Analytical study of wave propagation in micro polar elastic medium

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Abstract

In present investigation, propagation of wave in micropolar elastic medium at non-free surface is discussed. The amplitude ratio’s of longitudinal displacement wave (LD wave), coupled transverse displacement (CD-I) and transverse rotational wave (CD-II) are obtained for incident waves.

Key Words: Micropolar elastic medium, non-free surface, wave propagation, amplitude ratio.

1 Introduction

Classical theories of elasticity are not able to examine the behaviour of materials having brous or course grain structure etc. When the microstructure of the material was considered to be rigid, it leads to the micropolar theory. This theory is more dependable for geological materials like solis and rocks as it accounts the intrinsic rotation and estimates the inner structure of the material. Eringen [1] introduced a new formulation of equations in thermoelasticity which was known as the equations for the micropolar elastic theory. Sharma [2] investigated impact of relaxation times and two temperatures on coe cients of re ection in a half-space of micropolar thermoelastic solid.

Fu and Wei [3] investigated the transmission and re ection problem at the imperfect interface of the coupled transverse displacement and transverse rotational waves between two dissimilar micropolar solids. They discussed the impact of imperfect degree of interface on the transmission and re ection coe cients. Khurana and Tomar [4] observed propagation of plane waves (two longitudinal waves and two sets of coupled transverse waves) for a nonlocal isotropic micropolar solid and derived re ection coe cients and energy ratios when these waves incidents at stress-free boundary. Singh et.al [5] considered problem on Rayleigh wave for an rotating half-space in an orthotropic micropolar material and solved equations for the surface wave in the half space. They obtained the results to show the influence of orthotropy, rotation and non-dimensional frequency of the Rayleigh wave.

Zhang et.al [6] calculated the amplitude ratios of re ected waves for di erent incident waves and also, re ection coe cients in terms of energy ux ratios at non-free surface of a micropolar elastic half-space. Hassanpour and Heppler [7] reviewed the linear isotropic theory of micropolar elasticity with special attention on the notation, which are used for the representation in the micropolar elastic moduli and the experimental actions are taken to measure them. Videla and Atroshchenko [8] derived the analytical solution subjected to a remote uni-axial tension for the problem of a circular micropolar inhomogeneity in an in nite micropolar plate in homogeneous imperfect interface. They showed dependence of stress concentration factors on the micropolar material constants .

Gade and Ragunath [9] explored reduced micropolar theory to replicate ground motion during an earthquake. They calculated the expressions of ground displacement and rotational motions analytically for the case of buried seismic source. Singh [10] investigated a problem on Rayleigh surface...
wave in an isotropic micro-polar elastic solid half-space with impedance boundary conditions and derived a secular equation for non dimensional speed of the Rayleigh wave, which depends upon various parameters of material, frequency, micro-rotation and impedance parameters. Fan and Cheng [11] presented a elastic model set based on micromechanics in the framework of micropolar theory having two-phase FGMs to study the impact of size on the effective properties of the FGM and compared those results with experimental data.

2 Field Equations

Following Eringen [1], the basic equations and constitutive relations in micropolar elastic medium are:

\[
\begin{align*}
\left( +k+2 \right) \left( u \right) + k\left( \mathbf{i} \right) + \left( \mathbf{k}+\mathbf{i} \right) & = \frac{\partial u}{\partial t} \quad \text{(1)} \\
\frac{\partial u}{\partial r} \frac{\partial u}{\partial r} & = \left( \mathbf{t}_i \right) \left( \mathbf{u} \right) + \frac{k}{2} \left( \mathbf{u} \right) \quad \text{(2)} \\
\mathbf{t}_{ij} & = \mathbf{u}_{r;ij} + \left( \mathbf{u}_{j;i} + \mathbf{u}_{i;j} \right) + \mathbf{k} \left( \mathbf{ijm} \mathbf{m} + \mathbf{ijl} \mathbf{l} \right) \quad \text{(3)} \\
\mathbf{m}_{ij} & = \mathbf{r}_{ij} + \mathbf{r}_{ij} + \mathbf{k} \quad \text{(4)}
\end{align*}
\]

