Response of fractional order theories of thermoelasticity due moving load at the surface of micro polar thermoelastic half space

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Abstract

The present study concerned with the effect of moving load at the surface of micropolar thermoelastic half space under fractional order theories of thermoelasticity. The problem has been solved by using Eigen-value approach after using the Fourier transform. Analytic expressions for the displacement, microrotation, stresses and temperature distribution are derived in the transformed domain. The transformed expressions are inverted numerically to obtain the results in physical domain and numerically computed results are presented graphically.

Keywords: *Micropolar thermoelasticity, Fractional order derivative, Eigen-value approach, Fourier transform.*

1. Introduction

Fractional order theory of thermoelasticity based upon fractional calculus is well known and being applied to study the physical processes. Several research studies included fractional order theory thermoelasticity to study the heat conduction in different type of materials. Povstenko [1] generalized Fourier law to obtained fractional order heat conduction law by using fractional order derivative with respect to time and space variable. Sherief et al [2] proposed fractional ordered theory thermoelasticity where Lord-Shulman [3] theory of thermoelasticity is obtained as a special case. Youssef [4] used fractional order derivative and proposed another theory of thermoelasticity with wide range of fractional order parameter values. Ezzat [5] on the basis of Taylor series expansion obtained one more fractional order heat conduction equation and applied to number of problems of heat transfer in materials. The fractional order theories have also been employed to study the heat conduction in micropolar thermoelastic material. The articles [6-7] used fractional ordered theory of thermoelasticity to study various problems in micropolar thermoelastic materials.

The moving load is encounter in several technical and geophysical problems. For example, in solid dynamics the stresses during ground motions can be produced by blast waves and deformation field around drilling and mining tremors into the earth's interior surface. The articles [8-9] can be referred for problems of a moving load on the half spaces of different types of materials.

In the present article, a micropolar thermoelastic solid with fractional order heat conduction is considered for interaction under moving load. Fourier transform followed by eigen value approach. The fractional order theories proposed by Sherief et al [2], Ezzat [5] and Youssef [4] have been used and a combined result has been obtained for displacement, stress and temperature distribution. The inversion of Fourier transform have been carried out numerically and the results obtained are shown graphically.

2. Basic equations

The governing differential equations of motion of a homogeneous, isotropic micropolar thermoelastic solid [10] are

$$(\mu + K)\nabla^2\vec{u} + (\lambda + \mu)\nabla(\nabla.\vec{u}) + K\nabla\times\vec{\phi} - \nu\nabla \mathrm{T} = \rho\frac{\partial^2\vec{u}}{\partial t^2}, \qquad (1)$$

$$(\alpha + \beta + \gamma)\,\nabla\left(\nabla.\vec{\phi}\right) - \gamma\nabla\times\left(\nabla\times\vec{\phi}\right) + K\nabla\times\vec{u} - 2K\vec{\phi} = \rho \mathrm{j}\frac{\partial^2\vec{\phi}}{\partial t^2}. \qquad (2)$$

Following [2], [4] and [5] , the fractional order heat conduction is

$$K^*\nabla^2 T = \rho C^*\left(\frac{\partial^{p_1}}{\partial t^{p_1}} + \frac{\tau_0^{p_2}}{p_2!}\frac{\partial^{\alpha'+1}}{\partial t^{\alpha'+1}}\right)T + \nu T_0\left(\frac{\partial^{p_1}}{\partial t^{p_1}} + \frac{\tau_0^{p_2}}{p_2!}\frac{\partial^{\alpha'+1}}{\partial t^{\alpha'+1}}\right)\nabla.\vec{u}\ . \qquad (3)$$

In the above equation $\mathrm{p}_1 = 1, \mathrm{p}_2 = 1$ correspond to Sherief theory, for $\mathrm{p}_1 = 1, \mathrm{p}_2 = \alpha'$ we can obtain Ezzat Theory, $\mathrm{p}_1 = \alpha', \mathrm{p}_2 = 1$ gives Youssef theory and for $\mathrm{p}_1 = 1, \mathrm{p}_2 = 1, \alpha' = 1$ gives Lord-Shulman theory.

