minimum computational cost.

The layout of this paper is arranged as follows. In section 2, a brief note of telegraph equation is given. In section 3, Haar wavelets and their integrals are introduced. In section 4, we applied Haar wavelet method to the telegraph equation. In section 5, the introduced method is applied to numerical examples and also to demonstrate the efficiency of the proposed method, comparison with the other methods is given. Finally a conclusion is written in section 6.

2. Telegraph Equation

Telegraph equation is the hyperbolic partial differential equation arising in many fields such as vibrational systems, wave propagation of electrical signals in a cable transmission line, pulsating blood flow in arteries, continuous-time random walk, etc. In order to understand the mathematical derivation of telegraph equation, here below given explanation of model of a small portion of telegraph cable wire as an electrical circuit. As well as that a small telegraph wire and the long transmission line have similar characteristics. Let us consider that if the cable is partially insulated then the pair of current and capacitance outflow to ground.
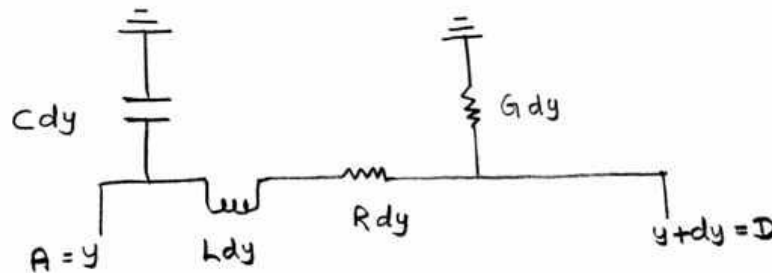

FIG1.Schematic diagram of telegraphic transmission line with leakage [19].

Suppose we are sending information at a distance $'y'$. If $g(y,t)$ is the voltage through the wire, similarly current at any time on the cable is $i(y,t)$ .Where R, C, L, G denotes the resistance, capacitance to the ground, inductance and conductance to the ground in the cable respectively.

By Ohm's law, the voltage around the resistor is

$$g = iR \ , \tag{2.1}$$

further, the voltage drip around the inductance and capacitance as follows

$$g = L\frac{\partial i}{\partial t} \ , \tag{2.2}$$

$$g = \frac{1}{C}\int idt \ . \tag{2.3}$$

To find the drip of voltage or loss of voltage between the end points A and D is,

voltage drip at the terminal $D =$ voltage at the terminal A-[drip of voltage along AD],

voltage drip at the terminal $D =$ voltage at the terminal A-[drip of voltage around the resistor

+ drip of voltage around the inductance],

voltage drip at the terminal D-voltage at the terminal $A = -$ drip of voltage around the

resistor - drip of voltage around the inductance,

$$dg = -iRdy - \frac{\partial i}{\partial t}Ldy .$$

(2.4)

Let us assume that also current can leak from the wire to ground, either by means of a resistor of conductance Gdy or as a result of a capacitor of capacitance Cdy. The load that outflows through resistor is $gGdy$. Because the charge on the capacitor is $q = gCdy$, the load that outflow from the capacitor is $q_t = g_tCdy$, where $q_t = \frac{\partial}{\partial t}q(y,t)$ and $g_t = \frac{\partial}{\partial t}g(y,t)$.

$$di = -gGdy - g_tCdy .$$

(2.5)

Dividing (2.4) and (2.5) by dy both sides and taking the limit $dy \to 0$, we get the following differential equations

$$g_y + iR + i_tL = 0 ,$$

(2.6)

$$Cg_t + Gg + i_y = 0.$$

(2.7)

By from $\frac{\partial}{\partial y}$ of (2.6) and $\frac{\partial}{\partial t}$ of (2.7) we obtain

$$g_{yy} + i_yR + i_{ty}L = 0 ,$$

(2.8)

$$Cg_{tt} + Gg_t + i_{yt} = 0.$$

(2.9)

From (2.7) and (2.9)

$$i_y = -Cg_t - Gg ,$$

(2.10)

$$i_{yt} = -Cg_{tt} - Gg_t .$$

(2.11)

Substituting (2.10) and (2.11) in (2.8) we get

$$g_{yy} = g_t[RC + GL] + RGg + LCg_{tt} .$$

(2.12)

Let us consider $\alpha = c^2[RC + GL]$, $\beta = c^2RG$ and $c^2 = \frac{1}{LC}$ then equation (2.12) reduces to

$$g_{tt} + \alpha g_t + \beta g = c^2 g_{yy} .$$

(2.13)

Where $g(y,t)$ is voltage at position $'y'$ and time $'t'$. Also by using $\dfrac{\partial}{\partial t}$ of (2.6) and $\dfrac{\partial}{\partial y}$ of (2.7), we obtain

$$i_{tt} + \alpha i_t + \beta i = c^2 i_{yy} .$$

(2.14)

Where $i(y,t)$ is current through the wire. The equations (2.13) and (2.14) both are called telegraph equations.

Consider the telegraph equation given in [5]

$$\frac{\partial^2 g}{\partial t^2} + \alpha \frac{\partial g}{\partial t} + \beta g = \frac{\partial^2 g}{\partial y^2} + k(y,t), b \le y < d,\, t \ge 0,$$

(2.15)

subject to the initial conditions

$$g(y,0) = f(y),\, b \le y < d,$$

$$\frac{\partial}{\partial t} g(y,0) = f_1(y),\, b \le y < d,$$

and boundary conditions

$$g(b,t) = q(t),\; g(d,t) = q_1(t),\, t \ge 0.$$

Where α and β are constant coefficients related to resistance, inductance, capacitance and capacitor of the cable, for $\alpha > 0$, $\beta = 0$ equation (2.15) indicates a damped wave equation and $\alpha > 0, \beta > 0$ it is called telegraph equation. We assume that $f(y), f_1(y)$ and their derivatives are continuous functions of $'y'$ and $q(t), q_1(t)$ and their derivatives are continuous functions of $'t'$.

