Investigation of a Hybrid Multi-state system: A Reliability approach

Amit Kumar

Department of Mathematics, School of Chemical Engineering and Physical Sciences
Lovely Professional University, Phagwara, 144411 Punjab, India

Abstract

The interconnection of the components of a system, inside it, is one of the factors which affect the reliability of the same and hence at the time of designing of a system engineers pay key attention for optimal configuration of system’s components. The present paper investigates the reliability and associated parameter for a system in which the components are interconnected in a hybrid (mixed) configuration. The considered system is having three components namely A, B and C which are connected to each other in a combination of series and parallel configuration. The combine functioning of these components are vital for the optimal performance of the considered system. A mathematical model is developed and solved with the aid of Markov process and Laplace transformation for finding the availability and reliability of the system. Also the most significant unit (which affects the system performance most) of the system is identified through sensitivity analysis. Suggestions for the improvement of the system’s performance are also given.

Key words: Hybrid configuration; Markov process; sensitivity analysis; most significant unit;

1. Introduction

System configuration will always play a key role when someone is talking about the reliability measures of a complex system because many times a small change in system configuration will affect the performance of the same drastically. Basically there are many kind of system configuration including series, parallel, hybrid or mixed etc. which can be found easily in many systems including sugar mill plant, plastic industry, manufacturing industry and many more. Investigation of such system for finding its reliability measures through different techniques/methods has been in past by many authors including [1-5]. Ram and Kumar [6] investigated a system whose units are interconnected a mixed configuration with k-out-of-n redundancy to find
out the various reliability measures of the same. For this the authors used Markov process and Mathematical modelling. Sung and Cho [7] optimize the system reliability by incorporating multiple constraints at system level in a series system. The extension of the reliability concepts from a series-parallel system to a complex system has been done by Sarhan [8], for this purpose author take a radar system of an aircraft. Xia et al. [9] used a MAM-MTW methodology for dynamic maintenance decision making for a series parallel system. Nourelfath and Ait-Kadi [10] find out the minimal cost configuration for a MSS series-parallel system for a specified maintenance policy. The above research work motivated the author to construct a mathematical model for a MSS having components in a combination of series and parallel configuration and find out reliability and availability of the same under some suggestions for its performance improvement.

2. System description

The considered system is having three components namely A, B and C which are interconnected to each other in a hybrid configuration i.e. combination of series and parallel configuration. The component “A” which is having two parallel units is connected in series configuration with component “B”. Component “A” and “B” is further connected with component “C” in parallel configuration. The following Fig. 1 and Fig. 2 give more insight about the interconnection of the system and different possible state of the system, in which the considered system possibly occurs at any instant time “t”, respectively.

![Fig. 1 System Configuration](image-url)
3. Assumptions

- Simultaneous failure of more than one component is not possible.
- Repairman will always available to take care of a failed/degraded unit.
- After the failure of a component of the system it will goes in degraded or failed state.
- Repair makes component as a new one.

4. Nomenclature and state description

The following Table 1 and Table 2 give a brief description about different nomenclature and states which are used in this paper.

<table>
<thead>
<tr>
<th>$P_i(t)$; $i = 0,1,2,3,4,5,6$</th>
<th>The probability that the system is in state $S_i; i = 0,1,2,3,4,5,6$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_i(s)$</td>
<td>Laplace transform of the state $P_i(t)$.</td>
</tr>
<tr>
<td>$P_i(x,t); i = 7,8,9$</td>
<td>The probability that the system is in state $S_i; i = 7,8,9$.</td>
</tr>
<tr>
<td>$\lambda_1, \lambda_2, \lambda_3$</td>
<td>The failure rate of the components “A”, “B” and “C” respectively.</td>
</tr>
<tr>
<td>$\phi_1(x), \phi_2(x), \phi_3(x)$</td>
<td>The repair rate of the components “A”, “B” and “C” respectively.</td>
</tr>
</tbody>
</table>
Simultaneous repair rate of A-B/B-C/A-B-C component of the system.

Time unit/Laplace transform variable.