The constitutive relations

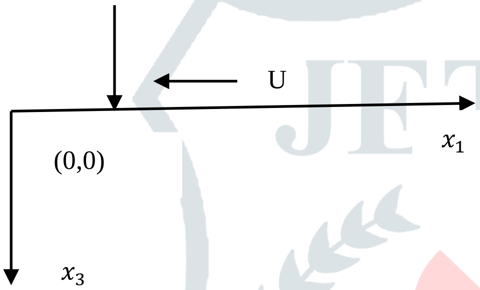
$$t_{ij} = \lambda u_{r,r}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + K\left(u_{j,i} - \epsilon_{ijr}\phi_r\right) - \nu T\delta_{ij} \qquad (5)$$

$$m_{ij} = \alpha\,\phi_{r,r}\delta_{ij} + \beta\,\phi_{i,j} + \gamma\,\phi_{j,i} \qquad (6)$$

where, $\vec{u}$ be the displacement vector, C^* stand for specific heat at constant strain, K^* represents coefficient of thermal conductivity, α' be a symbol of fractional order parameter, τ_0 represents thermal relaxation times, $\alpha, \beta, \gamma, \mu, \lambda$, K are material constant, j is the microinertia, $\vec{\phi}$ act for microrotation vector, ρ is the density, T characterize change in temperature at any time, T_0 is the reference temperature, $\nu = (3\lambda + 2\mu + K)\alpha_t$, α_t denote coefficient of thermal linear expansion.

3. Formulation of the problem

We consider a rectangular Coordinate system (x_1, x_2, x_3). It is assumed that a pressure pulse $F(x_1 + Ut)$ is moving at the free surface of a fractional ordered micropolar thermoelastic half space in the negative x_1-direction with constant speed U. We assume that a normal load is applied which is moving for infinite time and steady state condition prevails as seen by the observer moving with the load. The geometry of the problem is as given below



Geometry of the Problem

For two dimensional problem we assume

$$\vec{u} = (u_1, 0, u_3)\ ,\ \vec{\phi} = (0, \phi_2, 0) \qquad (6)$$

Following Fung [11], introducing Galilean transformation

$$x_1^* = x_1 + Ut\ ,\quad x_3^* = x_3\ ,\ t^* = t \qquad (7)$$

and it will change the boundary conditions independent of t^*. Let us assume following mentioned dimensionless variables

$$x_i' = \frac{\omega^*}{c_1}x_i^*,\ u_i' = \frac{\rho\omega^* c_1}{\nu T_0}u_i\,,\ \phi_2' = \frac{\rho c_1^2}{\nu T_0}\phi_2\,,\ m_{ij}' = \frac{\omega^*}{c_1 \nu\, T_0}m_{ij}\,,\ t' = \omega^* t^*\,,\ T' = \frac{T}{T_0}$$

$$\tau_0' = \omega^*\tau_0, \qquad t_{ij}' = \frac{t_{ij}}{\nu T_0}$$

where $\omega^* = \frac{\rho c^* c_1^2}{K^*}$, $c_1^2 = \frac{\lambda + 2\mu + K}{\rho}$ (8)

in equations (1) – (3) and after suppressing primes we get

$$\frac{\lambda+\mu}{\rho c_1^2}\frac{\partial}{\partial x_1}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) + \frac{\mu+K}{\rho c_1^2}\nabla^2 u_1 - \frac{K}{\rho c_1^2}\frac{\partial \phi_2}{\partial x_3} - \frac{\partial T}{\partial x_1} = \frac{U^2}{c_1^2}\frac{\partial^2 u_1}{\partial x_1^2} \qquad (9)$$

$$\frac{\lambda+\mu}{\rho c_1^2}\frac{\partial}{\partial x_3}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) + \frac{\mu+K}{\rho c_1^2}\nabla^2 u_3 + \frac{K}{\rho c_1^2}\frac{\partial \phi_2}{\partial x_1} - \frac{\partial T}{\partial x_3} = \frac{U^2}{c_1^2}\frac{\partial^2 u_3}{\partial x_1^2} \qquad (10)$$