In recent years, many researchers have worked on development of various numerical and analytical methods to solve telegraph equations. M. Abdou [1] developed Adomian decomposition method for solving the telegraph equation in charged particle transport. M. Javidi [8] used Chebyshev spectral collocation method for computing numerical solution of telegraph equation. A. Mohebbi, M. Dehghan [12] used high order compact solution to solve the telegraph equation. A numerical scheme is developed in [2,3] to solve telegraph equation using Quadratic B-spline collocation method and cubic B-spline quasi-interpolation method. R. K. Mohanty [13] applied a new technique to solve the linear one-space-dimensional hyperbolic equation which is unconditionally stable and is of second-order accurate in both time and space components. Also this author proposed in [14] an unconditionally stable finite difference formula for a linear second order one space dimensional hyperbolic equation with variable coefficients.

An effective way to get the solution of telegraph equation is the Haar wavelet method. In numerical analysis, Haar wavelet had become a fruitful tool for solving ordinary and partial differential equations [10,7 and 18]. U. Lepik [11] have solved evolution equation by using Haar wavelet method. Hariharan et al.[4] have solved numerically Fisher's equation. B.Naresh et al. [15] used Haar wavelet collocation method (HWCM) for computing numerical solution of wave equation. In this paper, we approximate the solution of telegraph equation by using Haar wavelet collocation method and compared our results with other methods [5, 6].

3. Haar wavelets and their integrals

Wavelet analysis is recently developed mathematical tool for numerical solution of differential equation, Integral equation, Stiff equation, etc. We have various types of wavelets like Haar, Coiflet and Daubechies, etc. Among the wavelets mathematically most uncomplicated wavelet is Haar wavelet because of orthogonality with localization, compact and scaling properties. Haar wavelet is intially introduced by a Hungarian mathematician named Alfred Haar in 1910. Haar wavelet is piecewise constant function which form an orthogonal basis for the subspace of $L^2[b,d]$ called Haar wavelet family.

Haar wavelet for $y \in [b,d)$ is as follows

$$h_1(y) = \begin{cases} 1, & b \le y < d, \\ 0, & \text{otherwise,} \end{cases}$$

this characterizes basic scale.

$$h_2(y) = \begin{cases} 1, & b \le y < \dfrac{b+d}{2}, \\ -1, & \dfrac{b+d}{2} \le y < d, \\ 0, & \text{otherwise,} \end{cases}$$

this characterizes basic wavelet shape.

Where $h_1(y)$ and $h_2(y)$ indicates the father and mother wavelets respectively.

Now, Haar wavelet family defined as

$$h_i(y) = \begin{cases} 1, & \text{for} \quad y \in [\lambda_1(i), \lambda_2(i)), \\ -1, & \text{for} \quad y \in [\lambda_2(i), \lambda_3(i)), \\ 0, & \text{otherwise.} \end{cases} \tag{3.1}$$

Where $\lambda_1(i) = \dfrac{k}{m}$, $\lambda_2(i) = \dfrac{k+0.5}{m}$ and $\lambda_3(i) = \dfrac{k+1}{m}$. $h_i(y)$ is true for $i > 2$. Haar wavelet family $h_i(y)$ constructed from dilatation and translation of a single function $h_2(y)$ (mother wavelet).

Where $m = 2^j (j = 0, 1, 2, ..., J)$ which indicates the level of the parameter. Where 'J' is the maximal level of resolution and $k = 0, 1, ..., m-1$ denotes translation parameter. The index 'i' is evaluated according to the formula $i = m + k + 1$. The minimal value of 'k' and 'm' are 0 and 1 respectively. The maximal value of $i = 2^{J+1} = 2L$. It is assumed that the interval $[b,d]$ will be divided into $2L$ subintervals, hence $\Delta y = \dfrac{d-b}{2L}$. Let us define the collocation points $y_l = \dfrac{l-0.5}{2L}$, where $l = 1, 2, ..., 2L$.

Any function which is having finite energy in $[b,d]$, i.e. $s \in L^2[b,d]$ can be decomposed as infinite sum of Haar wavelets:

$$s(y) = \sum_{i=1}^{\infty} a_i h_i(y)$$

Here, a_i's are Haar wavelet coefficients. If 's' is either piecewise constant or wish to approximate by piecewise constant on each subinterval, then the above infinite series will be determined at a finite number of terms.

We introduce the following notations are as follows

$$P_{1,i}(y) = \int_b^y h_i(y)dy , \tag{3.2}$$

$$P_{2,i}(y) = \int_b^y \int_b^y h_i(y)dy = \int_b^y P_{1,i}(y)dy,$$

....