<table>
<thead>
<tr>
<th>( \phi_{11}(x) / \phi_{24}(x) / \phi_{123}(x) )</th>
<th>( t/s )</th>
</tr>
</thead>
</table>

Table 1 Nomenclatures

- **\( S_0 \)**: Full working state i.e. all the components are working without any failure.
- **\( S_1 \)**: Represents the state in which one of the units of component “A” is failed.
- **\( S_2 \)**: Represents the state in which component “A” is completely failed.
- **\( S_3 \)**: Represents the state in which one of the units of component “A” and component “C” is failed.
- **\( S_4 \)**: Represents the state in which component “B” is failed.
- **\( S_5 \)**: Represents the state in which component “C” is failed.
- **\( S_6 \)**: Represents the state in which one of the units of component “A” and component “B” is failed.
- **\( S_7 \)**: Represents the failed state in which one of the units of component “A”, component “B” and component “C” is failed.
- **\( S_8 \)**: Represents the failed state in which component “A” and component “C” is failed.
- **\( S_9 \)**: Represents the failed state in which component “B” and component “C” is failed.

Table 2 State descriptions

5. Mathematical formulation of the problem

Critically analyzing the different failure and repair which can occurs in the system during its functioning at any instant time “\( t \)”, a state transition diagram in developed as given in Fig. 2. On the basis of Fig. 2 the following set of equation has been developed.
\[
\left(\frac{\partial}{\partial t} + 2\lambda_i + \lambda_2 + \lambda_3\right)P_i(t) = \phi_i(x)P_i(t) + \phi_2(x)P_2(t) + \phi_3(x)P_3(t) + \int_0^\infty \phi_{123}(x)P_9(x,t) dx + \int_0^\infty \phi_{123}(x)P_9(x,t) dx
\] (1)

\[
\left(\frac{\partial}{\partial t} + \lambda_1 + \lambda_2 + \lambda_3 + \phi_i(x)\right)P_1(t) = 2\lambda_1P_1(t) + \phi_2(x)P_2(t) + \phi_3(x)P_3(t)
\] (2)

\[
\left(\frac{\partial}{\partial t} + \lambda_3 + \phi_i(x)\right)P_2(t) = \lambda_3P_2(t)
\] (3)

\[
\left(\frac{\partial}{\partial t} + \lambda_2 + \phi_i(x) + \phi_3(x)\right)P_3(t) = 2\lambda_2P_3(t) + \lambda_4P_4(t)
\] (4)

\[
\left(\frac{\partial}{\partial t} + \lambda_1 + \phi_i(x)\right)P_4(t) = \lambda_4P_4(t)
\] (5)

\[
\left(\frac{\partial}{\partial t} + 2\lambda_1 + \lambda_2 + \phi_i(x)\right)P_5(t) = \lambda_5P_5(t) + \phi_6(x)P_6(t)
\] (6)

\[
\left(\frac{\partial}{\partial t} + \lambda_3 + \phi_2(x)\right)P_6(t) = \lambda_2P_6(t)
\] (7)

\[
\left(\frac{\partial}{\partial t} + \phi_i(x)\right)P_7(x,t) = 0
\] (8)

\[
\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \phi_{13}(x)\right)P_7(x,t) = 0
\] (9)

\[
\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \phi_{23}(x)\right)P_9(x,t) = 0
\] (10)

Initial condition \(P_i(t) = \begin{cases} 1; & \text{when } i = t = 0 \\ 0; & \text{otherwise} \end{cases}\) (11)

Boundary conditions

\[
P_i(0,t) = \lambda_iP_e(t) + \lambda_2P_9(t)
\] (12)
\[ P_s(0,t) = \lambda_1 P_1(t) \]  
\[ P_s(0,t) = \lambda_1 P_1(t) + \lambda_2 P_2(t) \]  

The transition state probabilities of the considered system can be obtained by solving the set of equation from (1) to (14) by the aid of Laplace transform as follow.