$$\nabla^2\phi_2 + \frac{Kc_1^2}{\gamma\omega^{*2}}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) - \frac{2Kc_1^2}{\gamma\omega^{*2}}\phi_2 = \frac{\rho j\,U^2}{\gamma}\frac{\partial^2\phi_2}{\partial x_1^2} \qquad (11)$$

$$\nabla^2 T-(\omega^*)^{p_1-1}\frac{U^{p_1}}{c_1^{p_1}}\left(\frac{\partial^{p_1}}{\partial x_1^{p_1}}+\frac{\tau_0^{p_2}}{p_2!}(\omega^*)^{\alpha'+1-(p_1+p_2)}\frac{U^{\alpha'+1-p_1}}{c_1^{\alpha'+1-p_1}}\frac{\partial^{\alpha+1}}{\partial x_1^{\alpha+1}}\right)T$$

$$-\frac{\nu^2 T_0(\omega^*)^{p_1-2}}{\rho K^*}\frac{U^{p_1}}{c_1^{p_1}}\left(\frac{\partial^{p_1}}{\partial x_1^{p_1}}+\frac{\tau_0^{p_2}}{p_2!}(\omega^*)^{\alpha'+1-(p_1+p_2)}\frac{U^{\alpha'+1-p_1}}{c_1^{\alpha'+1-p_1}}\frac{\partial^{\alpha'+1}}{\partial x_1^{\alpha'+1}}\right)\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_3}{\partial x_3}\right)=0 \qquad (12)$$

Applying the Fourier transform defined by

$\tilde{f}(\xi\,,x_3)=\int_{-\infty}^{\infty} f(x_1\,,x_3)e^{i\xi x_1}dx_1.$

on equations (9)-(12) we obtain

$$D^2\tilde{u}_1=a_{11}\xi^2\tilde{u}_1+i\xi a_{12}D\tilde{u}_3+a_{13}D\tilde{\phi}_2+i\xi a_{14}\tilde{T}\,, \qquad (13)$$

$$D^2\tilde{u}_3=a_{21}i\xi D\tilde{u}_1+a_{22}\xi^2\tilde{u}_3+a_{23}i\xi\tilde{\phi}_2+a_{24}D\tilde{T}\,, \qquad (14)$$

$$D^2\tilde{\phi}_2=-a_{31}D\tilde{u}_1-i\xi a_{31}\tilde{u}_3+(\xi^2 a_{33}+2a_{31})\tilde{\phi}_2\,, \qquad (15)$$

$$D^2\tilde{T}=-a_{42}i\xi\tilde{u}_1+a_{42}D\tilde{u}_3+(\xi^2+a_{41})\tilde{T}\ . \qquad (16)$$

where

$$a_{11}=\frac{\lambda+2\mu+K-\rho U^2}{\mu+K}\ ,\ a_{12}=\frac{\lambda+\mu}{\mu+K}\ ,\ a_{13}=\frac{K}{\mu+K}\ ,\ a_{14}=-\frac{\rho c_1^2}{\mu+K}\ , a_{21}=\frac{\lambda+\mu}{\lambda+2\mu+K},$$
$$a_{22}=\frac{\mu+K-\rho U^2}{\lambda+2\mu+K}\ ,\ a_{23}=\frac{K}{\lambda+2\mu+K}\ , a_{24}=\frac{\rho c_1^2}{\lambda+2\mu+K}\ ,\ a_{31}=\frac{K c_1^2}{\gamma\omega^{*2}}\ ,\ a_{33}=1-\frac{\rho j U^2}{\gamma}$$
$$a_{41}=(\omega^*)^{p_1-1}\frac{U^{p_1}}{c_1^{p_1}}\left((-i\xi)^{p_1}+\frac{\tau_0^{p_2}}{p_2!}(\omega^*)^{\alpha'+1-(p_1+p_2)}\frac{U^{\alpha'+1-p_1}}{c_1^{\alpha'+1-p_1}}(-i\xi)^{\alpha'+1}\right)$$
$$a_{42}=\frac{\nu^2 T_0(\omega^*)^{p_1-2}}{\rho K^*}\frac{U^{p_1}}{c_1^{p_1}}\left((-i\xi)^{p_1}+\frac{\tau_0^{p_2}}{p_2!}(\omega^*)^{\alpha'+1-(p_1+p_2)}\frac{U^{\alpha'+1-p_1}}{c_1^{\alpha'+1-p_1}}(-i\xi)^{\alpha'+1}\right), D=\frac{d}{dx_3}. \qquad ...(17)$$