Integrating the Haar functions for 'α' times .We get

$$P_{\alpha,i}(y) = \int_b^y \int_b^y ....\int_b^y h_i(r)dr^{\alpha},$$

$$P_{\alpha,i}(y) = \frac{1}{(\alpha-1)!} \int_b^y (y-r)^{\alpha-1} h_i(r)dr. \tag{3.3}$$ For   i=1

equation (3.3) becomes

$$P_{\alpha,1}(y) = \frac{1}{(\alpha)!}(y-b)^{\alpha}. \tag{3.4}$$

Taking into account of $h_i(y)$ these integrals can be calculated analytically by doing it we obtain

$$P_{\alpha,i}(y) = \frac{1}{\alpha!} \begin{cases} 0, & \text{if } y \in \left[b, \lambda_1(i)\right), \\ \left[y-\lambda_1(i)\right]^{\alpha}, & \text{if } y \in \left[\lambda_1(i),\lambda_2(i)\right), \\ \left[\left[y-\lambda_1(i)\right]^{\alpha} - 2\left[y-\lambda_2(i)\right]^{\alpha}\right], & \text{if } y \in \left[\lambda_2(i),\lambda_3(i)\right), \\ \left[\left[y-\lambda_1(i)\right]^{\alpha} - 2\left[y-\lambda_2(i)\right]^{\alpha} + \left[y-\lambda_3(i)\right]^{\alpha}\right], & \text{if } y \in \left[\lambda_3(i), d\right), \end{cases} \tag{3.5}$$

equation (3.5) is valid for $i \geq 2$.

4. Method for solving telegraph equation

Consider a telegraph equation (2.15) with the initial conditions $g(y,0) = f(y)$, $\dot{g}(y,0) = f_1(y)$, $0 \le y < 1$ and the boundary conditions $\qquad\qquad g(0,t) = q(t), g(1,t) = q_1(t)$, $t \ge 0$.

Now based on Haar wavelet method [10,11], the solution of the telegraph equation (2.15) can be expanded as

$$\ddot{g}''(y,t) = \sum_{i=1}^{2M} a_i h_i(y). \tag{4.1}$$

Where '$\cdot\cdot$' and '$''$' means differentiation with respect to 't' and 'y' respectively. Where a_i's are Haar wavelet coefficients in the interval $t \in [t_n, t_{n+1}]$ and $h_i(y)$ is Haar wavelet family.

On twice integration of equation (4.1) w.r.t. to t from t_n to t and w.r.t. y from 0 to y, following equations are obtained

$$\int_{t_n}^{t} \ddot{g}''(y,t)dt = \int_{t_n}^{t} \sum_{i=1}^{2M} a_i h_i(y)dt, \tag{4.2}$$

$$\dot{g}''(y,t) = \dot{g}''(y,t_n) + (t - t_n)\sum_{i=1}^{2M} a_i h_i(y), \tag{4.3}$$

$$g''(y,t) = g''(y,t_n) + \frac{(t-t_n)^2}{2}\sum_{i=1}^{2M} a_i h_i(y) + \dot{g}''(y,t_n)(t-t_n), \tag{4.4}$$

$$g'(y,t) = \frac{(t-t_n)^2}{2}\sum_{i=1}^{2M} a_i P_{1,i}(y) + (t-t_n)\left[\dot{g}'(y,t_n) - \dot{g}'(0,t_n)\right] + g'(y,t_n) - g'(0,t_n) + g'(0,t), \tag{4.5}$$

$$g(y,t) = \frac{(t-t_n)^2}{2}\sum_{i=1}^{2M} a_i P_{2,i}(y) + (t-t_n)\left[\dot{g}(y,t_n) - \dot{g}(0,t_n) - y\dot{g}'(0,t_n)\right] + g(y,t_n) - g(0,t_n) - y\left[g'(0,t_n) - g'(0,t)\right] + g(0,t),$$

(4.6)

$$\dot{g}(y,t) = (t-t_n)\sum_{i=1}^{2M} a_i P_{2,i}(y) + \left[\dot{g}(y,t_n) - \dot{g}(0,t_n) - y\dot{g}'(0,t_n)\right] + y\dot{g}'(0,t) + \dot{g}(0,t),$$

(4.7)

$$\ddot{g}(y,t) = \sum_{i=1}^{2M} a_i P_{2,i}(y) + y\ddot{g}'(0,t) + \ddot{g}(0,t). \tag{4.8}$$

Put y=1 in the equations (4.6) and (4.8) and we have

$$g'(0,t_n) - g'(0,t) = \frac{(t-t_n)^2}{2} \sum_{i=1}^{2M} a_i P_{2,i}(1) + (t-t_n)\left[\dot{g}(1,t_n) - \dot{g}(0,t_n) - \dot{g}'(0,t_n)\right] + g(1,t_n) -$$
$$g(0,t_n) + g(0,t) - g(1,t), \tag{4.9}$$

$$\ddot{g}'(0,t) = -\sum_{i=1}^{2M} a_i P_{2,i}(1) - \ddot{g}(0,t) + \ddot{g}(1,t). \tag{4.10}$$

