Two layer supply chain with one retailer and two suppliers under random demand, supply disruption and promotional effort

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Abstract

This paper studies the coordination issue of a supply chain consisting of one retailer and two suppliers, one is main supplier and another is backup supplier. The main supplier's yield is subject to disruption and the retailer faces a random demand. We determine the retailer's optimal ordering policy and the main supplier's production quantity that maximize expected profit of the centralized supply chain. Numerical examples are given to gain some qualitative insights.

Keywords: Supply chain management; Random supply; Uncertain demand; Disruption.

1 Introduction

This paper studies the coordination issue of a supply chain among one retailer and two suppliers, one is main supplier and another is backup supplier under . Few researchers already have done important work in the area of random demand along with supply disruption. We are going to consider the supplier's random yield, along with supply disruption, during which the output of the production is zero in this research. Apart from this what we are trying to do new here is that we consider the effect of promotional effort in the supply chain along with the supplier's random yield and supply disruption. We also propose an overproduction risk sharing and buy-back contracts with a side payment for supply chain coordination. Our work aims upon two areas of research: random yield with disruption and promotional effort and buy-back contracts in a supply chain with demand uncertainty.

In any production or logistics process random yield is somewhat a common issue. It is related to the fact that putting equal amount of input, the output of the production varies. Due to damage that occurs during transferring, any transportation process can also be viewed as random yield process. Some random yield process are for example, if the cost of tracking partial orders is high or the transportation costs are high, the supply contract may specify delivery in a single shipment with the uncertainty in delivery time, one can refer to the papers of H. Gurnani, R. Akella, and J. Lehoczky,(1996); in other situations, the manufacturer may agree to accept partial shipment of the order quantity like it is mentioned in K.Moinzadeh an H. L. Lee, (1989).

In the last few decades many researchers have done work in the area of random yield which is worth to mention. There is a widespread literature in the field of random yield. Mukhopadhyay and Ma (2009) developed a single-period model to evaluate the optimal procurement and production decisions with uncertain demand and random yield of the used parts under three different cases. Gerchak and Grosfeld-Nir
(1998) analysed the trade-off between set-up cost and production cost when making batch production decisions, where both the random yield and random demand follow a general discrete distribution. He and Zhao (2012) investigated the ordering policy of the retailer, raw material planning decision of the supplier, and the optimal contracts for a three-level supply chain with random yield and demand.

Now apart from random demand, supply chain disruption has also caught the attention of the researchers. Industries across the world have experienced losses from a variety of disruptions; the major disruptions include massive oods, hazardous chemical explosions, industrial strikes, and extreme weather condition or natural calamity. When the supply chain (SC) reacts to these disruptions by implementing a mitigation and recovery strategy, the main goals are to maintain or resume the continuity of the operation while meeting customer expectations and minimizing potential negative impacts. This complex optimization problem has motivated many researchers to study on supply chain disruption management (SCDM). A literature review by Ivanov et al. (2016) analysed quantitative studies that focused on reactive approach to face supply chain disruption. According to the authors, three basic risks to be considered are production, supply and transportation disruptions with common problems whether there are measure for recovery or without any.

As we have already mentioned that in this paper the two main areas of our research are random yield with disruption and promotional effort and buy back contracts in a supply chain with demand uncertainty. Many researcher have worked on different kind of contracts. For example in the model of Guler and Bilgic (2009), they studied the coordination of an arbitrary number of suppliers with random demand and random yield and established the concavity of expected supply chain profit and proposed two mixed type of contracts to coordinate the chain under forced compliance. Under the wholesale price contract, Keren (2009) analysed a two-tier supply chain, where a distributor facing a deterministic demand procures a product from a producer confronting a random production yield. And an analytical solution to the distributor’s ordering decision is derived when the production yield follows the uniform distribution. Li et al. (2012) extended and provided new results on the supply chain model with producer’s random yield proposed by Keren (2009). They derived analytic solutions of the supply chain decisions under generalized yield distribution.