The set of equations (13) – (16) can be written as

$$DW(\xi,x_3)=A(\xi)W(\xi,x_3)\ . \qquad (18)$$

where

$$W=\begin{bmatrix}V\\DV\end{bmatrix}\ ,\ A=\begin{bmatrix}O & I\\A_1 & A_2\end{bmatrix}, V=\begin{bmatrix}\tilde{u}_1\\\tilde{u}_3\\\tilde{\phi}_2\\\tilde{T}\end{bmatrix},\ A_1=\begin{bmatrix}a_{11}\xi^2 & 0 & 0 & i\xi a_{14}\\0 & a_{22}\xi^2 & i\xi a_{23} & 0\\0 & -i\xi a_{31} & \xi^2 a_{33}+2a_{31} & 0\\-i\xi a_{42} & 0 & 0 & \xi^2+a_{41}\end{bmatrix}, A_2=$$
$$\begin{bmatrix}0 & i\xi a_{12} & a_{13} & 0\\i\xi a_{21} & 0 & 0 & a_{24}\\-a_{31} & 0 & 0 & 0\\0 & a_{42} & 0 & 0\end{bmatrix}. \qquad ...(19)$$

and I is identity matrix and O is null matrix.

To solve Equation (18), we assume

$$W(\xi,x_3)=X(\xi)e^{qx_3}\ . \qquad (20)$$

which gives an eigen value problem. The characteristic equation of matrix A is

$$|A - qI| = 0\,. \tag{21}$$

Equation (20) on expansion gives

$$q^8 + \lambda_1 q^6 + \lambda_2 q^4 + \lambda_3 q^2 + \lambda_4 = 0. \tag{22}$$

where

$$\lambda_1 = \xi^2(1 - a_{11} - a_{22} - a_{33} + a_{12}a_{21}) - a_{24}a_{42} - a_{13}a_{31} + a_{41} - 2a_{13},$$

$$\lambda_2 = \xi^4\eta_1 + \xi^2\eta_2 + \eta_3,\ \lambda_3 = \xi^6\eta_4 + \xi^4\eta_5 + \xi^2\eta_6,$$

$$\lambda_4 = \xi^8\eta_7 + \xi^6\eta_8 + \xi^4\eta_9,$$

$$\eta_1 = a_{11}(1 + a_{33} + a_{22}) + a_{22}(a_{33} + 1) + a_{12}a_{21}(1 - a_{33}),$$

$$\eta_2 = a_{12}a_{21}(a_{41} - 2a_{31}) + a_{13}a_{31} + a_{24}a_{42}(a_{11} + a_{33} - a_{12}) - a_{14}a_{42}(a_{21} + 1) - a_{23}a_{31}(1 + a_{12}) + a_{41}(a_{11} + a_{22}) + 2a_{31}(a_{22} + a_{11}),$$

$$\eta_3 = a_{13}a_{31}(a_{41} - a_{24}a_{42}) + 2a_{31}a_{24}a_{42},$$

$$\eta_4 = a_{11}(-1 - a_{22} - a_{22}a_{33}) + a_{33}(a_{12}a_{21} + a_{22}),$$

$$\eta_5 = a_{12}a_{21}(a_{33}a_{41} + 2a_{31}) + a_{13}a_{31}(a_{22} + a_{21}) + a_{14}a_{42}(a_{22} + 1) + a_{24}a_{42}(a_{12} - a_{11}) - a_{11}a_{22}(2a_{31} + a_{41}) - a_{33}a_{42}a_{14}a_{21} + a_{23}a_{31}(-1 + a_{12}) + a_{22}(2a_{31} + a_{41}a_{33}) + a_{11}(a_{23}a_{31} - a_{41} - 2a_{31}),$$