If the equation (4.9) is substituted into equation (4.6), we obtain

$$g(y,t) = \frac{(t-t_n)^2}{2} \sum_{i=1}^{2M} a_i P_{2,i}(y) + (t-t_n)\left[\dot{g}(y,t_n) - \dot{g}(0,t_n)\right] + g(y,t_n) - g(0,t_n) - \frac{y(t-t_n)^2}{2} \sum_{i=1}^{2M} a_i P_{2,i}(1)$$
$$- y(t-t_n)\left[\dot{g}(1,t_n) - \dot{g}(0,t_n)\right] - y\left[g(1,t_n) - g(0,t_n) + g(0,t) - g(1,t)\right] + g(0,t), \tag{4.11}$$

$$g'(y,t) = \frac{(t-t_n)^2}{2} \sum_{i=1}^{2M} a_i P_{1,i}(y) + (t-t_n)\left[\dot{g}'(y,t_n)\right] + g'(y,t_n) - \frac{(t-t_n)^2}{2} \sum_{i=1}^{2M} a_i P_{2,i}(1) -$$
$$(t-t_n)\left[\dot{g}(1,t_n) - \dot{g}(0,t_n)\right] - \left[g(1,t_n) - g(0,t_n) + g(0,t) - g(1,t)\right], \tag{4.12}$$

$$g''(y,t) = \frac{(t-t_n)^2}{2} \sum_{i=1}^{2M} a_i h_i(y) + (t-t_n)\dot{g}''(y,t_n) + g''(y,t_n), \tag{4.13}$$

$$\dot{g}(y,t) = (t-t_n) \sum_{i=1}^{2M} a_i P_{2,i}(y) + \left[\dot{g}(y,t_n) - \dot{g}(0,t_n)\right] - y(t-t_n) \sum_{i=1}^{2M} a_i P_{2,i}(1) -$$
$$y\left[\dot{g}(1,t_n) - \dot{g}(0,t_n)\right] - y\left[\dot{g}(0,t_n) - \dot{g}(1,t_n)\right] + \dot{g}(0,t), \tag{4.14}$$

$$\ddot{g}(y,t) = \sum_{i=1}^{2M} a_i\left[P_{2,i}(y) - yP_{2,i}(1)\right] - y\left[\ddot{g}(0,t) - \ddot{g}(1,t)\right] + \ddot{g}(0,t). \tag{4.15}$$

The equations from (4.11) to (4.15) are discritized by assuming y→y_1 and t →t_{n+1}

$$g\left(y_1, t_{n+1}\right) = \frac{\left(t_{n+1}-t_n\right)^2}{2} \sum_{i=1}^{2M} a_i P_{2,i}\left(y_1\right) + \left(t_{n+1}-t_n\right)\left[\dot{g}\left(y_1,t_n\right)-\dot{g}\left(0,t_n\right)\right] + g\left(y_1,t_n\right) -$$

$$g\left(0,t_n\right) - \frac{y_1\left(t_{n+1}-t_n\right)^2}{2} \sum_{i=1}^{2M} a_i P_{2,i}\left(1\right) - y_1\left(t_{n+1}-t_n\right)\left[\dot{g}\left(1,t_n\right)-\dot{g}\left(0,t_n\right)\right] -$$

$$y_1\left[g\left(1,t_n\right)-g\left(0,t_n\right)+g\left(0,t_{n+1}\right)-g\left(1,t_{n+1}\right)\right] + g\left(0,t_{n+1}\right), \tag{4.16}$$

$$g'\left(y_1, t_{n+1}\right) = \frac{\left(t_{n+1}-t_n\right)^2}{2} \sum_{i=1}^{2M} a_i P_{2,i}\left(y_1\right) + \left(t_{n+1}-t_n\right)\left[\dot{g}'\left(y_1,t_n\right)\right] + g'\left(y_1,t_n\right)$$

$$- \frac{\left(t_{n+1}-t_n\right)^2}{2} \sum_{i=1}^{2M} a_i P_{1,i}\left(1\right) - \left(t_{n+1}-t_n\right)\left[\dot{g}\left(1,t_n\right)-\dot{g}\left(0,t_n\right)\right]$$

$$- \left[g\left(1,t_n\right)-g\left(0,t_n\right)+g\left(0,t_{n+1}\right)-g\left(1,t_{n+1}\right)\right], \tag{4.17}$$

$$g''\left(y_1, t_{n+1}\right) = \frac{\left(t_{n+1}-t_n\right)^2}{2} \sum_{i=1}^{2M} a_i h_i\left(y_1\right) + \left(t_{n+1}-t_n\right)\dot{g}''\left(y_1,t_n\right) + g''\left(y_1,t_n\right), \tag{4.18}$$

$$\dot{g}\left(y_1, t_{n+1}\right) = \left(t_{n+1}-t_n\right) \sum_{i=1}^{2M} a_i P_{2,i}\left(y_1\right) + \left[\dot{g}\left(y_1,t_n\right)-\dot{g}\left(0,t_n\right)\right] - y_1\left(t_{n+1}-t_n\right) \sum_{i=1}^{2M} a_i P_{2,i}\left(1\right) -$$

$$y_1\left[\dot{g}\left(1,t_n\right)-\dot{g}\left(0,t_n\right)\right] - y_1\left[\dot{g}\left(0,t_{n+1}\right)-\dot{g}\left(1,t_{n+1}\right)\right] + \dot{g}\left(0,t_{n+1}\right), \tag{4.19}$$

$$\ddot{g}\left(y_1, t_{n+1}\right) = \sum_{i=1}^{2M} a_i\left[P_{2,i}\left(y_1\right)-y P_{2,i}\left(1\right)\right] - y\left[\ddot{g}\left(0,t_{n+1}\right)-\ddot{g}\left(1,t_{n+1}\right)\right] + \ddot{g}\left(0,t_{n+1}\right). \tag{4.20}$$

5.