Now we discuss about the promotional effort in the supply chain. In recent times, several researchers have been working on the cooperative advertising policy in a manufacturer-retailer channel. This type of collaboration between two members of the supply chain can be defined as nancial agreement in which manufacturer agrees upon to share cost of promotional effort and offers to bear either certain part or the entire advertising expenditure of his retailer. Advertising is one of the most powerful and major tools used by the companies to target large number of buyers and populations. It consists of impersonal forms of communication conducted through paid media under clear sponsorship. Advertising can be used to build up a long term image for a product or to embark quick sale Kotler,(2001). Battberg and Neslin (1989) have discussed about promotion effects the sale to what extent. Krishnan et al.(2004) determined the promotional effort to maximize the revenue and explained that promotional efforts may include anything starting from offering free gifts to customer then price cuts, discounts in price, special services and many such more attractive incentives. Abad (2003) considered the retailer’s pricing and lot sizing policies under supplier’s trade promotion. Kurata and Liu (2007) contemplated how a retailer reasonably decides upon the price discount promotion. Szmerekovsky and Zhang (2009) considered the pricing decisions and two-tier advertising levels between one manufacturer and one retailer where the customer demand depends on the retail price and
advertisement by a manufacturer and a retailer. Xie and Wie (2009) and Xie and Neyret (2009) determined
the optimal cooperative advertising strategies and equilibrium pricing in a two-echelon distribution channel.

Supply disruption along with promotional e ort is not considered by the above-cited literature on ran-dom
yield. However, in practical reality, a supplier may be unable to satisfy the production order for a variety
of reasons, such as equipment failures, damaged facilities, problems in procuring the necessary raw materials,
and so forth. With more and more enterprises starting to realize that supply disruption severely a ects their
ability to successfully manage their own supply chains, supply disruption manage-ment has received
increasing attention. Many researchers have devoted much e ort to studying this issue. Hendricks and
Singhal (2013) estimated the short-term e ects of supply disruption such as pro-duction or shipment delays
on shareholder value. Taking into account the disruption frequency and the loss of market share, Pochard
(2003) analysed the value and the bene ts of dual sourcing. Sarkar and Mohapatra (2009) considered the
risks of supply disruption due to occurrence of super, semi super, and unique events and determined the
optimal size of supply base. Since double marginalization J. Spengler (1950) will directly lead to ine cient
performance of the supply chain, coordination of activities among the di erent members in the supply chain
is necessary for the whole supply chain’s e ective manage-ment. A great deal of e ort has been devoted
to the research of the supply chain coordination issues. All kinds of popular contracts have been
explored in the literature for the supply chain coordination, such as buy-back contracts or returns
revenue sharing contracts Y. Gerchak and Y. Wang (2004) and G. P. Cachon and M. A. Lariviere
(2005), wholesale price contracts M. A. Lariviere and E. L. Porteus (2001), risk sharing contracts C. L.
Li and P. Kouvelis (1999), quantity discount policies C. Corbett and X. Groote (2000), quantity exibility
contracts A. A. Tsay (1999), sales rebate contracts T. A. Taylor (2002), and so on. These explored a
variety of other combined contracts and found which can be applied properly.

The rest of the paper is organized as follows: Section 2 explains the fundamental assumptions and
notations and assumptions, Section 3 provides mathematical formulation and analysis of the model. In
section 4 we have discussed some numerical examples, section 5 is dedicated to illustrate the
sensitivity analysis, the last section i.e, section 6 is for to draw conclusion on the ndings of the paper.

2 Fundamental Notations and Assumptions

The following notations and assumptions are made to develop the proposed model:

2.1 Notation

(i) $c_m$ = Production cost ($/unit) of the main supplier’s.

(ii) $c_b$ = Production cost ($/unit) per unit of the backup supplier.

(iii) $c_u$ = Shortage cost ($/unit) per unit of the retailer.

(iv) $s$ = Unit selling price (retail price) ($/unit) of the retailer.

(v) $c_m$ = Marginal cost incurred due to event of disruption in supply.

(vi) $v$ = Unit salvage value/return price ($/unit) of unsold goods of the retailer provided by the
manufacturer.

(vii) $w_m$ = The primary supplier whole price($/unit).
web= The secondary supplier whole price($/unit).

(ix) p= the probability of disruption of yield.

(x) x= A part of demand quantity (units/month) during a period, which is a random variable following probability distribution.

(xi) f(x)= Probability density distribution function of x.

(xii) F (x)= Cumulative distribution function of x.

(xiii) L= The EPQ (economic production quantity) of the primary supplier (i.e., how many product to be produced by the primary supplier).

(xiv) Q= The EPQ (economic production quantity) of the secondary supplier (i.e., how many product to be produced by the secondary supplier).

(xv) Y = A random yield variable and positive support on [0,1] which satisfies P fY = 0g = p and P f0 < Y < 1g = \int_0^1 g(y)dy = 1 p = q.

