EFFECT OF SIZE, SHAPE AND ORIENTATION OF INCLUSIONS ON HOMOGENIZED PARAMETERS OF HETEROGENEOUS MATERIALS

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***Abstract* :** From the last few decades there is tremendous development in science and technology for heterogeneous materials. The physical properties of heterogeneous materials vary all through their microstructures. In this work the micromechanical analysis of heterogeneous materials is done and the effects of shape, size and orientation of inclusions on its properties is identified. This work is based on the study and analysis of homogenization based micromechanics model incorporating finite element methods. Steel fibre reinforced concrete and boron fibre aluminium composite are considered for analysis, i.e. the effect of shape, size and orientations of different morphologies on homogenized parameters is identified.

***Index Terms* - Heterogeneous materials, finite element mesh, inclusions, steel fibre reinforced concrete, boron aluminium matrix composite.**

I. INTRODUCTION

The determination of the properties of the homogeneous material that approximates the behaviour of the original heterogeneous material is called homogenization. Micromechanics deals with determining unknown properties of the heterogeneous material based on known properties of the particulate and the matrix. It is used to extrapolate existing composite property data to different particulate volume fraction or void content. The nature of distribution and properties of phases in a heterogeneous material is called its microstructure. The arrangement of these phases with respect to each other is also very important. A concept related to arrangement is that of connectedness of a phase. For example in a particle-matrix heterogeneous material the binder phase is connected and one can move relatively any distances through the material without leaving this phase. On the other hand the particles are disconnected and they do not form a connected phase because one can at most travel from one side of a particle to other. The nature of connectedness does not seem to be very clear in case of certain phase distributions because one can travel relatively large distances but not any distance. The most basic microstructural characterization tool is the volume fraction of a phase.

The microstrucrural analysis is also known simply as micromechanics. It is assumed that microstructure is well characterized. Multi-scale analysis not only aims to understand the behavior of the microstructure, but also explores the relationship between microstructure and analysis of microstructure. In this work the main interest lies not only on microstructural characterization but also in microstructural analysis or simply micromechanics. The related subject of multiscale analysis aims to understand the behavior of the microstructure and additionally investigates the relationship between the microstructure and the analysis of the macrostructure. The determination of properties of a homogeneous material that approximates the behavior of the original heterogeneous problems termed as effective properties or macroscopic properties is performed by homogenization technique. These properties called effective because these approximates the effects of the microscale features on the macroscale. These methods aim at the extraction of macroscopic properties. In the present work, only effective elastic properties are attempted for determination. The macroscopic elastic stiffness tensor is the property for which one looks for in linear elasticity. The effective elastic constants are affected due to the change in volume fraction of the heterogeneity and due to the change of the different morphologies, sizes and orientations of the heterogeneities.

II. GOVERNING EQUATIONS AND FORMULATION

In the homogenization problem, the original heterogeneous material of the microstructure is replaced by a homogeneous one, i.e. the effective material. The procedure is introduced by taking an example homogenization problem to demonstrate it for infinitesimal deformations.

Consider a microstructure that is heterogeneous on microscale. Let's denote heterogeneous material by M which can have multiple phases i.e. $M^{(I)}$. A solid mechanics problem postured on the structure is of the form,
Determine $u(x,t)$ so that

$$div(\sigma) + \rho b = \rho \ddot{u} \quad \text{in } R \tag{2.1}$$

with boundary conditions

$$u = \bar{u} \text{ On } \partial R^{u}, \quad t = \sigma n = \bar{t} \text{ on } \partial R^{t} \tag{2.2}$$

and constitute equation $\sigma = \hat{\sigma}(X, \in)$. (2.3)

The explicit form of the constitutive equation $\hat{\sigma}$ and density ρ oscillate from phase to phase due to the presence of heterogeneities. These can be denoted as $\hat{\sigma}^{(I)}$ and $\rho^{(I)}$. In such a case, the task to get the solution of the problem is challenging because the stress and strain fields are highly oscillating. A homogenization methodology is needed to construct a homogeneous effective material u^*. The approximate solution is obtained and then the same boundary conditions are applied to R with u replaced by u^*.

Homogenized problem can be represented for M^* as,

Determine $u^*(x,t)$ so that

$$div(\sigma^*) + \rho^* b = \rho^* \ddot{u}^* \text{ in } R \tag{2.4}$$

With boundary conditions

$$u^* = \bar{u} \text{ on } \partial R^u,\ t^* = \sigma^* n = \bar{t} \text{ on } \partial R^t, \tag{2.5}$$

and a constitutive equation

$$\sigma^* = \hat{\sigma}^*(\in^*) \tag{2.6}$$

If an expression for ρ^* i.e. effective density is available and $\hat{\sigma}^*$ i.e. effective constitutive equation then the solution can be obtained. The subject of homogenization is the determination of such effective quantities. The quality of approximation depends on the quality with which the approximations are determined.