Numerical studies

In this section, we apply the HWCM to few of the telegraph equation problems whose exact solutions are known. The potency of proposed method is presented for each example in the form of graph and table. All computations are carried out by MATLAB software. To show the preciseness of the present technique, the L_∞, L_2 errors and Root-Mean-Square (RMS) of errors are calculated using the following definitions:

$$L_\infty = \max_{1 \le p \le N}\left|g(p) - g_{exat}(p)\right|, \ \ L_2 = \sqrt{\sum_{p=1}^{N}\left|g(p) - g_{exat}(p)\right|^2},$$

$$RMS = \sqrt{\frac{1}{N}\left(\sum_{p=1}^{N}\left|g(p) - g_{exat}(p)\right|^2\right)}.$$

Where N =number of nodes, $g(p)$ =approximate solution, $g_{exat}(p)$ =exact solution for $p = 1, 2, .., N$.

Example 1. Consider the linear homogeneous telegraph equation (2.15) with the following conditions [5] ,

$$\alpha = 1, \beta = 1, 0 \leq y < 1, k(y, t) = 0, f(y) = e^y, f_1(y) = -e^y, \ q(t) = e^{-t}, q_1(t) = e^{1-t}.$$

The exact solution of this equation is

$$g(y, t) = e^{y-t}.$$

When **t=0.4,** the comparison of absolute errors of the Haar and Sinc solutions are represented in **Figure 1.** In **Table 1**, the comparisons of the Exact, Haar and Sinc solutions along with the absolute errors of Haar and Sinc solutions are inserted [5]. The L_∞, L_2 and RMS errors of the Sinc and Haar solutions are calculated in **Table 2**.

TABLE 1: Comparison of the values of Haar, Sinc solutions with their absolute errors for example 1.

| y	Exact solution	Sinc solution[5]	Haar solution	Absolute error (Sinc solution)[5]	Absolute error (Haar solution)
0.01	0.677	0.674	0.677	0.003	0.000139
0.02	0.683	0.679	0.684	0.004	0.000239
0.03	0.690	0.684	0.691	0.006	0.000310
0.04	0.697	0.689	0.698	0.008	0.000362
0.05	0.704	0.694	0.705	0.010	0.000403
0.06	0.711	0.700	0.712	0.011	0.000437
0.07	0.718	0.705	0.719	0.013	0.000466
0.08	0.726	0.711	0.727	0.015	0.000490
0.09	0.733	0.717	0.734	0.016	0.000512
0.1	0.740	0.723	0.741	0.017	0.000532

plot of absolute errors of both Haar and Sinc solution at t=0.4(ex1)
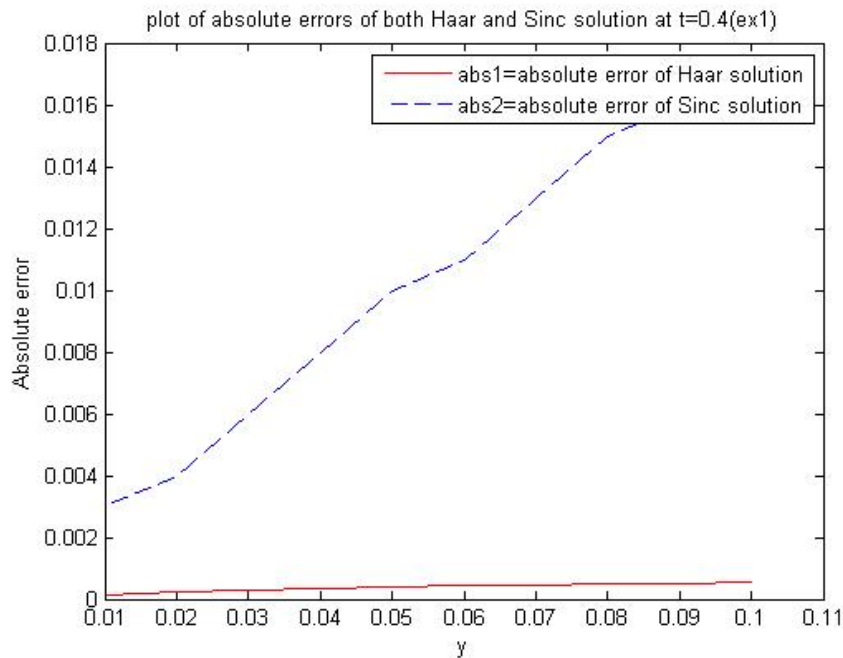
Figure 1: Comparison of absolute errors at t=0.4 of example 1.

TABLE 2 : The estimated numerical errors for example1 with dy=0.01,dt=0.1 at t=0.4.

	L_∞ - error	L_2 - error	RMS
SCM	0.017	0.0359	0.011336
HWCM	0.000532	0.0013	0.0004073

Also when **y=0.01**, we compared Haar wavelet collocation method with Sinc collocation method with their absolute errors in **Table3**. In **Figure2** and **Figure3** indicates the comparisons of Exact, Haar and Sinc solutions along with their absolute errors respectively [5].

TABLE 3: Comparison of the values of Haar, Sinc solutions at y=0.01 with their absolute

errors for example 1.