(ix) g(y)= Probability density distribution function of y. Here g(y) is not probability density function unless p = 0.

(xvi) = Promotional/advertising e ort (units/month).

(xvii) D(x; ; )= Demand (units/month) which is a function of the promotional e ort.

(xviii) E_c(Q; L; )= Expected profit ($/month) function of the chain in centralized model.

2.2 Assumptions

(i) The supply chain model is developed for a single period item.

(ii) The model associated with two-echelon supply chain comprising one retailer and two suppliers.

(iii) The Demand rate of the chain is assumed to be the function of promotional e ort cost.

(iv) Among the two suppliers one is primary/main supplier and another is secondary/backup.

(v) Primary supplier is not reliable with cheaper wholesale price whereas secondary supplier is not reliable with expensive wholesale price.

(vii) The secondary supplier’s production has a perfect yield as secondary supplier can convert similar products in the inventory to satisfy the order. The chain is with buyback policy.

(viii) The lead time is negligible, and replenishment rate is instantaneously in nite but its size is nite.

(ix) This depicts the supply chain having o -shoring situation.

(x) The customer demand rate is partly dependent on promotional e ort and uncertain factors. So, it is a combination of a promotional e ort variable and an uncertain variable.

3 Mathematical Formulation and Analysis of the Model

In this model we have considered a two layer supply chain with two supplier and one retailer. In this two-echelon supply chain the primary supplier sells its product through one retailer. We consider that the retailer may face the same supply disruption. That is why there is another supplier who is known as secondary supplier. We assume that the secondary supplier’s production has a perfect yield, for
example, the secondary supplier can convert similar or better products from his inventory to satisfy the order. There is a buyback policy between the supplier and the retailer. To ensure that each member of the chain has a positive profit so we assume the following inequality

\[ C_b > C_m > v \]
\[ S > W_b > C_b \]
\[ S > W_b > C_m. \]

The retailer and the suppliers are two risk-neutral rms that are controlled by a centralized decision maker. The retailer incurs some promotional e ort cost to increase the different product sales on the market. So, the market demand is influenced by the advertising expenditure incurred by the retailer that results in promoting the retail sales.

We define the demand function as

\[ D(x; \theta) = x \theta \]

where \( x \) is a random variable that follows the p.d.f \( f(x) \)

\[ \frac{1}{1 + (1 + (1 + \theta)^2)^2} \]

Here, \( \theta \) is a decision variable (the e ort for promotional activities) and is a positive constant which is estimated from previous data by any curve fitting method; \( \theta \) is an increasing function of because

\[ G(\theta) = k^m \]

where \( k \) is a scale parameter and \( m \) is an elasticity parameter, and both are positive constants.

3.1 Centralized Supply Chain

In Centralized model of this system, both supplier and retailer make decision after consulting among themselves. Therefore, important strategies such as deciding on the optimal order quantity, the optimal production quantity and optimal promotional e ort are determined by the both supplier and retailer as a joint venture. The prime objective of the members of the chain is to maximize the integrated expected profit of the system. To establish a performance benchmark, we rst analyse the optimal solution of an integrated supply chain. So, the expected integrated supply chain profit in the centralized model is given by

\[ E_c(Q; L; \theta) = sE \min(x(\theta); Ly + Q) + vE(Ly + Q X(\theta))^+ \
+ c_d E(X(\theta) Y L Q)^+ \]

\[ + c_b Q \]

\[ + c_m L G(\theta) \]

\( (3) \)
where \( = p + q \). The rst term in (3) is the expected revenue from sales, the second term is the salvaged value, the third term is the opportunity cost due to the lost sales, forth and fth term are the production costs and last term related to promotional e ort.