III. RESULTS AND DISCUSSION

3.1 Variation of homogenized parameter with volume fraction

The material to be analyzed is steel fibre reinforced concrete. The values of Young's Modulus and Poisson ratio for steel fibre and concrete matrix are:

Steel fibre:

Young's modulus (E_c): 210 GPa

Poisson ratio (v_c): 0.3

Concrete matrix:

Young's modulus (E_m): 25 GPa

Poisson ratio (v_m): 0.2

Two distinct distributions are considered in this example. In one case, the base cell consists of a centrally located single circular fibre, the diameter of which is changed corresponding to various volume fractions. In other case, a random dispersion of circular fibres of different volume fractions constitutes the microstructure. Homogenized material constants of the base cell are conducted for an orthotropic material characteristic under plane stress conditions.

Table 3.1: Homogenized material properties with different volume fractions

Volume Fraction (%)	Young's modulus (GPa)	
	single inclusion	random dispersion
30	36.832	37.132
40	42.095	42.988
50	51.498	51.056
60	76.118	61.661

As is evident, the effective Young's Modulus keeps on increasing with increase in volume fractions for both the cases of single inclusion and multi-inclusions.

3.2 The effect of size of inclusions

The size of inclusions also has a profound effect on the homogenized coefficients. To analyze the same the effect of size of inclusion on the homogenized coefficient is analyzed in this section. The value of homogenized coefficient is determined for small, medium and large sized inclusions.

Two different morphologies are considered for the analysis, one is circular inclusion and other is diamond shaped inclusion or rhombus inclusion. The analysis is performed with single as well as random inclusion for both the types of inclusion. The material taken for the analysis is steel fibre reinforced concrete. The values of the Young's Modulus and Poisson ratio for steel fibre and concrete matrix are already presented. The results of effective Young's Modulus are shown in Table 3.2.

Table 3.2: Representing the values of homogenized coefficient, the effective Young's modulus for different sizes of the inclusions

Size of inclusion	Single Circular	Random Circular	Single Rhombus	Random Rhombus
Small	27.76	27.61	26.04	27.09
Medium	29.70	29.32	28.72	28.2
Large	42.09	42.99	35.34	35.35

From above table, it is observed that the effective Young's Modulus increases with the increase in the size of the inclusions. However the values of effective Young's Modulus are found to be lower for the diamond or rhombus shaped inclusions than that of the circular ones.

3.3 The effect of orientation of inclusions

The particulates in the material have a profound effect on the homogenized properties. The inclusions or particulates can have different shapes, sizes and concentration. Following are the orientations for which the homogenized coefficients are determined

a) Elliptical inclusion with random orientation.
b) Elliptical inclusions with horizontal major axis.
c) Circular inclusions.
d) Elliptical inclusions with vertical major axis.

The material taken for the analysis is boron fibre aluminium matrix. Constituent material properties are:

Boron fiber:

Young's modulus (E_c): 400 GPa
Poisson ratio (v_c): 0.3

Aluminum matrix:

Young's modulus (E_m): 72.0 GPa
Poisson ratio (v_m): 0.3333.

Table 3.3: Homogenized material properties for different orientations of inclusions

Orientation of inclusions	Micromechanical model (GPa)	VCFEM model (GPa)
Elliptical inclusion with random orientation	88.87	89.22
Elliptical inclusion with horizontal orientation	87.77	89.53
Elliptical inclusion with vertical orientation	87.77	88.65
Circular inclusions with random orientation	87.93	89.78

3.4 The effect of shape of inclusions

The effect of different morphologies of the inclusions on maximum numerical volume fraction is discussed. Five different types of morphologies of the inclusions are analyzed for its maximum volume fractions which include

(a) Circular
(b) Square
(c) Star shaped
(d) Elliptical
(e) Hexagonal

In this analysis the numbers of particles of the inclusion are kept fixed to 20 value and the random distribution of these particles are considered. The maximum theoretical volume fraction of the composite is taken as 0.62. The results obtained for maximum volume fraction for 20 numbers of particles and maximum theoretical volume fraction of 0.62 are presented in Table 3.4

Table 3.4: Maximum volume fraction for different shapes of the inclusion phase

Shape of the inclusions	Max. numerical volume fraction
Circular	0.6159
Square	0.3892
Star	0.122
Elliptical	0.3117
Hexagonal	0.3059

From the results presented in Table 3.4, it is observed that the maximum volume fraction near the theoretical one is obtained only in case of circular inclusions whereas the star shape inclusions represents the minimum value of numerical volume fraction. The above results indicate that there is a significant effect of the shape or morphology of the inclusion on the volume fraction of the composite material.

IV. CONCLUSION

In this work the micromechanical analysis of heterogeneous materials is done and the effect of shape, size and orientation of inclusions on its properties is identified. This work is based on the study and analysis of homogenization based micromechanics model incorporating finite element methods. From the above results it is clear that the effective Young's Modulus keeps on increasing with increase in volume fractions for both the cases of single inclusion and multi-inclusions. The effective Young's Modulus increases with the increase in the size of the inclusions. However the values of effective Young's Modulus are found to be lower for the diamond or rhombus shaped inclusions than that of the circular ones. The effective young's modulus shows slight variation with the change in orientation of the inclusions, however it is highest for elliptical inclusions with random orientations. It is observed that the maximum volume fraction near the theoretical one is obtained only in case of circular

inclusions whereas the star shape inclusions represents the minimum value of numerical volume fraction. There is significant effect of shape, size and orientation of inclusions on homogenized parameters.

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