A Review On Two-Temperature Theory Of Generalized Thermoelasticity

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Abstract
In this paper, a brief study on the theory of thermoelasticity with two temperature (T-T) is reviewed and work by different researcher is also discussed. The basic equations for the generalized theories of thermoelasticity are also given.

Keywords: Thermoelasticity, two temperature, generalized theories.

1. Introduction
Thermoelasticity is the change in the size and shape of a solid object as the temperature is applied on it. Most problem in thermoelasticity depend on the supposition that temperature field is represented by the first law of thermoelasticity of rigid body. In nineteenth century, Duhamel [1] and Neumann [2] had given the theory named as uncoupled thermoelasticity (UCT). Duhamel [1] had derived the equations with temperature gradients for the strain in an elastic body and Neumann [2] also came up with the same result. This theory depicts two phenomena which are unrealistic with physical observations. First phenomena, it does not have an elastic term and next phenomena is that, the heat equation is parabolic type, which depicts infinite speeds of propagation for the heat waves.

To cope up the first shortcoming, Biot [3] had given the theory of coupled thermoelasticity (CT). The governing equations of this theory is also coupled and eliminating the first paradox of the CT theory. Although, both theories have the second short coming as the heat equation for the CT is also parabolic in nature.

One of the generalizations to the CT was commenced by Lord and Shulman [4], they acquire a wave-type heat equation by defining a fresh law of heat-conduction to exchange the classical Fourier’s rule. Since, heat equation of given theory is of the wave-type and it warrants the finite speeds of
propagation for heat and elastic waves. The left behind governing equations of given theory, are constitutive relations and equations of motion remained identical for the both theories (CT and UCT).

Chen and Gurtin [5], and Chen et.al [6] introduced a conjecture of heat conduction in deformable bodies, it depends on two distinct temperatures, first one is conductive temperature ($\phi$) and other one is thermodynamic temperature (T). In time-dependent situations, the distinction among these $T$-$T$ are directly proportional to the heat supply and without heat bring in, the given $T$-$T$ are identical for both wave propagation and time-dependent problems in absence of heat supply.

In year 1972, Green and Lindsay [7] derived a temperature rate dependent thermoelasticity theory by giving relaxation time factors. The recently emerged theory admit fixed speed of heat propagation is now consigned as the hyperbolic thermoelasticity theory. Since, the heat equation is of hyperbolic type differential equation and it ensures the finite speed of propagation. The G-L theory includes two constants which acts as relaxation time and modified every equations of the CT theory and that of traditional Fourier’s law of heat conduction.


2. Review of literature

Youssef and Al-Lehaibi [11] presented a numerical model of an elastic material with the help of cylindrical cavity. The governing equations had taken for the fractional order generalized thermoelasticity theory. Bala [12] briefly examined about the development of the theory of T-$T$ thermoelasticity. Likewise, the basic equations of T-$T$ thermoelasticity are reviewed with regards to L-S and G-N theories of generalized thermoelasticity.

He and Guo [15] investigated about the dynamic response of a one-dimensional problem for a thermoelastic rod with finite length in the fraction order theory of thermoelasticity.

Kar [16] obtained expression for the temperature distribution, stress and displacement components in Laplace transform domain in infinite medium having a spherical cavity by using approach of eigen value in the context of theory of thermoelasticity with two relaxation time. Zenkour and Abouelrega [18] established theory of nonlocal thermoelasticity in which they consider the new heat-conduction equation with fractional orders. The closed form expression for the transverse vibrations of a homogenous isotropic, thermoelastic thin beam with variable thermal conductivity, based Euler–Bernoulli theory has been determined.

AI- Lehaibi [19] examined the elastic infinite medium with cylindrical cavity in context of T-T thermoelasticity by using Laplace transform. Othman et al. [20] studied the deformation of thermoelastic solid half-space to study the impact of rotation and hydrostatic initial stress with T-T. The normal mode analysis technique is worked to get the analytical rotation of the stresses, displacement components, T and ϕ.