\[
\bar{P}_0(s) = \frac{1}{s + 2\lambda_1 + \lambda_2 + \lambda_3 - \frac{H_3 \phi_1(x)}{H_2}} - \left( \phi_1(x) + \frac{\lambda_2 \phi_{23}(x)}{(s + \phi_{23}(x))} \right) \left( \frac{\lambda_2}{(s + \lambda_3 + \phi_2(x))} \right) \\
- \left( \phi_1(x) + \frac{\lambda_2 \phi_{23}(x)}{(s + \phi_{23}(x))} \right) \left( \frac{\lambda_2}{(s + 2\lambda_1 + \lambda_2 + \phi_1(x))} + \frac{H_4 \phi_1(x)}{(s + 2\lambda_1 + \lambda_2 + \phi_1(x))} \right) \\
- \left( \frac{\lambda_2 \lambda_3 H_4 \phi_{13}(x)}{H_2(s + \lambda_3 + \phi_1(x))(s + \phi_{13}(x))} \right) - \left( \frac{\lambda_2 \lambda_4 H_1 \phi_{23}(x)}{H_2(s + \lambda_3 + \phi_2(x))(s + \phi_{23}(x))} \right) \\
- \left( \frac{\lambda_2 H_4 \phi_{123}(x)}{(s + \phi_{123}(x))} \right) 
\]  

(15)

\[
\bar{P}_1(s) = \frac{H_1}{H_2} \bar{P}_0(s) \]  

(16)

\[
\bar{P}_2(s) = \frac{\lambda_1 H_3}{H_2(s + \lambda_3 + \phi_1(x))} \bar{P}_0(s) \]  

(17)

\[
\bar{P}_3(s) = \left( \frac{\lambda_1 \lambda_3}{H_1(s + 2\lambda_1 + \lambda_2 + \phi_1(x))} + \frac{\lambda_3 H_3}{H_1 H_2} \right) \bar{P}_0(s) \]  

(18)

\[
\bar{P}_4(s) = \frac{\lambda_2}{(s + \lambda_3 + \phi_2(x))} \bar{P}_0(s) \]  

(19)

\[
\bar{P}_5(s) = \left( \frac{\lambda_3}{(s + 2\lambda_1 + \lambda_2 + \phi_1(x))} + \frac{\phi_3(x) H_4}{(s + 2\lambda_1 + \lambda_2 + \phi_1(x))} \right) \bar{P}_0(s) \]  

(20)

\[
\bar{P}_6(s) = \frac{\lambda_2 H_3}{H_2(s + \lambda_3 + \phi_2(x))} \bar{P}_0(s) \]  

(21)

Where
The inverse Laplace transformation of these transition state probabilities (equation (15)-(21)) gives the time dependent transition state probabilities of the states, in which the system can be present at any instant time \( t \), which further can be used to evaluate the probability of working \( P_{up}(t) \) of the considered system by using the following equation (22).

\[
P_{up}(t) = P_o(t) + P_1(t) + P_2(t) + P_3(t) + P_4(t) + P_5(t)
\]  

(22)

Now by using different state probability in (22) and putting the values of different failure and repair rates in it one can obtain the working probability of the system.

6. Performance Assessment

6.1. Availability

The availability of the considered system will be obtained, by putting the numerical value of different failure rates as \( \lambda_1 = 0.10, \lambda_2 = 0.15, \lambda_3 = 0.20 \) and repair rates as one in equation (22), as follows.

\[
A(t) = \begin{cases} 
0.1666667 \exp(-1.2000000 \ t) - 0.2715561411 \exp(-1.35000 \ t) + 0.00341952766 \exp(-1.749565860 \ t) + 0.1358364314 \\
+ 0.004857324063 \exp(-0.8534304994 \ t) + 0.00395939268 \exp(-2.600879832 \ t) + 0.9606643355 
\end{cases}
\]  

(23)
The numerical behavior of the availability of the considered system will be obtained by varying $t$ in (23) as given in following Table 3 and in Fig. 3.

<table>
<thead>
<tr>
<th>Time Unit ($t$)</th>
<th>Availability $A(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000000000</td>
</tr>
<tr>
<td>3</td>
<td>0.9652213473</td>
</tr>
<tr>
<td>6</td>
<td>0.9608738619</td>
</tr>
<tr>
<td>9</td>
<td>0.9606729108</td>
</tr>
<tr>
<td>12</td>
<td>0.9606647110</td>
</tr>
<tr>
<td>15</td>
<td>0.9606643549</td>
</tr>
</tbody>
</table>