From (3), the integrated supply chain’s expected pro t function can rewritten as follows:

\[
E_c(Q; L; ) = p \left( \int_{0}^{1} [s(x) + q(L + Q - x)]f(x)dx \right) + c_u \left( \int_{0}^{1} f(x)dx \right)
\]

\[
+ \left( \int_{0}^{1} c_u(x) (L + Q)f(x)dx \right)
\]

\[
+ \left( \int_{0}^{1} s(L + Q)c_u(x) (L + Q)f(x)dx \right) \quad g(y)dy \quad c_p Q \quad c_m L \quad G( )
\]

(4)

Di erentiating \( E_c \) with respect to \( Q; L \) and , we have

\[
\frac{\partial E_c}{\partial Q} = \left( \int_{0}^{1} s(x) + q(L + Q) f(x)dx \right) + c_u \left( \int_{0}^{1} f(x)dx \right)
\]

\[
+ \left( \int_{0}^{1} c_u(x) (L + Q)f(x)dx \right) \quad g(y)dy \quad c_p Q \quad c_m L \quad G( )
\]

(5)

\[
\frac{\partial^2 E_c}{\partial Q^2} = \left( \int_{0}^{1} s(x) + q(L + Q) f(x)dx \right) + c_u \left( \int_{0}^{1} f(x)dx \right)
\]

\[
+ \left( \int_{0}^{1} c_u(x) (L + Q)f(x)dx \right) \quad g(y)dy \quad c_p Q \quad c_m L \quad G( )
\]

(6)

\[
\frac{\partial^2 E_c}{\partial Q \partial L} = \left( \int_{0}^{1} s(x) + q(L + Q) f(x)dx \right) + c_u \left( \int_{0}^{1} f(x)dx \right)
\]

\[
+ \left( \int_{0}^{1} c_u(x) (L + Q)f(x)dx \right) \quad g(y)dy \quad c_p Q \quad c_m L \quad G( )
\]

(7)

\[
\frac{\partial^2 E_c}{\partial Q \partial L} = \left( \int_{0}^{1} s(x) + q(L + Q) f(x)dx \right) + c_u \left( \int_{0}^{1} f(x)dx \right)
\]

\[
+ \left( \int_{0}^{1} c_u(x) (L + Q)f(x)dx \right) \quad g(y)dy \quad c_p Q \quad c_m L \quad G( )
\]

(8)

\[
\frac{\partial^2 E_c}{\partial Q^2} = \left( \int_{0}^{1} s(x) + q(L + Q) f(x)dx \right) + c_u \left( \int_{0}^{1} f(x)dx \right)
\]

\[
+ \left( \int_{0}^{1} c_u(x) (L + Q)f(x)dx \right) \quad g(y)dy \quad c_p Q \quad c_m L \quad G( )
\]

(9)

\[
\frac{\partial^2 E_c}{\partial Q^2} = \left( \int_{0}^{1} s(x) + q(L + Q) f(x)dx \right) + c_u \left( \int_{0}^{1} f(x)dx \right)
\]

\[
+ \left( \int_{0}^{1} c_u(x) (L + Q)f(x)dx \right) \quad g(y)dy \quad c_p Q \quad c_m L \quad G( )
\]

(10)

\[
\frac{\partial^2 E_c}{\partial Q \partial L} = \left( \int_{0}^{1} s(x) + q(L + Q) f(x)dx \right) + c_u \left( \int_{0}^{1} f(x)dx \right)
\]

\[
+ \left( \int_{0}^{1} c_u(x) (L + Q)f(x)dx \right) \quad g(y)dy \quad c_p Q \quad c_m L \quad G( )
\]

(11)
Here in above we have partially differentiated the profit function in Equation (4) to evaluate the optimal (maximum) profit of the coordinated or centralized environment. We can characterize the optimal planned order quantity $Q$, the optimal order quantity $L$ and Promotional/advertising effort through the rst-order conditions by equating the Equation (5), (6) and (7) to 0 and thereafter solved by numerically (using Mathematica 14.0). The following Proposition 3.1 states that the objective function in (4) is concave. Hence, the optimal production quantity, order quantity and promotional effort from the secondary supplier can be determined easily.

Proposition 3.1 The integrated supply chain expected profit function $E_c(Q; L; \gamma)$ is jointly concave in $Q; L$ and $\gamma$.

Proof To check whether the profit function $(E_c(Q; L; \gamma))$ is concave, we determine its Hessian matrix $H(E_c(Q; L; \gamma))$.

$$H(E_c(Q; L; \gamma)) = \begin{bmatrix}
\frac{\partial^2 E_c}{\partial Q^2} & \frac{\partial^2 E_c}{\partial Q \partial L} & \frac{\partial^2 E_c}{\partial Q \partial \gamma} \\
\frac{\partial^2 E_c}{\partial L \partial Q} & \frac{\partial^2 E_c}{\partial L^2} & \frac{\partial^2 E_c}{\partial L \partial \gamma} \\
\frac{\partial^2 E_c}{\partial \gamma \partial Q} & \frac{\partial^2 E_c}{\partial \gamma \partial L} & \frac{\partial^2 E_c}{\partial \gamma^2}
\end{bmatrix}$$