Othman and Tantawi [21] established the model of conditions generalized thermoelasticity in a semiconductor medium with T-T. The investigation was done under L-S theory with ORT. The normal mode analysis technique is applied to get the expression for the measured variables. Lata [22] presented the time harmonic deformation in 2-D homogeneous, transversely isotropic thermoelastic solids without energy dissipation and with T-T. The application of a time harmonic linearly distributed loads has been measured to demonstrate the effectiveness of the solution attained.

M. Zenkourl and Abouelregal [23] presented another model of nonlocal thermoelasticity beam theory with phase-lags believing the thermal conductivity to be variable. A nanobeam subjected to a harmonically varying heat was considered. The nonlocal theory of CT and generalized with ORT can be extracted as limited and special cases from the present model. Ailawalia et al. [24] discussed 2-D deformation in a generalized thermoelastic media with micro temperatures having an internal heat source, subjected to a mechanical force.

Abdou et al. [25] discussed about the physical quantities in generalized thermoelastic medium with double porosity under L-S theory. The impact of rotation and gravity has been introduced. The half-space is considered to be isotropic homogeneous thermoelastic material. Alshaikh [26] presented the
generalized thermoelastic medium for an isotropic solid cylinder. The material of the cylinder is taken as homogenous isotropic both thermally and mechanically. All equations are written in the form of vector matrix differential in the Laplace transform domain, later on solved by an eigen value approach. Abbas [27] discussed about the fractional order derivative on a 2-D problem because of thermal shock with weak, ordinary and solid conductivity. The governing equations are taken in the context of Green and Naghdi of sort III model under fractional order derivative. Magana et.al [28] re-searched the well-posedness and the stability of the solutions for several Taylor approximations of the phase-lag T-T equations and give equations on the parameters which ensure the presence and uniqueness of solutions as well as the stability and the instability of the solutions for each approximation. Kumar et.al [29] investigated the propagation of plane waves for an isotropic thermoelastic media with void and T-T with regards to three-stage lag theory of thermoelasticity. Youssef et. al. [30] studied about numerical model of 3-D generalized thermoelasticity which is enhanced using L-S theory. The governing equations on non-dimensional form are connected to a 3-D half-space subjected to heat source and traction free surface by utilizing the double Fourier and Laplace Transfrom.

3. Basic Equation

The field equations and constitutive relations are given by Youseff [9] in generalized thermoelastic body having TT as:

\[
(\lambda + 2\mu)\nabla(\nabla \cdot \vec{u}) - \beta \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) \nabla \theta - \mu (\nabla \times \nabla \times \vec{u}) = \rho \frac{\partial^2 \vec{u}}{\partial t^2} \tag{1}
\]

\[
K^* = \rho C^* \left( 1 + \tau_0 \frac{\partial}{\partial t} \right) \frac{\partial \theta}{\partial t} + \beta \theta_0 \left( \frac{\partial}{\partial t} + \eta \tau_0 \frac{\partial^2}{\partial t^2} \right) \nabla \cdot \vec{u} \tag{2}
\]

\[
t_y = \lambda u_{k,k} \delta_{ij} + \mu (u_{i,j} + u_{j,i}) - \beta \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) \theta \delta_{ij} \tag{3}
\]

where

\[
\theta = (1 - a \nabla^2) \phi \tag{4}
\]

\(\lambda, \mu\) - Lame's constants, \(t\)-time, \(\beta = (3\lambda + 2\mu) \alpha_t, \alpha_t\) - coefficient of linear thermal expansion, \(\rho, C^*\) - density and specific heat respectively, \(\phi\) - conductive temperature, \(\theta\) Temperature distribution, \(K^*\) - thermal conductivity, \(t_y\) - components of stress tensor, \(\tau_0, \tau_1\) the relaxation times, \(\delta_{ij}\) - Kronecker delta, \(\vec{u}\) - displacement vector, \(\eta\) - the constant, \(a\)-two temperature parameter, \(\theta_0\) reference temperature.

\(\eta = 1, \quad \tau_1 = 0, \quad L-S \text{ theory}\)
\[ \eta = 0, \quad \tau_i > 0, \quad G-L \text{ theory} \]

\[ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \]

REFERENCES