$$\eta_6 = a_{23}a_{31}(a_{12}a_{41} - a_{14}a_{42}) + 2a_{21}a_{31}(a_{12}a_{41} - a_{14}a_{42}) + a_{13}a_{31}a_{41}(a_{22} + a_{21}) + a_{24}a_{42}a_{31}(2a_{12} - 2a_{11} + a_{13}) - a_{31}a_{41}(a_{23} - 2a_{22}) - 2a_{31}(a_{11}a_{41} - a_{14}a_{42}),$$

$$\eta_7 = a_{11}a_{22}a_{33}\,,\ \ \eta_8 = a_{22}a_{33}(a_{11}a_{41} - a_{14}a_{42}) + a_{11}a_{31}(2a_{22} - a_{23}),$$

$$\eta_9 = (a_{11}a_{41} - a_{14}a_{42})(2a_{31}a_{22} - a_{31}a_{23}). \tag{23}$$

The roots of equation (21) give the characteristics roots of the matrix A and for Eigen value q_s the corresponding eigen vectors $X(\xi)$ can be evaluated by solving the following equation

$$[A - qI]X(\xi) = 0 \tag{24}$$

The set $X_s(\xi)$ of eigen vectors are given by

$$X_s(\xi) = \begin{bmatrix} X_{s1}(\xi) \\ X_{s2}(\xi) \end{bmatrix}, \quad s = 1, 2\ldots 8$$

where

$$X_{s1}(\xi) = \begin{bmatrix} -i\xi a_s' \\ q_s \\ i\xi q_s b_s' \\ c_s' \end{bmatrix}, \quad X_{s2}(\xi) = \begin{bmatrix} -i\xi a_s' q_s \\ q_s^2 \\ i\xi b_s' q_s^2 \\ c_s' q_s \end{bmatrix}, \quad q = q_s\,;\ \ s = 1,2,3,4 \tag{25}$$

$$X_{j1}(\xi) = \begin{bmatrix} -i\xi a_s' \\ -q_s \\ -i\xi q_s b_s' \\ c_s' \end{bmatrix}, \quad X_{j2}(\xi) = \begin{bmatrix} i\xi a_s' q_s \\ q_s^2 \\ i\xi b_s' q_s^2 \\ -c_s' q_s \end{bmatrix}, \quad j = s + 4\,;\ q = -q_s\,; s = 1,2,3,4 \tag{26}$$

and

$$a_s' = \frac{1}{\Delta}\{(\xi^2 a_{33} + 2a_{31} - q_s^2)[a_{21}a_{24}q_s^2 - a_{14}(a_{22}\xi^2 - q_s^2)] + a_{31}(a_{13}a_{24}q_s^2 + \xi^2 a_{23}a_{14})\},$$

$$b_s' = \frac{1}{\Delta}\{a_{31}[a_{21}a_{24}q_s^2 - a_{14}(a_{22}\xi^2 - q_s^2)] - [a_{24}(a_{11}\xi^2 - q_s^2) + \xi^2 a_{21}a_{14}]\},$$

$$c_s' = \frac{-\xi^2 a_{42}a_s + a_{42}q_s^2}{q_s^2 - \xi^2 - a_{41}},$$

$\Delta = a_{31}(a_{13}a_{24}q_s^2 + \xi^2 a_{23}a_{14}) + (\xi^2 a_{33} + 2a_{31} - q_s^2)[a_{24}(a_{11}\xi^2 - q_s^2) + \xi^2 a_{21}a_{14})$.

The solution of equation (20) is given by

$$W(\xi, x_3) = \sum_{s=1}^{4}[B_s X_s(\xi)\exp(q_s x_3) + B_{s+4}X_{s+4}(\xi)\exp(-q_s x_3). \quad (27)$$

where $B_i(i = 1,2 \ldots 8)$ are arbitrary constants.
The relation (27) represents the general solution of FMTH problem in the Fourier transform domain.
The transformed displacements, microrotation and temperature distribution satisfying the regular conditions as $x_3 \to \infty$ are given by

$$\tilde{u}_1 = -i\xi(a_1' B_5 e^{-q_1x_3} + a_2' B_6 e^{-q_2x_3} + a_3' B_7 e^{-q_3x_3} + a_4' B_8 e^{-q_4x_3}), \quad (28)$$

