Abstract—The main objective of the paper is to design a robust PI controller for a bioreactor. A biochemical reactor can be used in a variety of processes, from waste treatment to alcohol fermentation. Biomass (cells) consume substrate like sugar or chemicals and produce more cells. The performance of bioreactor depends on the type of controller and control techniques such as adaptive, robust, cascade or inferential. The performance of the bioreactor in open loop and using conventional controllers was analyzed and the time response performance was improved by designing a robust PI controller using Kharitonov Theorem.

Keywords—Bioreactor, Robust PI controller, Kharitonov theorem

I. INTRODUCTION

It is well-known that a highly non-linear process is one of the most challenging control problems. In case of mild linearity, linear control based on local linearization may provide a satisfactory performance. Inspired by this realization there have been considerable interests in non-linear control in the recent years. As a result of this, an exact linearized method has emerged. By using non-linear co-ordinate transformations and state space model methods, a wide range of non-linear processes can now be linearized and these methods are said to have been accepted globally by all. Exact linearization control provides a promising method to the non-linear control of processes [1].

A bioreactor model developed by Agarwal and Lim [1] was taken for consideration. The open loop performance is found to be unstable. They are difficult to control, which may be attributed to its nonlinear dynamic behaviour. The model parameters of the bioreactor also vary in an unpredictable manner. The complexity of the biochemical processes inhibits the accurate modelling and also the lack of suitable sensors makes the process stability difficult to characterize. This paper is mainly concerned in controlling the bioreactors using different controllers.

A linearized bio-reactor model developed by Sigurd Skogestad [2] was also referred. He emphasized on restricting the exact linearization control that limits the possibility of unstable zero dynamics and singular points. A complete analysis of zero dynamic error, singular point and disturbance decoupling is done six SISO and multivariable control structures. This study shows the importance of selection of control structure in exact linearization control.

A parallel cascade controller for tuning unstable FOPTD systems proposed by M. Chidambaram [3] was taken into consideration. They used proportional (P) controller for secondary loop and proportional integral (PI) for primary loop. Robust stability analysis using the complimentary sensitivity function was carried out.

Fig. 1: Continuous fermenter

Mathematical Model Description

Chemostat is a very popular mode of operating a continuous biological reactor. In this open-loop mode, the flow rate of nutrient to the bioreactor is held constant so that the dilution rate is less than the maximum specific growth rate. The medium contains excess of all but one nutrient. A steady state results when the specific growth rate of microorganism balances exactly with the dilution rate. The steady state is said to be asymptotically stable if the reactor returns to its steady state after small perturbations. However the attainment and maintenance of the desires steady state may be difficult.

II. DESCRIPTION OF THE PROCESS

Fermenter model

Many models have been proposed for fermentation processes. Structured models attempt to describe the individual organisms in detail but are usually mathematically too complex to be useful for controller design. Significantly simpler unstructured models can be obtained by assuming that the fermented culture consists of a single, homogeneously growing organism. These models usually consist of a few nonlinear ordinary differential equations and are particularly well-suited to the nonlinear control strategies.

A schematic of a continuous fermenter is shown in Figure. 1. We assume that the fermenter has a constant volume, its contents are well mixed, and the feed is sterile. The dilution rate D and the feed substrate concentration S are available as manipulated inputs. The effluent cell-mass or biomass concentrations X, substrate concentration S, are the process state variables.
NON-LINEAR EQUATION OF BIOREACTOR

\[ \frac{dx}{dt} = (\mu - D)x \quad \ldots 1 \]

\[ \frac{ds}{dt} = D(s_f - s) - \frac{\mu_y}{Y} x \quad \ldots 2 \]

where, x and s are state variables representing the bio mass concentration and substrate concentration respectively and D, Dilution rate and s_f is the substrate concentration in the feed stream. The kinetics of the cell mass is defined in terms of specific growth rate, \( \mu \) and yield of the cell mass, Y. Here \( \mu \) is also referred to as substrate inhibition.

\[ \mu = \frac{\mu_{max} \cdot s}{K_m + s + K_1 \cdot s^2} \ldots 3 \]

D – Dilution rate and is given by,

\[ D = \frac{F}{V} = \text{Volumetric flow rate}/\text{Reactor volume} \]

A. LINEARIZATION

The linearization is done by applying taylor series to the above non-linear equation. The linearized equation is then converted to a transfer function for which we would obtain the response curves in MATLAB.

The transfer function obtained is given by

\[ G(s) = -5.8644 \cdot e^{(-s)}/[5.888s - 1] \]

Where, \( K_c = -5.8644; L = 1; T = 5.888 \)

B. STATE SPACE MODEL OF A BIO REACTOR

The state space model of a bio reactor is obtained by using the following formula. The state space model gives an optimized value of the substrate concentration[3].

General form:

\[ A = [\mu_s - D_s \quad x_s \quad \mu_s - D_s - (x_s \cdot \mu_s / Y)] \]

\[ B = [ -x_{1s} \quad s_{i0} - s_i ] \]

The above matrix represents the generalized form of A and B[7]. Depending upon the value of x, we can classify into 3 operating points:

1. Stable steady state operating point.
2. Unstable operating point for a stable steady state model
3. Unstable steady state operating point.

III. DESIGN OF CONVENTIONAL PI CONTROLLER

Here, we look into the open loop and closed loop response of the transfer function

A. Open loop response of

\[ G(s) = -5.8644 \cdot e^{(-s)}/[5.888s - 1] \]

\[ \text{Inference:-} \]

Response is unstable and hence we require the use of controller to achieve stability.