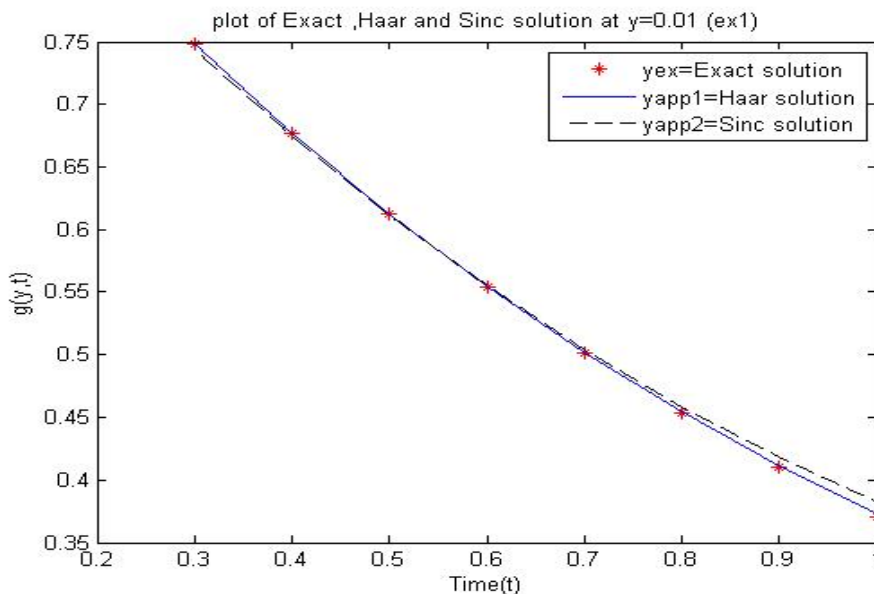| t	Exact solution	Sinc solution[5]	Haar solution	Absolute error (Sinc solution)[5]	Absolute error (Haar solution)
0.3	0.748	0.744	0.748	0.0040	0.000074
0.4	0.677	0.674	0.677	0.0030	0.000139
0.5	0.612	0.611	0.613	0.0010	0.000218
0.6	0.554	0.555	0.555	0.0010	0.000307
0.7	0.501	0.504	0.502	0.0030	0.000407
0.8	0.453	0.458	0.454	0.0050	0.000519
0.9	0.410	0.418	0.411	0.0080	0.000640
1	0.371	0.382	0.372	0.0110	0.000771

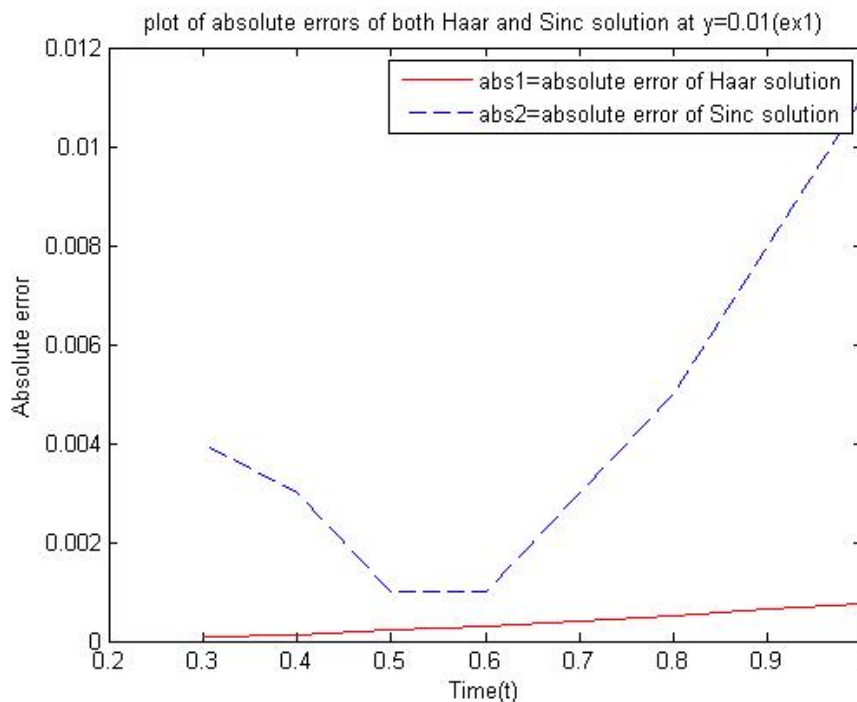**Figure 2: Comparison of Exact, Haar and Sinc solution for y=0.01 of example1.**

Figure 3: Comparison of absolute errors for y=0.01 of example1.

Example 2.

Consider the linear non-homogeneous telegraph equation as follows [5] ,

$$\frac{\partial^2 g}{\partial t^2} + \frac{\partial g}{\partial t} + g = \frac{\partial^2 g}{\partial y^2} + y^2 + t - 1, 0 \le y < 1, t \ge 0,$$

with the given initial and boundary conditions as follows

$$f(y)=y^2, f_1(y)=1,$$
$$q(t)=t, q_1(t)=1+t.$$

The exact solution of this equation is

$$g(y,t)=y^2+t.$$

When **t=0.3, Figure 4** represents the comparison of Exact, Haar and Sinc solution. **Figure 5** and **Figure 6** indicates the absolute errors of Sinc and Haar solutions respectively. In **Table 4** the comparisons of the Exact, Haar and Sinc solutions along with their absolute errors are shown [5]. The L_∞, L_2 and RMS errors of Sinc and Haar solutions are calculated in **Table 5**.

TABLE 4: Comparison of the values of Haar and Sinc solutions for t=0.3 of example2.