$$\tilde{u}_3 = -(q_1 B_5 e^{-q_1x_3} + q_2 B_6 e^{-q_2x_3} + q_3 B_7 e^{-q_3x_3} + q_4 B_8 e^{-q_4x_3}), \quad (29)$$

$$\tilde{\phi}_2 = -i\xi(q_1 b_1' B_5 e^{-q_1x_3} + q_2 b_2' B_6 e^{-q_2x_3} + q_3 b_3' B_7 e^{-q_3x_3} + q_4 b_4' B_8 e^{-q_4x_3}), \quad (30)$$

$$\tilde{T} = (c_1' B_5 e^{-q_1x_3} + c_2' B_6 e^{-q_2x_3} + c_3' B_7 e^{-q_3x_3} + c_4' B_8 e^{-q_4x_3}). \quad (31)$$

4. Boundary Conditions

We consider a concentrated normal line load $F(x_1 + Ut) = F\delta(x_1^*)$, where F is magnitude of force applied and $\delta(x_1^*)$ represents Dirac delta function. At the free surface $x_3 = 0$ the boundary conditions are taken as

$$t_{33} = -F\delta(x_1^*), \ t_{31} = m_{32} = T = 0 \ . \quad (32)$$

Applying Fourier transforms on equation (32) and using equations (4)-(8) and (27)-(31), we obtained

$$\tilde{t}_{33} = -\frac{\hat{F}}{\Delta}(d_{11}\Delta_1 e^{-q_1x_3} + d_{12}\Delta_2 e^{-q_2x_3} + d_{13}\Delta_3 e^{-q_3x_3} + d_{14}\Delta_4 e^{-q_4x_3}), \quad (33)$$
$$\tilde{t}_{31} = -\frac{\hat{F}}{\Delta}(d_{21}\Delta_1 e^{-q_1x_3} + d_{22}\Delta_2 e^{-q_2x_3} + d_{23}\Delta_3 e^{-q_3x_3} + d_{24}\Delta_4 e^{-q_4x_3}), \quad (34)$$
$$\tilde{m}_{32} = -\frac{a_8\xi\hat{F}}{\Delta}(d_{31}\Delta_1 e^{-q_1x_3} + d_{32}\Delta_2 e^{-q_2x_3} + d_{33}\Delta_3 e^{-q_3x_3} + d_{34}\Delta_4 e^{-q_4x_3}), \quad (35)$$
$$\tilde{T} = -\frac{\hat{F}}{\Delta}(c_1'\Delta_1 e^{-q_1x_3} + c_2'\Delta_2 e^{-q_2x_3} + c_3'\Delta_1 e^{-q_3x_3} + c_2'\Delta_2 e^{-q_4x_3}). \quad (36)$$

where

$\Delta = \begin{vmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{vmatrix}$, where Δ_i , $i = 1,2,3,4$ areobtained from Δ by interchanging i^{th}column by the column $[1,0,0,0]^T$

and

$$d_{1i} = \iota\xi a_6 q_i a_i' - \iota b_i' q_i - c_i', \quad d_{2i} = a_7 b_i'\xi + a_1 a_i'\xi^2 + a_2\xi, \quad d_{3i} = q_i \ , \ d_{4i} = c_i'$$

5. Inversion of the transform

To obtain all the results in real domain we have to invert all the quantities from transformed domain $f(\xi, x_3)$ to the physical domain in the form $f(x_1, x_3)$. The inversion of Fourier transform is obtained as

$$f(x_1, x_3) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{f}(\xi, x_3)e^{-i\xi x_1}d\xi$$
$$f(x_1, x_3) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\{[\cos(\xi x)]f_e - i\,[sin\xi x]f_o\}d\xi$$

Where f_e and f_o are respectively, the even and odd parts of the function $\bar{f}(\xi, x_3)$

6. Numerical Results and Discussion

Following Eringen [10] and Kumar et al [7] the relevant data for a magnesium crystal is taken as