Table 3 Availability of the system vs. time unit $t$

![Fig. 3 Availability of the system vs. time unit $t$](image)
6.2. Reliability

The reliability of the considered system, using equation (22), will be obtained as following

\[
R(t) = \left[ 4 \exp \left( \frac{-2\lambda_1 t}{2} - \lambda_2 - \lambda_3 t \right) \sinh \left( \frac{\lambda_1 t}{2} \right) + \frac{(4\lambda_1 + 3\lambda_2) \exp \left( -2\lambda_1 + \lambda_2 + \lambda_3 t \right)}{(2\lambda_1 + \lambda_3)} \right] \\
+ \exp \left( -\lambda_3 t \right) + \frac{\lambda_3 (3\lambda_1 + \lambda_3) \exp \left( -\lambda_3 t \right)}{(2\lambda_1 + \lambda_3) (\lambda_1 + \lambda_3)} + \frac{(-2\lambda_1 - 4\lambda_2) \exp \left( -\lambda_1 + \lambda_2 + \lambda_3 t \right)}{(\lambda_1 + \lambda_3)} \right] 
\]

(24)

Now putting the value of different failure rates as \( \lambda_1 = 0.10, \lambda_2 = 0.15, \lambda_3 = 0.20 \) in equation (24), the time dependent reliability of the system will be obtained as given in equation (25).  

\[
R(t) = \left[ 4 \exp \left( -0.500000 \right) \sinh \left( 0.050000 t \right) + 2.500000 \exp \left( -0.55 \right) \right] \\
+ \exp \left( -0.20 t \right) + 0.833333 \exp \left( -0.15 t \right) - 3.333333 \exp \left( -0.45 t \right) \right] 
\]

(25)

The graphical representation of the reliability of the considered system can be obtained by varying time unit in equation (25) as given in Fig. 2.

<table>
<thead>
<tr>
<th>Time Unit ((t))</th>
<th>Reliability (R(t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0000000000</td>
</tr>
<tr>
<td>3</td>
<td>0.8305397031</td>
</tr>
<tr>
<td>6</td>
<td>0.5688364950</td>
</tr>
<tr>
<td>9</td>
<td>0.3616443103</td>
</tr>
<tr>
<td>12</td>
<td>0.2231251029</td>
</tr>
<tr>
<td>15</td>
<td>0.1361892119</td>
</tr>
</tbody>
</table>

Table 4 Reliability of the system vs. time unit \( t \)
6.3. Sensitivity Analysis for system’s reliability

In the present paper, author have been done the sensitivity analysis for system’s reliability for finding the critical components of the same i.e. to identify that which unit’s failure affects its performance most. It is done by using equation (24). Table 5 and corresponding Fig. 5 show the sensitivity of system reliability w.r.t. time unit $t$.

<table>
<thead>
<tr>
<th>Time Unit ($t$)</th>
<th>Sensitivity of System’s Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>For $\lambda_1$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-0.106967034</td>
</tr>
<tr>
<td>6</td>
<td>-0.267723769</td>
</tr>
<tr>
<td>9</td>
<td>-0.2990362147</td>
</tr>
<tr>
<td>12</td>
<td>-0.2488008530</td>
</tr>
<tr>
<td>15</td>
<td>-0.1808934595</td>
</tr>
</tbody>
</table>

Table 5: Sensitivity analysis for system’s reliability vs. Time unit $t$
7. Result discussion and Conclusion

The present paper investigated a system whose components are interconnected in a mixed configuration for evaluating the availability, reliability and sensitivity analysis for the same. A mathematical model is developed and analyzed by using Markov process and mathematical modelling. The findings of the paper are

- The availability of the considered hybrid system at fifteen unit of time is 0.9606643549 (Fig. 3) and after that it seems to be constant.
- The reliability of the considered hybrid system at same time is obtained as 0.1361892119 (Fig. 4) i.e. there is almost 13% probability that the system will performs its intended task at fifteen unit of time. It has been observed that the reliability of the system decrease more rapidly as compared to availability.
- The sensitivity of the system reliability is shown in Fig. 5. It can be observed from it that the system’s reliability is highly sensitive w.r.t. the failure rate of component B and least sensitive w.r.t. the failure of components A. Hence the system reliability can be enhanced if the failure of B component of the system controlled (as much as possible).
Reference