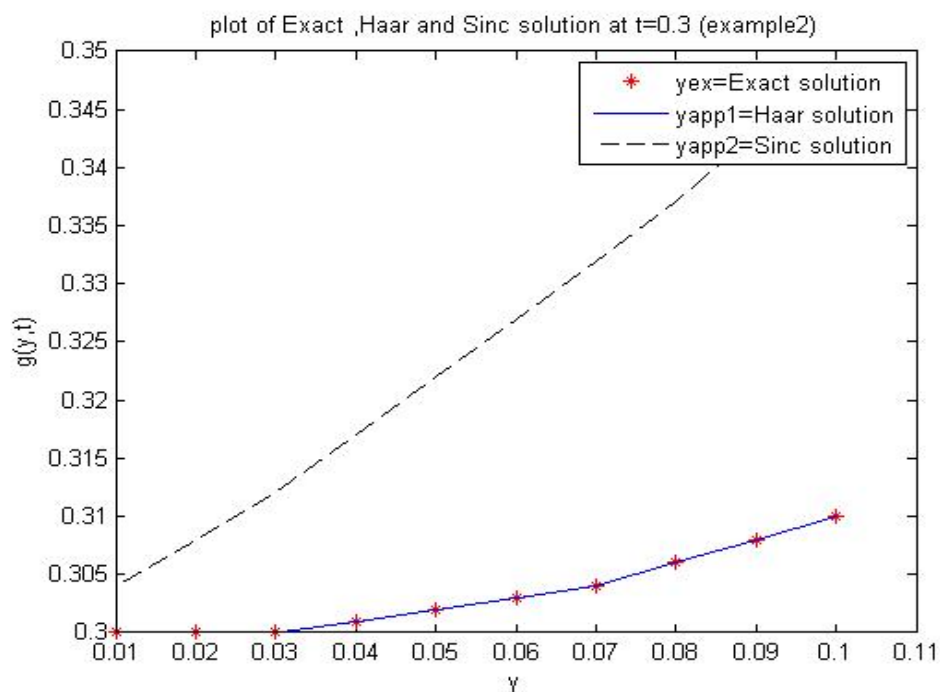| y	Exact solution	Sinc solution[5]	Haar solution	Absolute error (Sinc solution)[5]	Absolute error (Haar solution)
0.01	0.300	0.304	0.300	0.004	0
0.02	0.300	0.308	0.300	0.008	0
0.03	0.300	0.312	0.300	0.012	0
0.04	0.301	0.317	0.301	0.016	0
0.05	0.302	0.322	0.302	0.020	0
0.06	0.303	0.327	0.303	0.024	0
0.07	0.304	0.332	0.304	0.028	0
0.08	0.306	0.337	0.306	0.031	0
0.09	0.308	0.343	0.308	0.035	0
0.1	0.310	0.349	0.310	0.039	0

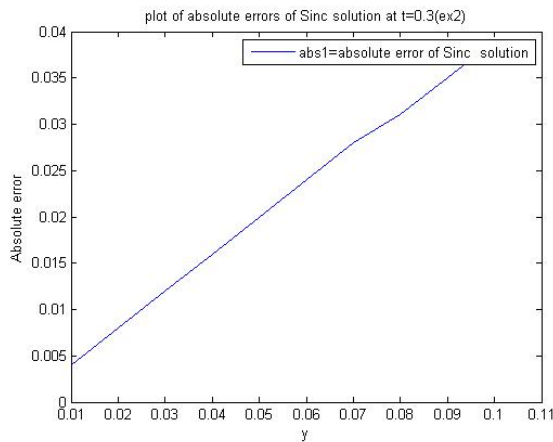**Figure 4: Comparison of Exact, Haar and Sinc solution for t=0.3 of example2.**

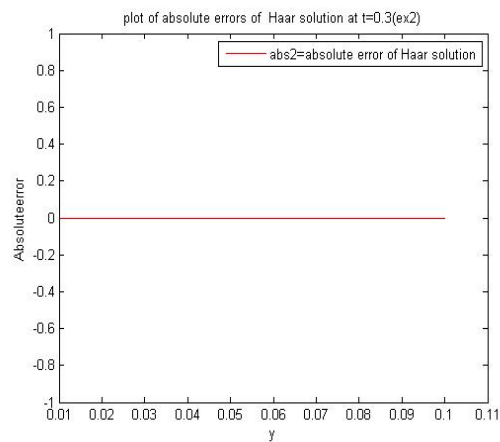**Figure 5: Absolute errors of Sinc solution for t=0.3 of example 2.**

Figure 6: Absolute errors of Haar solution for t=0.3 of example 2.

TABLE5: Comparison of L_∞, L_2 and RMS errors for example 2 with dy=0.01, dt=0.1 at t=0.3.

	L_∞ - error	L_2 - error	RMS
SCM	0.039	0.077117	0.024387
HWCM	0	0	0

Example 3.

Consider the non–linear and non-homogeneous telegraph equation as follows [6],

$$\frac{\partial^2 g}{\partial t^2} + 2\frac{\partial g}{\partial t} + g^2 = \frac{\partial^2 g}{\partial y^2} - 2e^{-t}\cosh(y) + \left[e^{-t}\cosh(y)\right]^2,\ 0 \le y < 1,\ t \ge 0,$$

with the given initial and boundary conditions as

$$f(y) = \cosh(y), f_1(y) = -\cosh(y),$$
$$q(t) = e^{-t},\ q_1(t) = e^{-t}\cosh(1).$$

The exact solution of this equation is

$$g(y,t) = e^{-t}\cosh(y).$$

The comparison of Exact, HWCM, ADM and MADM solutions are displayed in **Figure 7. Figure8** indicate the comparison of absolute error curve obtained by HWCM, ADM and MADM. In **Table6**, the comparisons of the Exact, HWCM, ADM and MADM along with their absolute errors are shown [6]. The L_∞, L_2 and RMS errors of ADM, MADM and HWCM are obtained in **Table 7**.