$\lambda = 9.4\,X10^{11}kgm^{-1}s^{-2}$, $\mu = 4.0X10^{11}kgm^{-1}s^{-2}$, $T_0 = 298K$, $K = 1.0X10^{11}kgm^{-1}s^{-2}$, $j = 0.2X10^{-19}m^2$, $\gamma = 0.779X10^{-9}kgms^{-2}$,

$\rho = 1.74X10^3kgm^{-3}$, $\alpha_t = 2.36X10^{-5}K^{-1}$, $C^* = 9.623X10^2Jkg^{-1}K^{-1}$,

$K^* = 2.510Wm^{-1}K^{-1}$, $\tau_0 = 0.02s$

The computations are carried out for $U < c_1$ at $t = 1$ on the surface of the plane $x_3 = 1$ in the range $0 \leq x_1 \leq 3$. The variations of stress components i.e. normal stress, tangential stress, tangential couple stress and temperature distribution under three fractional ordered theories of thermoelasticity given by Sherief (SH), Ezzat (EZ), Youssef (YU) and Lord-Shulman (LS) theory have been studied and the results are depicted graphically.

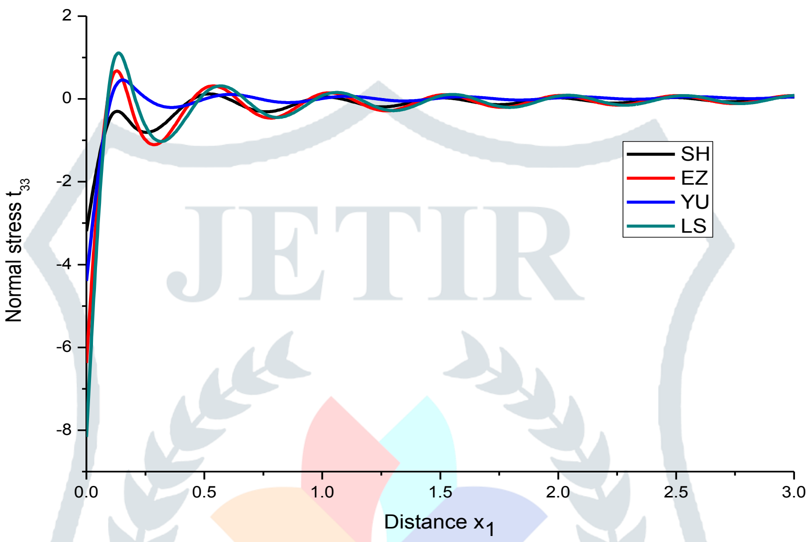


Figure 1. Normal stress Vs distance.

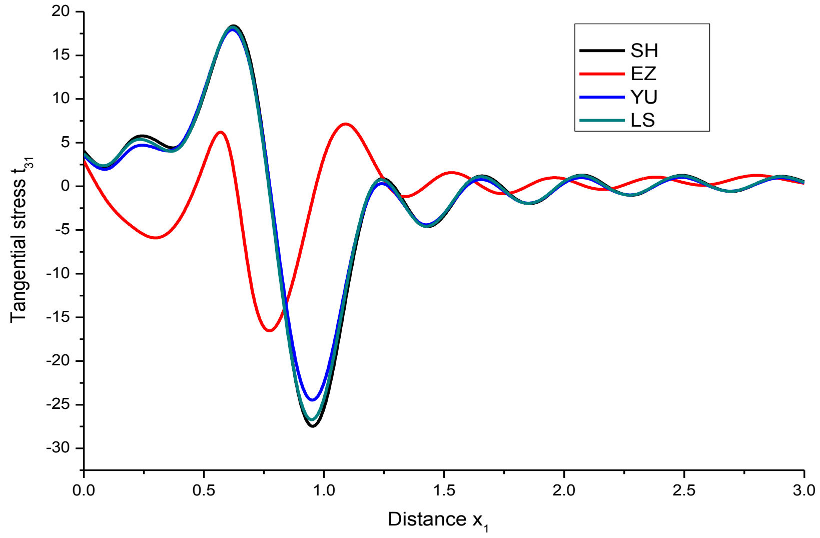


Figure 2. Tangential stress Vs distance.