\( \mathbf{t} \) - Lame's constants, \( \mathbf{t} \) - time, \( \mathbf{t} \) - coefficient of linear thermal expansion, \( \mathbf{C} \), - specific heat and density, \( \mathbf{t}_{ij} \) - components of stress tensor, \( \mathbf{t}_{ij} \) - Kronecker delta, \( \mathbf{u} \) - displacement vector, \( \mathbf{k} \), and \( \mathbf{t} \) are micropolar constants,
mi_j - couple stress tensor components, ij_m is alternating tensor, k is microrotation vectors.,

\[ r = i \hat{x} + j \hat{y} + k \hat{z} ; \quad r = \hat{x}^2 + \hat{y}^2 + \hat{z}^2 : \]

3 Formulation and the solution

We have taken a homogeneous, isotropic with micropolar in elastic half space on non free surface. The rectangular cartesian co-ordinate system \((x_1, x_2, x_3)\) having origin at interface \(x_3 = 0\) is considered along with \(x_3\)-axis pointing normally into medium. Plane waves in \(x_1, x_3\)-plane are considered in which wave front is parallel to \(x_2\)-axis, therefore all variables will depend on \(x_3, x_1\) and \(t\). Thus problem considered in two dimensional, so we take

\[ u = (u_1; 0; u_3) \]

To ease the solution, quantities having no dimensions are introduced as follows:

\[ x^0 = \frac{l_1}{c_1} x_1 ; \quad x^0 = \frac{l_1}{c_3} x_3 ; \quad u^0 = \frac{l_1}{c_1} u_1 ; \quad u^0 = \frac{l_1}{c_3} u_3 ; \]

\[ t^0 = \frac{l_1}{c_3} t_3 ; \quad t^0 = \frac{l_1}{c_3} t_3 ; \quad \theta = \frac{2}{c_1} \beta ; \quad m^0 = m^0_{ij} \]

where

\[ c_1^2 = 2 + k \quad \text{and} \quad l_1 = \frac{k}{j} : \]

The expression related to components of displacement are expressed by using Helmholtz decomposition, therefore \(u_3\) and \(u_1\) are related to the and (scalar potential functions) having no dimensions are given by

\[ u_1 = \frac{\hat{x}_3}{c_1} ; \quad u_3 = \frac{\hat{x}_3}{c_3} + \hat{x}_1 : \]

using equations (5)-(6) in (1)-(4) and assuming the motion to be harmonic and for solving the equations we assume solutions in the form

\[ (; ; 2) = (0; 0; 0) e^{ix_1 \sin x_3 \cos + t} , \]

where denoted as wave number, is known as iota, is angle of inclination and quantities such as 0, 0, 0 are arbitrary constants. Using the values of 0, 0, 2 we obtained following equations
\[(4 + A^2 + B)(2; ) = 0\]  
\[(2, 1) = 0\]

where \(= \) represents the velocity of various waves; \(1, 2, 3\) are velocities of the longitudinal displacement (LD) wave, coupled transverse displacement (CD-I) wave and transverse rotational (CD-II) wave respectively and

\[
an_1 = \frac{2k}{1}; \quad a_2 = \frac{+ k}{c_1}; \quad a_3 = \frac{k^2}{c_1^2}; \quad a_4 = \frac{1}{c_1}; \quad a_5 = \frac{c^2}{c_1^2};
\]

\[
a_6 = \frac{1}{c_1^2}; \quad a_7 = \frac{2 + k}{c_1^2}; \quad a_8 = \frac{1}{c_1^2}; \quad a_9 = \frac{a_2}{c_1^2}; \quad a_{10} = \frac{a_1}{c_1^2}; \quad a_{11} = \frac{1}{c_1^2}; \quad a_{12} = \frac{1}{c_1^2}; \quad A = \frac{a_2 a_6}{c_1^2}; \quad B = \frac{a_2}{a_6}; \quad a = \frac{1}{c_1^2}; \quad c_1^4 = \frac{1}{a^2}; \quad a_6 = \frac{1}{a^2};
\]