TABLE 6: Comparison of the values of HWCM, ADM and MADM with their absolute errors for example 3.

y	Exact solution	Adomian decomposition solution (ADM)[6]	Modified Adomian decomposition solution (MADM) [6]	Haar solution (HWCM)	Absolute error (ADM)[6]	Absolute error (MADM)[6]	Absolute error (HWCM)
0	0.90483742	0.91366178	0.91451972	0.90483742	0.00882436	0.00968230	0.00000
0.1	0.90936538	0.91825217	0.91914260	0.90959803	0.00888679	0.00977722	0.00023265
0.2	0.92299457	0.93215657	0.93305989	0.92345575	0.00916200	0.01006532	0.00046113
0.3	0.94586140	0.95545619	0.95641958	0.94652742	0.00959479	0.01055818	0.00066602
0.4	0.97819473	0.98848184	0.98946965	0.97898349	0.01028710	0.01127492	0.00078876
0.5	1.02031817	1.03152442	1.03256015	1.02114134	0.01120625	0.01224198	0.00082317

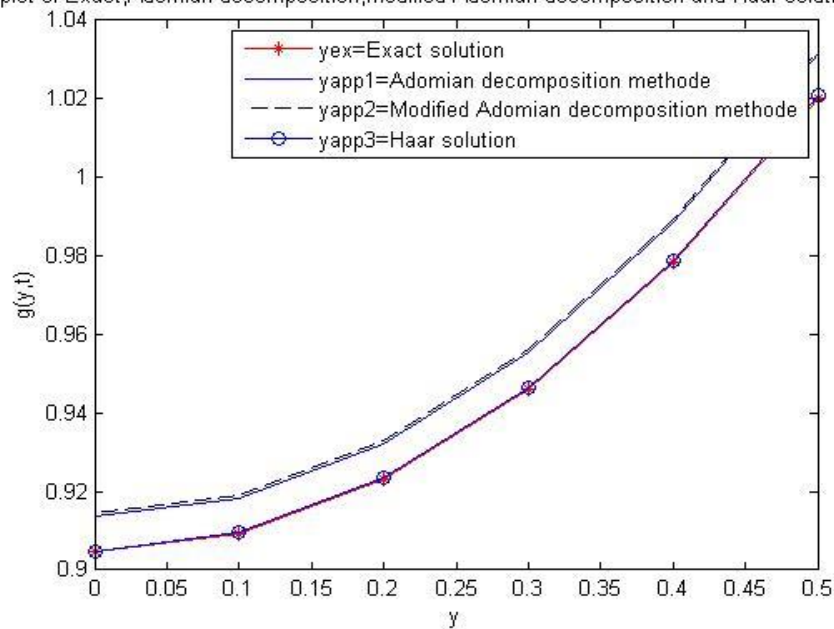**Figure 7: Comparison of Exact, HWCM, ADM and MADM for example3.**
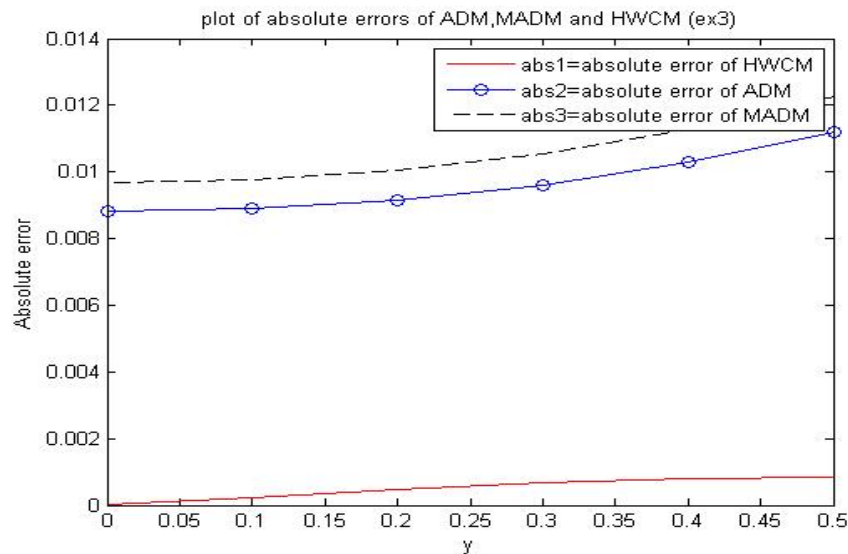
Figure 8: Comparison of absolute errors of HWCM, ADM and MADM for example3.

TABLE 7: Computational errors in L_∞, L_2 and RMS of example 3 with dy=0.1, dt=0.1 at t=0.1.

	L_∞ - error	L_2 - error	RMS
ADM	0.01121	0.02375	0.00970
MADM	0.01224	0.02606	0.01064
HWCM	0.00082	0.00142	0.00058

Conclusion

In this paper, we used Haar wavelet collocation (HWCM) for solving telegraph equation. HWCM is best shoot with less errors for solving the telegraph equations. HWCM has been verified on three test problems. The graphs and tables conformed that the numerical results obtained from HWCM are better than Sinc-collocation method, Adomian decomposition method and Modified Adomian decomposition method.In order to establish the accuracy of the proposed method L_∞, L_2 and RMS errors are calculated.It is observed that HWCM gives more accurate results in comparison with Sinc-collocation method, Adomian decomposition method and Modified Adomian decomposition method. From the examples we concluded that proposed method is very reliable, simple, fast, minimal computational costs and flexible

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