Here, the leading principle minors of $H(E_c(Q; L; \gamma))$ are

$$D_{11} = \left(\frac{\partial^2 E_c}{\partial Q^2}\right)_{Q, L} > 0; \quad 8s > v;$$

$$D_{22} = \left(\frac{\partial^2 E_c}{\partial L^2}\right)_{Q, L} = \left(\frac{\partial^2 E_c}{\partial L^2}\right)_{Q, L} > 0; \quad 8s > v;$$

$$D_{33} = \left(\frac{\partial^2 E_c}{\partial \gamma^2}\right)_{Q, L} > 0; \quad 8s > v;$$

$$D_{12} = \frac{\partial^2 E_c}{\partial Q \partial L} = \left(\frac{\partial^2 E_c}{\partial Q \partial L}\right)_{Q, L} > 0; \quad 8s > v;$$

$$D_{13} = \frac{\partial^2 E_c}{\partial Q \partial \gamma} = \left(\frac{\partial^2 E_c}{\partial Q \partial \gamma}\right)_{Q, L} > 0; \quad 8s > v;$$

$$D_{23} = \frac{\partial^2 E_c}{\partial L \partial \gamma} = \left(\frac{\partial^2 E_c}{\partial L \partial \gamma}\right)_{Q, L} > 0; \quad 8s > v;$$

$$D_{11} D_{22} D_{33} - D_{12} D_{13} D_{23} > 0; \quad 8s > v;$$
Here, if we assume that $A < 0$ and $B < 0$, then $AB C^2 > 0$ and $D_{33} < 0$. Again, if we assume that $A > 0$ and $B > 0$, then $AB C^2 > 0$ and $D_{33} > 0$. Therefore, in this policy we assume that $A < 0$ and $B<0$.

So, the values of the all leading principle minors $D_{11}$; $D_{22}$ and $D_{33}$ of the Hessian matrix $H(E_C(Q; L; )$ are alternate sign for optimal values of $Q; L$ and $\lambda$. Therefore, the Hessian matrix $H(E_C(Q; L; )$ is negative definite, and consequently the function $E_C(Q; L; )$ is concave.
Proposition 3.2 If
\[ A = \int_{0}^{1} g(y)dy; \quad B = \int_{0}^{1} \left( \frac{p f}{g(y)} \right) + \int_{0}^{1} \left( \frac{q}{g(y)} \right) g(y)dy \]
and \( s > v \), then optimal value of \( Q; L; \& \) are uniquely solved by
\[ p F \left( \frac{1}{h} \right) + \int_{0}^{1} F \left( \frac{1}{h} \right) g(y)dy = \frac{s v + c u}{s + c} \]
(15)
(16)
(17)

From the Proposition 3.2, we can obtain the maximum expected profit of the integrated supply chain. Determine the value of \( Q; L; \) and (using Mathematica 9.0). If the total profit function of the chain \( E_{C}(Q; L; \) attained maximum value for \( Q; L; \) and, then this values of \( Q; L; \) and are called optimal solution for this integrated supply chain model.

Here, in Proposition 3.1, we are going to see the objective function (3) is concave or not, and in Proposition 3.2, we shall determine the optimal (maximum) value of \( Q; L; \) and for the profit function \( E_{C}(Q; L; \).

4 Numerical example

When demand follows the function \( D(x, \gamma) = x^{*} (\gamma) \) and \( x \) follows the normal distribution i.e.,
\[ f(x) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left( -\frac{(x - \mu)^2}{2\sigma^2} \right); \quad \text{for} \quad -\infty < x < \infty \]

We consider the values of the key parameters in appropriate units as follows: \( m_x = 100, \), \( \sigma = 5, \) \( c = 0.35, \) \( d = 1, \) \( c_u = 5, \) \( s = 27, \) \( v = 5, \) \( c_b = 12, \) \( c_m = 4, \) \( \gamma = 5, \) \( \lambda = 0.25, \) \( m = 2, \) \( p = 0.25, \) \( q = 0.75, \) \( k = 7 \)
Then, the optimal solutions are \( L = 747.890 \) \( Q = 513.03 \), \( c = 8.6800 \) and the maximum profit is \( E_{C} = 9685.95 \).