4 Boundary conditions

Appropriates conditions at surface \(x_3 = 0\) are

\[
(i) \quad t_{13} = S_1 u_3;
\]

\[
(ii) \quad t_{31} = S_2 u_1;
\]

\[
(iii) \quad m_{32} = S_3 2
\]

We assume that the values of \(, 1, 2,\)

\[
= A_0 e^{k(x, \sin x_3 \cos + l)} + A_1 e^{k(x, \sin x_3 \cos + l)} (13)
\]

\[
= B_0 e^{k(x, \sin x_3 \cos + l)} + B_1 e^{k(x, \sin x_3 \cos + l)} (14)
\]

\[
= \frac{X_{di} B_0 e^{k(x, \sin x_3 \cos + l)} + B_1 e^{k(x, \sin x_3 \cos + l)}}{a_2 k^2}
\]

where

\[
d_i = \frac{a_2 k^2}{a_3}; \quad (i = 1, 2)
\]

where the values of \(d_i\) are coupling constants. \(B_0, B_1\) are the amplitude of incident coupled transverse displacement (CD-I) and transverse rotational wave (CD-II)
and $A_0$ is the amplitude of the incident L-D wave (Longitudinal Displacement wave). $B_i$ are the amplitude of the reflected coupled waves i.e transverse rotational and transverse displacement wave and $A_1$ is the amplitude of the reflected L-D wave (Longitudinal Displacement wave). Using Snell’s Law defined as follows

$$\sin \theta_0 = \sin \theta_1 = \sin \theta_2 = \sin \theta_3$$

(16)

where

$$1: 22 = 3: 1; \quad \text{at} \quad x = 0;$$

(17)

Taking the phase for the reflected waves, one can write the equations (16)-(17)

$$\cos j = 2^0_2 \sin^2 0 3_2$$

(18)

Following Schoenberg [12], if we write

$$\cos j = \cos j_0 + \frac{c_j}{2 \lambda}$$

(j = 1, 2, 3);

$$\cos j^0 = \Re e \cdot 8^2 \sin^2 0 9; \quad c_j = 2 \Im m \cdot \sin^2 0 2$$

(18)

where $j^0$, called real phase speed and $j^0$, known as reflection angle and are given by

$$\sin^2 0 + \Re e$$

and $c_j$ is knowns as attenuation in a depth and equals to $2 \cdot \lambda$ i.e. wavelength of incident wave

Making use of the equation (7) in the conditions given by (10)-(12) and with the use of equations given by (13)-(15), a homogenous system equations is obtained as follows

$$X$$

$$a_{ij}Z_j = Y_j; \quad (i; j = 1; 2; 3);$$

5
$$a_{1p} = (a_8 \ a_7) p \left( \begin{array}{c} 2 \ p \\ 0 \end{array} \right) \left[ \left( \begin{array}{c} 2 \ 0 \\ 0 \end{array} \right) \sin 0 \right] - \left( \begin{array}{c} 0 \ \\
0 \end{array} \right); \quad pS_1 = \sin 0$$

$$a_{2p} = \mu^{1/3} \left( \frac{3}{2} \right)^{2} \left( \begin{array}{c} 2 \ p \\ 0 \end{array} \right) \left[ \left( \begin{array}{c} 2 \ 0 \\ 0 \end{array} \right) \sin 0 \right] - \left( \begin{array}{c} 0 \ \\
0 \end{array} \right); \quad pS_2 = \sin 0$$

$$a_{3p} = \sigma \left( \begin{array}{c} 2 \ p \\ 0 \end{array} \right) \left[ \left( \begin{array}{c} 2 \ 0 \\ 0 \end{array} \right) \sin 0 \right] - \left( \begin{array}{c} 0 \ \\
0 \end{array} \right); \quad a_{33} = 0$$

where \((p = 1; 2)\):

5 Conclusion

In this investigation amplitude ratios are calculated numerically for non-free surface in homogenous isotropic micropolar elastic medium. The amplitude ratios are calculated for incident LD-Wave and Coupled waves, namely coupled transverse rotational wave and transverse displacement wave. The results of the problem can be useful to researcher working in the field of seismology.

References


