Numerical solution of 1D-Heat Equation

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Abstract

The physical models such as 1D-heat equation and its different versions widely applicable in many branches of physics and engineering. In this article, we have numerically solved the equations and present grid convergence study. To visualize the dynamical behaviour of the equations, we presented the solutions graphically. The presented solutions show that, the finite difference method is an effective and powerful method.

Key words: Partial Differential Equation, finite difference method, Active-Dissipative Systems.

1 Introduction

Numerical techniques are intended for the productive arrangement of scientific issues requiring specific numerical outcomes. A numerical technique is a complete and particular arrangement of strategies for the solution of a problem, together with process-able mistake estimates. There are numerous numerical methods in use, but in this work we are focused on finite difference method. The finite difference method was developed for finding the numerical solutions of various differential equations. It was first used by Euler, in somewhere in seventeenth century. For the discretization of the derivatives of the variables Taylor series expansions were used. The principle of finite difference methods is near the numerical plans used to tackle ordinary differential conditions. The domain is divided in space and in time. This error is known as the discretization truncation error.
2 Finite Difference Method.

As we have to find the solution of the heat equation [1] with the finite difference method [2], firstly in domain D we will define a set of grid points. Select $x = \frac{b-a}{N}$ and $t$ where $N$ is an integer, $x$ is a state step size and $t$ is the time step size. where $x_i = a + i \times \Delta x; i = 0; 1; 2; \ldots; N$; and $t_n = n \times \Delta t; n = 0; 1; 2; \ldots$.

If $D = [a; b]$ $[0; T]$ then select $t = \frac{T}{M}$ where $M$ is an integer and $t_n = n \times \Delta t; n = 0, 1, 2, \ldots, M$. The partial derivatives

$$
\frac{\partial u}{\partial x} \quad \text{and} \quad \frac{\partial^2 u}{\partial x^2}
$$

are always approximate by central difference ratios, that is

$$
\frac{u_i^n - u_{i-1}^n}{\Delta x} \quad \text{and} \quad \frac{2u_i^n + u_{i+1}^n - u_{i-1}^n}{2 \Delta x^2}
$$

at the grid point $(i; n)$. Here $u_i^n = u(x_i; t_n)$.

It depends on how $u_t$ is approximate, as we have the three basic methods: implicit, explicit and Crank-Nicolson method.

3 Numerical Solution of Heat Equation in One Dimensional

Here we consider the following heat equation [3,4,5],

$$
\frac{\partial u}{\partial x} + \frac{4 \partial^2 u}{\partial x^2} = 0
$$

for $0 < x < 4$. Where exact solution, initial and boundary conditions are,

$$
e^{t \sin \left( \frac{x}{2} \right)} + e^{t \sin \left( \frac{x}{4} \right)}
$$

$$
u(x; 0) = \sin(4x)(1 + 2\cos(4x))
$$

$$
u(0; t) = u(4; t) = 0; t > 0:
$$

We have solved the heat equation using central finite difference method and compare the exact solution with the numerical solution by using different grid sizes. We find that the smaller is the grid size, the greater accuracy of the solution.
Conclusion

We applied finite difference method (central) to solve heat equation (one dimensional case), we wrote the matlab code for solving it. For testing the correctness of the code we compare the numerical solution with the exact solution for different grid sizes and time steps.
4 Reference

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