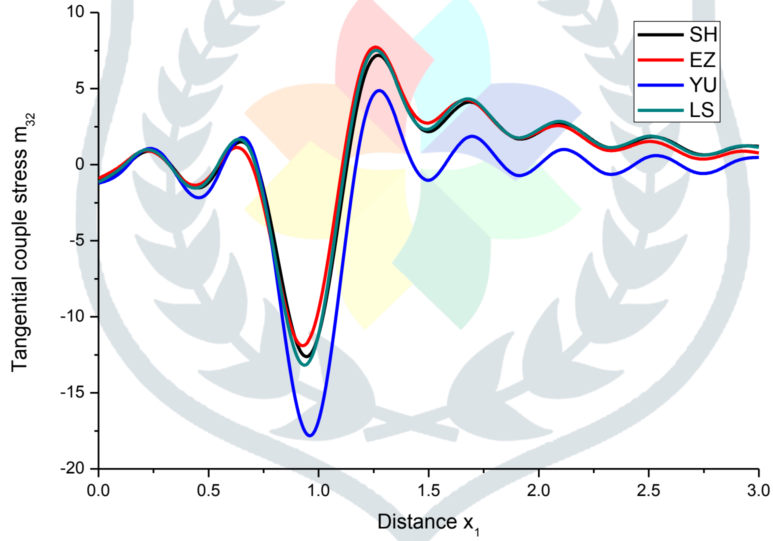


Figure 3. Tangential couple stress Vs distance.

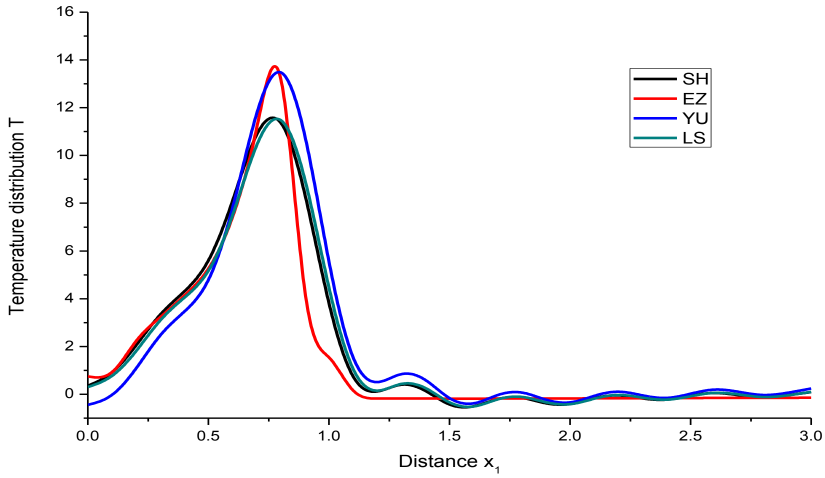


Figure 4 Temperature distribution Vs distance.

Figure 1 shows the variations of normal stress with respect to distance corresponding to four theories and as shown in figure it is noticed that normal stress increases initially near the source and then oscillates and finally vanishes with distance in all the theories. Figure 2 represents the variations in tangential stress with respect to distance. The oscillatory behavior of tangential stress field is noticed for all the theories and as distance increases tangential stress vanishes in the cases. Figure 3 display the variations of tangential couple stress verses distance and this field also shows oscillatory behavior for all theories. Values corresponding to fractional order theories are quite close to each other as compared to LS theory. Figure 4 represents the variations of temperature distribution and as shown in figure this field increases initially in cases of all the theories, then decreases and finally vanishes for higher distance values.

7. Conclusion

The response of fractional order theories of thermoelasticity under a moving load on the surface of micropolar thermoelastic half space has been investigated. Analytical expressions are obtained in the transformed domain for components of displacement, stress and temperature distribution. Numerical inversion techniques are employed to obtain the solution in physical domain. It is observed that under moving load boundary conditions the qualitative behavior of all the physical quantities (stress components and temperature distribution) is similar for Lord-Shulman as well fractional order theories of thermoelasticity. Small difference in magnitude of physical quantity is observed near the source and then vanishes with increase in distance for all the theories.

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