5 Sensitivity Analysis

Now we will study the sensitivity analysis of the key parameters and the features of analysis have been discussed below. Here we have studied the changes of optimal variables and profit with (30%; 20%; 10%; + 10%; +20%; +30%) changes in the key parameters. The key parameters that are considered here are the retail selling price per unit of retailer (s), unit production of the manufacturer (c_m), shortage cost per unit (c_u), unit salvage value (v)(cf. Table 3) and the unit production cost of the back-up supplier.
We observed the change in the nature of the optimal solution due to the sensitivity analysis and presented the results of sensitivity analysis for the parameters $s$ (cf. Table 1), $c_u$ (cf. Table 2), $v$ (cf. Table 3), $c_b$ (cf. Table 4) and $c_m$ (cf. Table 5) in the following tables:

### Table 1: Sensitivity Analysis of $s$ (Unit selling price of the retailer)

<table>
<thead>
<tr>
<th>Change of $s$ (%)</th>
<th>Optimal values of variables</th>
<th>$L$</th>
<th>$Q$</th>
<th>$E_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30 %</td>
<td>846.882</td>
<td>412.324</td>
<td>6.8850</td>
<td>4902.72</td>
</tr>
<tr>
<td>-20 %</td>
<td>804.823</td>
<td>453.169</td>
<td>7.5667</td>
<td>6466.06</td>
</tr>
<tr>
<td>-10 %</td>
<td>773.038</td>
<td>458.877</td>
<td>8.1567</td>
<td>8063.30</td>
</tr>
<tr>
<td>-0.0%</td>
<td>747.890</td>
<td>513.030</td>
<td>8.8800</td>
<td>9685.95</td>
</tr>
<tr>
<td>+10 %</td>
<td>727.334</td>
<td>536.184</td>
<td>9.1548</td>
<td>11328.5</td>
</tr>
<tr>
<td>+20 %</td>
<td>710.103</td>
<td>556.296</td>
<td>9.5889</td>
<td>12987.1</td>
</tr>
<tr>
<td>+30 %</td>
<td>695.378</td>
<td>574.041</td>
<td>9.9056</td>
<td>14659.1</td>
</tr>
</tbody>
</table>

### Table 2: Sensitivity Analysis of $c_u$ (Under stock cost of the retailer)

<table>
<thead>
<tr>
<th>Change of $c_u$ (%)</th>
<th>Optimal values of variables</th>
<th>$L$</th>
<th>$Q$</th>
<th>$E_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30 %</td>
<td>763.967</td>
<td>500.344</td>
<td>8.6937</td>
<td>9728.30</td>
</tr>
<tr>
<td>-20 %</td>
<td>758.387</td>
<td>504.712</td>
<td>8.6893</td>
<td>9713.81</td>
</tr>
<tr>
<td>-10 %</td>
<td>753.033</td>
<td>508.940</td>
<td>8.6850</td>
<td>9699.69</td>
</tr>
<tr>
<td>-0.0%</td>
<td>747.890</td>
<td>513.030</td>
<td>8.6800</td>
<td>9685.95</td>
</tr>
<tr>
<td>+10 %</td>
<td>742.950</td>
<td>517.011</td>
<td>8.6768</td>
<td>9672.54</td>
</tr>
<tr>
<td>+20 %</td>
<td>738.193</td>
<td>520.867</td>
<td>8.6728</td>
<td>9659.47</td>
</tr>
<tr>
<td>+30 %</td>
<td>733.611</td>
<td>524.613</td>
<td>8.6689</td>
<td>9646.72</td>
</tr>
</tbody>
</table>

### Table 3: Sensitivity Analysis of $v$ (Unit salvage value/return price of goods unsold by the retailer to manufacturer)

<table>
<thead>
<tr>
<th>Change of $v$ (%)</th>
<th>Optimal values of variables</th>
<th>$L$</th>
<th>$Q$</th>
<th>$E_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30 %</td>
<td>590.022</td>
<td>418.895</td>
<td>9.1740</td>
<td>8937.01</td>
</tr>
<tr>
<td>-20 %</td>
<td>577.526</td>
<td>419.208</td>
<td>8.6893</td>
<td>9248.89</td>
</tr>
<tr>
<td>-10 %</td>
<td>568.315</td>
<td>519.076</td>
<td>8.6088</td>
<td>9451.20</td>
</tr>
<tr>
<td>-0.0%</td>
<td>747.890</td>
<td>513.030</td>
<td>8.6800</td>
<td>9685.95</td>
</tr>
<tr>
<td>+10 %</td>
<td>1404.18</td>
<td>485.739</td>
<td>8.7768</td>
<td>10004.6</td>
</tr>
<tr>
<td>+20 %</td>
<td>3122.69</td>
<td>337.227</td>
<td>10.033</td>
<td>19825.4</td>
</tr>
<tr>
<td>+30 %</td>
<td>3808.55</td>
<td>225.199</td>
<td>13.629</td>
<td>27941.7</td>
</tr>
</tbody>
</table>