B. Closed loop response of

\[ G(s) = -5.8644 \cdot e^{(-s)}/[5.888s - 1] \]

using De Paor and O’Malley[3] control tuning method

Calculated PI control parameters–

\[ K_p = -0.7356 \]

\[ K_i = -0.1349 \]

IV. DESIGN OF ROBUST PI CONTROLLER

The primary and essential requirement of all applications is the stability of closed control loop. But due to the presence of Model Uncertainty the closed loop stability and performance may not be satisfactory. Model uncertainty can be caused due to the presence of one or more of the following: non-linear effects when a linear model is used, high order dynamics when model neglects such performance, slow varying parameters such as heat transfer coefficient or some unknown phenomenon[8].
Hence there is a need for robust stability analysis and controller synthesis for uncertain systems especially control systems with parametric uncertainty. There are several methods for designing a robust PI controller in which one of them is applying Kharitonov Theorem [4]. This theorem investigates the stability characteristics of the interval systems via four vortex polynomials with real coefficients varying in a bounded range.

<table>
<thead>
<tr>
<th>K</th>
<th>L</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5.6</td>
<td>-0.6</td>
<td>-5.6</td>
</tr>
<tr>
<td>-5.6</td>
<td>-0.6</td>
<td>-5.9</td>
</tr>
<tr>
<td>-5.6</td>
<td>-1.2</td>
<td>-5.6</td>
</tr>
<tr>
<td>-5.6</td>
<td>-1.2</td>
<td>-5.9</td>
</tr>
<tr>
<td>-5.9</td>
<td>-0.6</td>
<td>-5.6</td>
</tr>
<tr>
<td>-5.9</td>
<td>-0.6</td>
<td>-5.9</td>
</tr>
<tr>
<td>-5.9</td>
<td>-1.2</td>
<td>-5.6</td>
</tr>
<tr>
<td>-5.9</td>
<td>-1.2</td>
<td>-5.9</td>
</tr>
</tbody>
</table>

Table 1

The above table represents the chosen values for Kc, L and T. Using these values we obtain the following transfer functions:

- sys1 = (-5.6 * exp(-0.6*s))/(5.6*s-1);
- sys2 = (-5.6 * exp(-0.6*s))/(5.9*s-1);
- sys3 = (-5.6 * exp(-1.2*s))/(5.6*s-1);
- sys4 = (-5.6 * exp(-1.2*s))/(5.9*s-1);
- sys5 = (-5.9 * exp(-0.6*s))/(5.6*s-1);
- sys6 = (-5.9 * exp(-0.6*s))/(5.9*s-1);
- sys7 = (-5.9 * exp(-1.2*s))/(5.6*s-1);
- sys8 = (-5.9 * exp(-1.2*s))/(5.9*s-1);

By using this theorem, we obtain the range of values for Kc and the following graph shows that the range is [-0.1, -0.6].

In this section, we would see the response curves of each of the model and finally the optimized Kp and Ki values.

a. Open loop response of state space model with stable steady state operating point[5]:

X(0) = [1.5301 0.1745]

b. Closed loop response of state space model with stable steady state operating point:

X(0) = [1.5301 0.1745]

Since it has a stable operating point, the response obtained is stable and ideal. The values of Kc, overshoot and settling time are calculated from the above graph.

<table>
<thead>
<tr>
<th>Kc</th>
<th>Overshoot (%)</th>
<th>Settling time</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
<td>63</td>
<td>200</td>
</tr>
<tr>
<td>-0.55</td>
<td>66</td>
<td>159</td>
</tr>
<tr>
<td>-0.6</td>
<td>69.4</td>
<td>159</td>
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<tr>
<td>-0.58</td>
<td>68</td>
<td>155</td>
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<tr>
<td>-0.56</td>
<td>66.7</td>
<td>157</td>
</tr>
<tr>
<td>-0.525</td>
<td>64.4</td>
<td>165</td>
</tr>
<tr>
<td>-0.535</td>
<td>64.8</td>
<td>165</td>
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<tr>
<td>-0.515</td>
<td>63.9</td>
<td>180</td>
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<td>-0.2</td>
<td>44</td>
<td>270</td>
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<td>-0.1</td>
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</tr>
<tr>
<td>-0.005</td>
<td>63.2</td>
<td>167</td>
</tr>
<tr>
<td>-0.45</td>
<td>59.6</td>
<td>210</td>
</tr>
<tr>
<td>-0.445</td>
<td>58.7</td>
<td>208</td>
</tr>
<tr>
<td>-0.441</td>
<td>58.4</td>
<td>204</td>
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</tbody>
</table>

Table 2
From the above table, we can infer that $K_c = -0.441$ has the optimal values of overshoot and settling time.

<table>
<thead>
<tr>
<th>$K_c$</th>
<th>Overshoot(%)</th>
<th>Settling time</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.441</td>
<td>43.07</td>
<td>40</td>
</tr>
<tr>
<td>-0.5</td>
<td>40.38</td>
<td>25</td>
</tr>
<tr>
<td>-0.55</td>
<td>39.23</td>
<td>22</td>
</tr>
</tbody>
</table>

Table 3

From the above table we can conclude that, $K_c = -0.55$ has an optimal response with a relatively low overshoot and lesser settling time.

VI. RESPONSE AFTER APPLYING DISTURBANCE

Here, a disturbance is fed on the output side at $t=50$.

CONCLUSION

A response of bioreactor model was analyzed for open loop and closed loop condition utilizing conventional mode of controller. A robust PI controller was designed to optimize the performance. The results show considerable decrease in overshoot and settling time thereby enhancing the time domain specifications. Design of robust cascade controller for a bioreactor is under study.

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