### Table 4: Sensitivity Analysis of $c_b$ (Production cost of the backup supplier)

<table>
<thead>
<tr>
<th>Change of $c_b$ (%)</th>
<th>Optimal values of variables</th>
<th>$L$</th>
<th>$Q$</th>
<th>$E_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30 %</td>
<td>140.876</td>
<td>795.157</td>
<td>9.3270</td>
<td>11968.3</td>
</tr>
<tr>
<td>-20 %</td>
<td>367.573</td>
<td>670.618</td>
<td>9.0905</td>
<td>11095.7</td>
</tr>
<tr>
<td>-10 %</td>
<td>556.629</td>
<td>585.308</td>
<td>8.8768</td>
<td>10344.2</td>
</tr>
<tr>
<td>-0.0%</td>
<td>747.890</td>
<td>513.030</td>
<td>8.6800</td>
<td>9685.95</td>
</tr>
<tr>
<td>+10 %</td>
<td>958.752</td>
<td>445.065</td>
<td>8.5024</td>
<td>9111.23</td>
</tr>
<tr>
<td>+20 %</td>
<td>1205.07</td>
<td>375.935</td>
<td>8.3433</td>
<td>8618.24</td>
</tr>
<tr>
<td>+30 %</td>
<td>1513.43</td>
<td>299.391</td>
<td>8.2074</td>
<td>8211.88</td>
</tr>
</tbody>
</table>
Table 5: Sensitivity Analysis of $c_m$ (Production cost of the main supplier)

<table>
<thead>
<tr>
<th>Change of $c_m$ (%)</th>
<th>Optimal values of variables</th>
<th>L</th>
<th>Q</th>
<th>$E_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30 %</td>
<td></td>
<td>1333.57</td>
<td>825.309</td>
<td>7.3354</td>
</tr>
<tr>
<td>-20 %</td>
<td></td>
<td>1186.77</td>
<td>973.271</td>
<td>9.9623</td>
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<tr>
<td>-10 %</td>
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<td>2072.06</td>
<td>427.710</td>
<td>8.7951</td>
</tr>
<tr>
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<td></td>
<td>747.890</td>
<td>513.030</td>
<td>8.6800</td>
</tr>
<tr>
<td>+10 %</td>
<td></td>
<td>463.104</td>
<td>556.799</td>
<td>8.6226</td>
</tr>
<tr>
<td>+20 %</td>
<td></td>
<td>308.934</td>
<td>591.343</td>
<td>8.5842</td>
</tr>
<tr>
<td>+30 %</td>
<td></td>
<td>196.045</td>
<td>623.242</td>
<td>8.5588</td>
</tr>
</tbody>
</table>

6 Conclusion

This paper makes several theoretical and practical contributions applied in industries. This paper studies the coordination issue of a supply chain consisting of one retailer and two suppliers, one is main supplier and another is backup supplier. It generalizes the framework for finding the economic order quantity where the main supplier’s yield is subject to disruption and the retailer faces a random demand. We determine the retailer's optimal ordering policy and the main supplier’s production quantity that maximize expected profit of the centralized supply chain. Supply disruption and random yield of the manufacturer along with the buyback contract and promotional effort have been the core area of concentration in this research work. While considering the sales price and advertising/promotional costs shared by the manufacturer and the retailer for uncertain demand of the end customers are considered altogether. It also demonstrates how the coordinating contract of incentives controls the overall performance of the chain. The generalized model is tested by usual distribution function applied in industries. These results are very useful for the manufacturer and the retailer to decide how to coordinate the supply chain.

For further research interest, it is worth considering some other situations such as multiobjective supply chain in the system or taking the demand to be deterministic type or introducing some possibilities of deteriorating items etc. To add to the list, the proposed model can even be further extended in several ways. For example, we may generalize the model to allow idle times, finite replenishment rates, variable production rates etc.
References


